Week 1 Notes

Information:
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I'll post my discussion notes here. There is also a link where you can let me know if there is anything you want to do in discussion.
The course webpage can be found at:
http://www.math.ucla.edu/~das/32a.2.19s/

There will be weekly quizzes starting next week which we will do at the beginning of each discussion.

Today: Vectors and Geometry.
There are two equivalent ways to think about vectors.

1) as a pair or triple of numbers for a vector on the plane or 3-space respectively
2) something that has both direction and length.
We will focus on the second way today.

Reminder:
1. A vector \( \vec{v} \) isn't fixed anywhere, we can move it around as long as we don't change its length or direction.
2. Given two vectors \( \vec{v} \) and \( \vec{w} \), we can add them:
   \[
   \vec{v} + \vec{w} = \vec{v + w}
   \]
   and subtract them:
   \[
   \vec{v} - \vec{w} = \vec{v} - \vec{w}
   \]
3. We can scale by a constant
   \[
   \alpha \vec{v}, \quad \frac{1}{\alpha} \vec{v}, \quad 2\vec{v}
   \]
4. \(-\vec{v}\) has the same length as \(\vec{v}\) but opposite direction.
   \[
   \vec{v}, \quad -\vec{v}
   \]
Given points $P, Q$. $\overrightarrow{PQ}$ is the vector starting at $P$ and ending at $Q$. \textbf{Warning: vectors aren't fixed. So the square $ABCD$}

\begin{center}
\begin{tikzpicture}
  \node (A) at (0,0) {A};
  \node (B) at (2,0) {B};
  \node (C) at (2,2) {C};
  \node (D) at (0,2) {D};
  \draw (A) -- (B);
  \draw (B) -- (C);
  \draw (C) -- (D);
  \draw (D) -- (A);
\end{tikzpicture}
\end{center}

Then $\overrightarrow{AB}$ and $\overrightarrow{CD}$ both have the same length and direction and so are equal as vectors. This is why we usually replace $\overrightarrow{AB}$ with a letter, i.e. $\vec{v} = \overrightarrow{AB}$ and as square, $\vec{v} = \overrightarrow{CD}$ also.

We can do geometry with vectors.

\textbf{Example.}

\begin{center}
\begin{tikzpicture}
  \node (A) at (0,0) {A};
  \node (B) at (2,2) {B};
  \node (C) at (4,0) {C};
  \node (D) at (2,0) {D};
  \draw (A) -- (B);
  \draw (B) -- (C);
  \draw (C) -- (D);
  \draw (D) -- (A);
\end{tikzpicture}
\end{center}

Here $\overrightarrow{ABC}$ is a $\overrightarrow{D}$, $\overrightarrow{BC} \parallel \overrightarrow{DE}$ and $\overrightarrow{D}$ bisects $\overrightarrow{AB}$. \textbf{E} bisects $\overrightarrow{AC}$. We will show that $\overrightarrow{BC} = 2 \overrightarrow{DE}$. In particular, $||\overrightarrow{BC}|| = 2 ||\overrightarrow{DE}||$.

\textbf{Proof:}
proof:

We have \( \overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} \) by subtraction

\[ = 2\overrightarrow{AE} - 2\overrightarrow{AD} \quad \text{as midpoint} \]

\[ = 2(\overrightarrow{AE} - \overrightarrow{AD}) \]

\[ = 2\overrightarrow{DE} \quad \text{by subtraction} \]

Since this is the first week, I want to do some questions as a warm-up.

**Questions:**

1) Given \( \vec{v}, \vec{w} \) as follows. Sketch: \( -\vec{v}, 2\vec{w} + \vec{v}, \vec{v} - \vec{w} \).

![Diagram](attachment:image.png)

2) Given trapezoid \( WXYZ \):

- when \(ZY: WX = 3:2\).
- Let \( s = \overrightarrow{WX}, t = \overrightarrow{WT} \).
a) What is $\overrightarrow{XY}$? In terms of $s, t$.

b) What is $X'Y'$? In terms of $s, t$.

3) Is it always true that $||\overrightarrow{v} + \overrightarrow{w}|| \leq ||\overrightarrow{v}|| + ||\overrightarrow{w}||$? why/why not?

4) Given a parallelogram $ABCD$. Prove that the diagonals bisect each other.

**Answers:**

1)
2) Since \( EY: WX = 3:2 \) and \( \overrightarrow{WX} = \frac{3}{2} s \) and \( \overrightarrow{EY} \) in the same direction, we have \( \overrightarrow{EY} = \frac{3}{2} s \).

As for \( \overrightarrow{XY} \), this is the same as going around the trapezium in the opposite direction, so
\[
\overrightarrow{XY} = \overrightarrow{XW} + \overrightarrow{WZ} + \overrightarrow{ZY}
\]
\[
= -s + t + \frac{3}{2} s
\]
\[
= t + \frac{1}{2} s.
\]

3) Yes. This is called the triangle inequality.

\[\overrightarrow{V}, \overrightarrow{W}, \overrightarrow{V+W}\] form the edges of a triangle and the sum of any two side lengths is always greater than the third side length.

4) (This one was harder)
let the bisection point be \( E \). The first step in these kind of questions is to reinterpret the geometry question into one about vectors.

The diagonals bisect each other if from \( A \), we go half way to \( C \) (lands us) in the same place as if we started at \( B \) and went halfway to \( D \). i.e.

we want to show

\[
\frac{1}{2} \overrightarrow{AC} = \overrightarrow{AB} + \frac{1}{2} \overrightarrow{BD} \quad \ldots \quad (1)
\]

half way
to \( C \) from \( A \).
get \( L \)
get halfway
\( \overrightarrow{BD} \) from \( \overrightarrow{AB} \) to \( D \) from \( B \).

we will let \( \overrightarrow{v} = \overrightarrow{AB} \), \( \overrightarrow{w} = \overrightarrow{AP} \). Then \( \overrightarrow{AC} = \overrightarrow{v} + \overrightarrow{w} \)

by addition and \( \overrightarrow{BD} = \overrightarrow{w} - \overrightarrow{v} \) by subtraction.

Now, in (1): 
\[
LBD = \frac{1}{2} (\overrightarrow{v} + \overrightarrow{w})
\]
\[ \text{RHS} = \vec{v} + \frac{1}{2}(\vec{w} - \vec{v}) \]
\[ = \frac{1}{2}(\vec{w} + \vec{v}) \]
\[ = \text{LHS}. \]

Hence (1) is true and we are done.