Questions

**Question 1.** Compute without calculator:

(a) \(\arcsin(\sin \frac{\pi}{3})\)  
(b) \(\arcsin(\sin \frac{4\pi}{3})\)  
(c) \(\arctan(\tan \frac{3\pi}{4})\)

**Solution to Question 1.**

(a) Since the range of \(\arcsin\) is \([\frac{\pi}{2}, \frac{\pi}{2}]\), we have that \(\arcsin(\sin \frac{\pi}{3}) = \frac{\pi}{3}\).

(b) Since \(\frac{4\pi}{3}\) is not in the range of \(\arcsin\), we cannot use the previous solution. We want to find a value \(y \in [\frac{\pi}{2}, \frac{\pi}{2}]\) such that \(\sin(\frac{4\pi}{3}) = \sin(y)\), since this will then give us \(\arcsin(\sin 4\pi/3) = \arcsin(\sin y) = y\).

In order to do this, we will use the identity \(\sin(x) = \sin(\pi - x)\) (note, the general version of this identity is \(\sin(x) = \sin((-1)^n(x - n\pi))\), \(n \in \mathbb{Z}\) which can be used in the general case).

Now, we have that \(\sin(\frac{4\pi}{3}) = \sin(\pi - \frac{4\pi}{3}) = \sin(-\frac{\pi}{3})\) and as \(-\frac{\pi}{3}\) is in the range of \(\arcsin\) we have that \(\arcsin(\sin 4\pi/3) = \arcsin(\sin -\pi/3) = -\pi/3\).

(c) Similarly to the previous question. \(\frac{3\pi}{4}\) is not in the range of \(\arctan\). However, we do have that \(\tan(x)\) is \(\pi\) periodic (\(\tan(x) = \tan(x + n\pi)\) for all \(n \in \mathbb{Z}\)) and so \(\arctan(\tan 3\pi/4) = \arctan(\tan(3\pi/4 - \pi)) = \arctan(\tan -\pi/4) = -\pi/4\).

**Question 2.** Compute without calculator:

1. \(\cos(\arctan(x))\)
2. \(\cot(\sec^{-1}(x))\) for \(x \geq 1\)

**Solution to Question 2.**

1. We use a triangle method. We have the triangle

```
          x
          |
          |
          |
  \sqrt{x^2 + 1}  |
  |
  |
  |
  1
```

\[\arctan(x)\]

1
Hence, we have that \( \cos(\arctan(x)) = \frac{1}{\sqrt{x^2 + 1}} \).

2. The triangle method can be used also. Another method is by trig identity \( \tan^2(x) = \sec^2(x) - 1 \).

\[
\cot(\sec^{-1}(x)) = \frac{1}{\tan(\sec^{-1}(x))} = \frac{1}{\pm \sqrt{\sec^2(\sec^{-1}(x)) - 1}} = \frac{1}{\pm \sqrt{x^2 - 1}}
\]

Note that since cot is positive between \([0, \pi/2]\) we take the positive root. \( \cot(\sec^{-1}(x)) = \frac{1}{\sqrt{x^2 - 1}} \)

**Question 3.** Find the derivatives of the following functions:

(a) \( \arcsin(e^x) \)

(b) \( \arccos(\ln(x)) \)

(c) \( \sec^{-1}(t + 1) \)

(d) \( \tan^{-1}\left(\frac{1 + t}{1 - t}\right) \).

**Solution to Question 3.**

(a) \( \frac{e^x}{\sqrt{1 - e^{2x}}} \)

(b) \( \frac{-1}{x\sqrt{1 - \ln^2(x)}} \)

(c) \( \frac{1}{t + 1}\sqrt{(t + 1)^2 - 1} \)

(d) \( \frac{1}{1 + t^2} \).

**Question 4.** Evaluate the following integrals:

(a) \( \int \frac{dt}{\sqrt{1 - 16t^2}} \)

(b) \( \int \frac{dx}{x\sqrt{x^4 - 1}} \)

(c) \( \int \frac{\ln(\cos^{-1}(x))dx}{(\cos^{-1}(x))\sqrt{1 - x^2}} \).

**Solution to Question 4.**

(a) \( \frac{1}{4} \arcsin(4t) + C \). Use substitution \( u = 4t \).

(b) \( \frac{1}{2} \sec^{-1}(x^2) + C \). Use substitution \( u = x^2 \).

(c) \( -(\ln(\arccos(x)))^2 + C \). Use substitution \( u = \arccos(x) \).

**Homework Questions**

7.8.8, 7.8.57, 7.8.60, 7.9.55, 9.4.1, 9.4.15