FROM QUADRATIC FORMS TO ALGEBRAIC GROUPS

ASCONA MEETING, 18-23 FEBRUARY 2007

TITLES AND ABSTRACTS

MINICOURSES

Nikita Karpenko – *Canonical dimension.*

Abstract : We review results of several last years (including the most recent ones) on canonical dimension (absolute as well as at a prime p) of projective homogeneous varieties and of split linear algebraic groups.

Jean-Pierre Serre – *Cohomology invariants, Witt invariants and trace forms.*

Abstract : I plan to discuss the properties of these invariants, and to determine them explicitly for the symmetric groups, the alternating groups and the groups with a 2-Sylow isomorphic to the dihedral group of order 8. This will be applied to Galois trace forms à la Bayer-Lenstra.

Alexander Vishik – *Elementary discrete invariant of quadrics and u-invariant of fields.*

TALKS

Jean-Louis Colliot-Thélène – *Obstructions de réciprocité pour les points entiers des espaces homogènes et représentation des formes quadratiques entières.*

Résumé : Une forme quadratique entière peut être représentée par une autre forme quadratique entière sur tous les anneaux d'entiers p-adiques et sur les réels, sans l'être sur les entiers. On en trouve de nombreux exemples dans la littérature. Dans un travail avec XU Fei (Beijing), nous montrons qu'une partie de ces exemples s'explique au moyen d'une obstruction de type Brauer-Manin pour les points entiers.

Philippe Gille – *Lower bounds for the essential dimension of reductive groups.*

Abstract : In this joint work with Z. Reichstein (Vancouver), we provide new lower bounds for the essential dimension of complex reductive groups. It can be achieved in Reichstein-Youssin's framework but also by a new way which does not involve the resolution of singularities. In particular, that provides partial results in the positive characteristic case.

Detlev Hoffmann – *Isomorphism criteria for Witt rings of real fields and reduced Witt rings.*

Abstract : Let K and L be fields (of characteristic $\neq 2$) and denote by WK and WL their Witt rings. In 1970, Harrison published a famous criterion for WK and WL to be isomorphic (in which case K and L are said to be Witt equivalent) in terms of an isomorphism of the square class groups of the two fields which is in a certain sense compatible with the isotropy behaviour of 2-fold Pfister forms over these two fields. Fields of characteristic 2 have been treated by Baeza-Moresi. Various authors have refined these and given new criteria (such as a 'Hilbert-symbol equivalence') in the case of global fields (Perlis-Szymiczek-Conner-Litherland, Czogala, Szymiczek, Szczepanik and others). Koprowski studied the case of function fields of curves over a real closed field and derived criteria analogous to that for global fields. In particular, he showed that if K and L are function fields of curves over a real closed field, then $WK \cong WL$ iff both K and L are real or both are nonreal. In his thesis, Nicolas Grenier-Boley generalized Harrison's criterion to isomorphism of Witt rings (resp. Witt modules) of hermitian forms over quadratic field extensions (resp. over quaternion division algebras). In this talk, we will give a survey of these results and of recent joint work with Grenier-Boley, where we study Witt equivalence of real fields whose (generalized) u-invariant is at most 2 in terms of their spaces of orderings and their square class group (resp. their 'positive' square class group $(\sum F^2)^*/F^{*2}$ if the fields are SAP), thus giving a new proof of Koprowski's result. Criteria for isomorphisms of reduced Witt rings and of Witt rings (resp. Witt modules) of hermitian forms over quadratic extensions of real fields of u-invariant ≤ 6 (resp. quaternion division algebras over real fields of u-invariant ≤ 14) are also obtained.

Bruno Kahn – *The Brauer group and indecomposable (2,1) cycles.*

Hanspeter Kraft – *Essential dimension: An approach via classical invariant theory.*

Alexander Merkurjev – *Unramified cohomology of algebraic varieties.*

Abstract : Let X be a complete algebraic variety over a field F. We show that the functor taking a cycle module M over F to the group of unramified elements in $M(F(X))$ is represented by a cycle module. A relation to the rationality problem of algebraic varieties will be discussed.

Ivan Panin – *On relation of the algebraic cobordism to the Quillen K-theory.*

Abstract : Quillen's K-theory is reconstructed via the algebraic cobordism of Voevodsky. More precisely, for a ground field k the algebraic cobordism T-spectrum MGL of Voevodsky is considered as a commutative ring T-spectra. Setting $\mathrm{MGL}^i = \oplus_{2q-p=i} \mathrm{MGL}^{p,q}$ we consider the bigraded theory $\mathrm{MGL}^{p,q}$ as just a graded theory. There is a unique ring morphism $\phi : \mathrm{MGL}^0(k) \to \mathbb{Z}$ which takes the class $[X]_{\mathrm{MGL}}$ of a smooth projective k-variety X to the Euler characteristic $\chi(X, \mathcal{O}_X)$ of the structure sheaf $\mathcal{O}_X$. Our main theorem states that there is a canonical grade preserving isomorphism of the following ring cohomology theories (for X smooth and $U \subset X$ open):

$$\varphi : \mathrm{MGL}^*(X,U) \otimes_{\mathrm{MGL}^0(k)} \mathbb{Z} \xrightarrow{\simeq} \mathrm{K}^{TT}_{-*}(X,U) = \mathrm{K}'_{-*}(X-U),$$

where K^{TT}_* is the Thomason-Trobough K-theory and K'_* is Quillen's K'-theory. This theorem is a result of a joint work with K.Pimenov and O.Röndigs.

Parimala – *Symplectic discriminant.*

Markus Rost – *Galois cohomology and cycles.*

Abstract : For a function field $F(X)$ there is a classical theorem which describes the elements of the Brauer group which split over $F(X)$ in terms of the Picard group of X. This relation extends to symbols in the higher Galois cohomology mod p involving then certain (classical) Chow groups. The construction is mainly due to Voevodsky. It uses Steenrod operations in motivic cohomology and the Bloch-Kato conjecture in the previous weight. For a norm variety, the resulting cycles can be used to split off a certain motive.

Jean-Pierre Tignol – *Octagons of Witt groups.*

Abstract : 8-term exact sequences of Witt groups coiling into octagons were observed in various situations related to quadratic extensions by Lewis, Warshauer, Grenier-Boley-Mahmoudi, ... The aim of this talk is to set up a general framework in terms of categories with duality to explain the occurrence of these exact sequences. (Joint work with David Lewis.)

Charles Walter – *Witt groups of quadrics.*