Worksheet 1

(1) Consider the plane \( \frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1 \).

Where does this plane intersect the \( x \)-, \( y \)-, and \( z \)-axes? Call the intersection points \( P \), \( Q \), and \( R \), in that order. Find the vectors \( v = \overrightarrow{PQ} \) and \( w = \overrightarrow{PR} \) and the cross-product \( v \times w \). Is \( v \times w \) a normal vector for the plane? Does that make sense in this situation?

Intersect at \((2,0,0), (0,3,0), (0,0,4)\).

\[ \begin{align*}
P &= Q & R \\
\vec{v} &= (2,3,0) & \vec{w} &= (-2,0,4) \\
\end{align*} \]

Yes it's normal as \( \vec{v}, \vec{w} \) tangent to the plane by construction (and not parallel).

(2) Find all values of \( b \) such that the vectors \((4, -2, 7)\) and \((b^2, b, 0)\) are orthogonal.

\[ \langle 4, -2, 7 \rangle \cdot \langle b^2, b, 0 \rangle = 0 \iff \text{ orthogonal.} \]

\[ 4b^2 - 2b = 0 \]

\[ 2b (2b - 1) = 0 \]

So \( b = 0, \frac{1}{2} \).

If \( b = 0 \), I guess this is still technically orthogonal.

(3) Parametrize the curve of intersection of the surfaces \( z = x^2 \) and \( x^2 + y^2 = 1 \). (It’s the outline of a Pringle!)

\[ \mathbf{r}(t) = \left( \cos t, \sin t, \cos^2 t \right) \]
(4) Find the unit vector at \( P = (0, 0, 1) \) pointing in the direction along which the function 
\[ f(x, y, z) = xz + e^{-x^2+y} \]
increases most rapidly.

\[ \nabla f = (z - 2x e^{-x^2+y}, e^{-x^2+y}, x) \]

Hence \( \nabla f(0, 0, 1) = \langle 1, 1, 0 \rangle \).

(5) Is there a function \( f \) such that \( \nabla f = (y^2, x) \)?

\[ f = \int y^2 \, dx = y^2x + C(y) \]
\[ f_y = 2xy + C'(y) = x. \] This is impossible.

Alternatively, if such an \( f \) existed, as
\[ \frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} \Rightarrow 2y = 1 \text{ which is correct.} \]

(6) Identify the following surfaces in 3D (sketch them, don’t worry if you can’t remember their names):

1) \( z = x^2 + y^2 \)
2) \( z^2 = x^2 + y^2 \)
3) \( \frac{x^2}{4} + \frac{y^2}{9} = 1 \)
4) \( z = y^2 \)
5) \( x^2 + y^2 + z^2 = 16 \)
6) \( y = 9 - x^2 - z^2 \)

1) paraboloid  3) elliptic cylinder  5) sphere  
2) cone  4) parabola sheet  6) paraboloid.