Information

Discussion Questions

Question 1.
(a) What is the point of numerical integration?
(b) Describe in your own words what the Midpoint ($M_N$), Trapezoidal ($T_N$) and Simpson’s ($S_N$) Rules are.
(c) What are the formulas for these rules? (Can you do so without looking at your notes?)

Question 2. Consider the definite integral $\int_{2}^{5} \frac{1}{x} \, dx$. In this question we will investigate how well the Trapezoidal Rule ($T_N$) approximates this integral. The error bound is given by the formula

$$\text{error}(T_N) \leq \frac{K_2(b-a)^3}{12N^2}.$$

(a) Do you expect the Trapezoidal Rule to over or underestimate the definite integral? If so, why?

(b) Let $f(x) = \frac{1}{x}$, the constant $K_2$ is any number such that $|f''(x)| \leq K_2$ for all $x$ in the interval we are integrating over. However we usually take it to be the the absolute value of the maximum of the second derivative, $\max_{x \in [a,b]} |f''(x)|$. Find the maximum of $f''$ and set $K_2$ to be the absolute value of this value.

(c) In the formula $b - a$ is the length of the interval we are integrating over. In this case we have $b - 1 = 5 - 2 = 3$. Use this and the previous part to find a value of $N$ for which $\text{error}(T_N) < 10^{-6}$.

Question 3. Compute the arc length of $y = \ln(e^x + 1 - e^x - 1)$ over the interval $[1, 3]$.

Question 4. Compute the surface area of revolution about the $x$-axis for $y = \frac{1}{4}x^2 - \frac{1}{2}\ln(x)$ over the interval $[1, e]$.

Homework Questions

Section 8.9
12, 16, 34, 36, 38, 40

Section 9.1
2, 9, 14, 18, 21, 23, 28, 40, 42
Extra Questions

**Question 5.** Evaluate the following integrals

(a) \( \int \frac{dx}{x^2 + 2x + 5} \)

(b) \( \int \sin^5(x) \cos^2(x) \, dx \)

(c)* \( \int \sin^4(x) \cos^2(x) \, dx \)

(d) \( \int \sqrt{1 + \sqrt{x}} \, dx \)

(e)* \( \int \frac{1}{\sech(x)} \, dx \).

(f)** \( \int_0^{\pi/2} \frac{\sin(x)}{\cos(x) + \sin(x)} \, dx \)

Hint for (f): Remember the trig identities \( \sin(\pi/2 - x) = \cos(x) \).

**Question 6.** Find the surface area of the torus obtained by rotating the circle \( x^2 + (y - b)^2 = r^2 \) about the \( x \)-axis.