Midterm 2 Practice
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Most of these problems come from Sang Truong’s Midterm 2 problem set. I’ll be posting answers to these at http://www.math.ucla.edu/~ben.szczesny/MATH32A-S19/coursehome.html

**Question 1.** The involute of a circle has parameterisation given by

\[ \vec{r}(\theta) = \langle R(\cos(\theta) + \theta \sin(\theta)), R(\sin(\theta) - \theta \cos(\theta)) \rangle \]

Find the arclength parameterisation.

**Question 2.** Show that the curvature at an inflection point of a plane curve \( y = f(x) \) is zero.

**Question 3.** Given a frenet frame \((\vec{T}, \vec{N}, \vec{B})\) with arclength parameterisation.

(a) Show \( \frac{d\vec{B}}{ds} = \vec{T} \times \frac{d\vec{N}}{ds} \) and conclude that \( \frac{d\vec{B}}{ds} \) is orthogonal to \( \vec{T} \).

(b) Show that \( \frac{d\vec{B}}{ds} \) is orthogonal to \( \vec{B} \). Hint: Differentiate \( \vec{B} \cdot \vec{B} = 1 \).

(c) Show that \( \frac{d\vec{B}}{ds} \) is a multiple of \( \vec{N} \).

**Question 4.** A particle has orbit given by

\[ \vec{r}(t) = (\ln(t), t, t^2/2) \quad \text{for} \quad t \geq 0. \]

Find the equation for the osculating plane to this particle at \( t = 1 \)

**Question 5.** Show that for a vector function \( \vec{r}(t) \), both \( \vec{r}'(t) \) and \( \vec{r}''(t) \) lie in the osculating plane. Hint: differentiate \( \vec{r}'(t) = v(t)\vec{T}(t) \).

**Question 6.** Find the domain for the following functions

(a) \( f(x, y) = \frac{1}{\sqrt{x^2 + y^2} - 1} \)

(b) \( f(x, y) = \frac{y \sin(x)}{1 + y} \)

(c) \( f(x, y) = -\frac{1}{\sin(xy)} \)