Week 1 Notes
Monday, April 2, 2018  8:35 PM

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Crash Course on Inverse Functions.
(for Tuesday)

- A function \( f: D \rightarrow R \) is a rule that takes something from domain \( D \) to range \( R \)

- A function is one-to-one (or injective) if you can't get the same output from two different inputs, i.e. \( f(x) = x^2 \)
two different inputs, i.e. \( f(x) = x^2 \) is not one-to-one.

a) \( f(-2) = (-2)^2 = 4 = f(2) \)

An inverse function is the function you get by doing the opposite rule: i.e., \( f(x) = x^3 \), the opposite is taking cube roots.

\[ f^{-1}(x) = x^{1/3} \] is the inverse function.

A function has an inverse iff it is one-to-one on its domain \( D \). This is what the first question is about.

**Example:**

Does the function \( f(x) = x^2 + 4x \) have an inverse? One way to show a function is one-to-one is to show it's strictly increasing/decreasing. We can do this by looking at its derivative (probably the most common...
(a) Differentiation for most functions

way to do this)
\[ f'(x) = 27x^2 + 4 > 0 \text{ for all } x. \]
Hence \( f \) is strictly increasing and so is one-to-one and has an inverse.

**Example**

Find the inverse function of \( f(x) = 3x - 9 \).
We do so by solving \( y = f(x) \) for \( y \).

\[ y = 3x - 9 \]
\[ y + 9 = 3x \]
\[ \Rightarrow \quad \frac{y + 9}{3} = x \]

We then interchange \( x, y \) to get inverse:
\[ f^{-1}(x) = \frac{x + 9}{3}. \]

**Example:**

For functions not one-to-one we can...
restrict the domain so the function is one-to-one on the new smaller domain and then get an inverse for the new function, i.e. \( f(x) = x^2 \) not one-to-one on \( \mathbb{R} \), but it \( \text{is} \) when domain is \( [0, \infty) \) and its inverse is then \( f^{-1}(x) = \sqrt{x} \).

If we instead restricted the domain to \( (-\infty, 0) \), the inverse would be \( f^{-1}(x) = -\sqrt{x} \).

**Log Rules**

**Definition of \( \log_b a \):** The number that you raise \( b \) by to get \( a \).

\[ b^{\log_b a} = a \]

- \( \log_b(xy) = \log_b(x) + \log_b(y) \)
- \( \log_b(x/y) = \log_b(x) - \log_b(y) \)
- \( \log_b(x^n) = n \log_b(x) \)
\[ \log_b(x^n) = n \log_b(x) \]
\[ \log_b 1 = 0 \quad \log_b b = 1 \]

**Derivatives of \( e^x/\ln x \)**

\[ \frac{d}{dx} e^x = e^x \]
\[ \frac{d}{dx} \ln(x) = \frac{1}{x} \]

**when** \( \ln x = \log_e(x) \)

\( e \) is a super special number, Ingeniel

\[ \frac{d}{dx} (b^x) \neq b^x \quad , \quad \frac{d}{dx} \log_b(x) \neq \frac{1}{x} \]

**Remember the chain rule!**

\[ \frac{d}{dx} g(f(x)) = g'(f(x))f'(x) \]

**Example**

Derivative of \( f(x) = e^{2x^2} \), \( g(x) = \ln(2x) \)

For first one we use chain rule:
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\[ f(x) = e^{g(x)} \]
\[ f'(x) = e^{g(x)} \cdot g'(x) \]

so in our case

\[ f'(x) = 4x e^{2x^2} \]

In second case, chain rule give in general

\[ g(x) = \ln(h(x)) \]
\[ g'(x) = \frac{h'(x)}{h(x)} \]

so in our case

\[ g'(x) = \frac{2}{2x} = \frac{1}{x} \]