Problem 1 For these questions, consider vectors $\mathbf{a}$ and $\mathbf{b}$ in the plane $\mathbb{R}^2$.

(a) Use the parallelogram law to explain geometrically why $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$. That is, the order in which you add vectors doesn’t matter.

(b) Is it true that $\mathbf{a} - \mathbf{b} = \mathbf{b} - \mathbf{a}$? Why, why not?

(c) Suppose that $||\mathbf{a}|| = 5$. What is the length of $-5\mathbf{a}$?

(d) Suppose that $\mathbf{a} = \langle 2, 1 \rangle$ and $\mathbf{b} = \langle -1, 3 \rangle$ and both are based at the origin. Compute the vector that connects the head of $\mathbf{a}$ to the head of $\mathbf{b}$.

Problem 2 For these questions, consider vectors $\mathbf{u}$ and $\mathbf{v}$ in the plane $\mathbb{R}^3$.

(a) When is it true that $||\mathbf{u}|| + ||\mathbf{v}|| = ||\mathbf{u} + \mathbf{v}||$?

(b) What about $||\mathbf{u}||^2 + ||\mathbf{v}||^2 = ||\mathbf{u} + \mathbf{v}||^2$?

(c) Suppose we have that $||\mathbf{u}|| + ||\mathbf{v}|| = ||\mathbf{u} + \mathbf{v}||$, $||\mathbf{u} + \mathbf{v}|| = 15$ and $\mathbf{u} = \langle 4, 3, 0 \rangle$. What is $\mathbf{v}$?

Problem 3 Here we find the parametric equations for a line in $\mathbb{R}^3$ passing through the points $\mathbf{a} = \langle 1, 0, 1 \rangle$ and $\mathbf{b} = \langle 2, 1, -1 \rangle$.

(a) Find a vector $\mathbf{u}$ in the same direction as the line.

(b) Let $\mathbf{c}$ be any point on the line. Explain why $\mathbf{c} + t\mathbf{u}$ gives a parametric equation for the line. Write down this equation.

(c) Can you get more than one parametric equation for the same line through these methods?

Problem 4 (Additional, harder problem) Consider two vectors $\mathbf{u} = \langle a, b \rangle$ and $\mathbf{v} = \langle c, d \rangle$. We will consider the number $D = ac + bd$. Hint: Consider Question 2

(a) Show that $D = 0$ if and only if $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

(b) Show that $D = ||\mathbf{u}|| ||\mathbf{v}||$ if $\mathbf{u}$ and $\mathbf{v}$ point in the same direction.

(c) Use the law of cosines to find a formula for $D$ in terms of the lengths of $\mathbf{u}$, $\mathbf{v}$ and the angle $\theta$ between them.

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