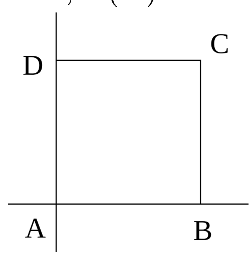
Sample midterm

(In the original version, **1-5** were on separate pages with space for answers. Each problem counts 10 points. See reverse for **1**.)

2. Calculate the projection on the x, y-plane of the cube $(\pm 1, \pm 1, \pm 1)$ as viewed from $pt(3, 2, 1, 0)_h$. Your method should use a transformation of some kind. (Calculate images of vertices. No sketch is required.)

3. Let A, B, C, D be the vertices of the standard unit square, listed counter-clockwise. Find a matrix for a projective transformation T on $\mathbf{P}_2$ such that $T(A) = B$, $T(B) = C$, $T(C) = D$, $T(D) = A$.



4. Find a matrix for a rotation of $90°$ in $\mathbf{R}^3$ about an axis through the points $(1, 1, 1)$ and $(1, 2, 2)$. (You may leave your answer as a product of matrices each with explicit entries. A rotation of $90°$ in either direction is OK.)

5. Short-answer questions:

(a) Give an example of an orthogonal matrix that is neither a reflection nor a rotation.

(b) Let T be a projective transformation in three dimensions that moves $(0, 0, 2)$ to the point at infinity on the z-axis while keeping all points of the x, y-plane fixed. Find $T(1, 1, 1)$.

(c) For an $n \times n$ invertible matrix M and vector $\mathbf{b}$ in $\mathbf{R}^n$, $\begin{bmatrix} M & \mathbf{0} \\ \mathbf{b} & 1 \end{bmatrix}^{-1} = \begin{bmatrix} M^{-1} & \mathbf{0} \\ -\mathbf{b} & 1 \end{bmatrix}$ for [choose one] (i) all such $\mathbf{b}$, M, (ii) some but not all $\mathbf{b}$, M, (iii) no $\mathbf{b}$, M.

(d) Find the area of the triangle in $\mathbf{R}^2$ with vertices $(0, 0)$, $(1, 2)$, $(2, 1)$.

(e) Suppose the plane $2x + 2y + z = 0$ is given a coordinate frame $\mathbf{v}, \mathbf{w}, \mathbf{n}$ for the up-vector $\mathbf{k}$. Find the entries of $\mathbf{v}$ numerically.

1. For each image below, identify the kind of projection by main classification and subclassification. Consider lines parallel if they do not meet on the page when extended. Write each answer near its picture.