(800, #8) Find $\dfrac{\partial z}{\partial s}$ **and** $\dfrac{\partial z}{\partial t}$ **for** $z = \sin(x)\cos(y),\ \ x = (s-t)^2,\ y = s^2 - t^2$

Solution:

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = \cos(x)\cos(y)2(s-t) - \sin(x)\sin(y)2s =$$

$$2(s-t)\cos\!\left((s-t)^2\right)\cos\!\left(s^2 - t^2\right) - 2s\sin\!\left((s-t)^2\right)\sin\!\left(s^2 - t^2\right)$$

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$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = \cos(x)\cos(y)(-2(s-t)) - \sin(x)\sin(y)(-2t)$$

$$-2(s-t)\cos\!\left((s-t)^2\right)\cos\!\left(s^2 - t^2\right) + 2t\sin\!\left((s-t)^2\right)\sin\!\left(s^2 - t^2\right)$$