

Assignment #5

Office hours No office hour on Monday, Feb. 10; extra office hour 1:00-2:00 on Tuesday, Feb. 11.

Assignment due in lecture on Friday, Feb. 14:

To do but not hand in:

S-1, S-4;

T-2;

U-1.

To hand in:

S-3, S-5, S-6;

T-1 (below);

U-2.

Note. Three harder problems are M-5, N-11, T-3. An original proof of any of these (not a proof from other sources) can be traded for equivalent effort on other homework problems, now or later—ask if you are interested.

Problem T-1. For the category \mathcal{D} of finite distributive lattices with 0,1-homomorphisms (homomorphisms f with $f(0) = 0, f(1) = 1$), and the category \mathcal{P} of finite partially ordered sets with isotone maps, show a natural duality. Define explicitly whatever functors are needed, state whatever facts are needed, point out those that have already been proved during the course, and prove the rest, briefly.

Problem T-2. Under the natural duality of the previous problem, show that a surjection (“onto” map) in \mathcal{D} corresponds to an injection (one-to-one map) in \mathcal{P} and vice-versa.

Problem T-3. [not assigned!] Let L be the power set of $\omega = \{0, 1, 2, \dots\}$ modulo finite subsets. In other words, consider two subsets of ω to be equivalent if they differ by finitely many elements. Show that any partially ordered set of cardinality \aleph_1 or less can be embedded in L .