Math 149 W02 NN.

# Study Guide

In addition to the general topics listed below, be able to do all homework problems, including those "not to hand in". In general, where formulas are developed you should know them, and if there is a proof in a homework problem you should know it. The exam will cover the whole course, but will have an extra emphasis on topics past the midterm.

Review of vectors and linear transformations

- The projection of a vector on a line
- Cross products
- Relation to matrices; "mapping the house"; determinants; invertibility

Orthogonal matrices and transformations

- Meaning and characterization of orthogonal matrices
- Rotation matrices
- Reflections—enough to do homework problems
- "Three-step" method for problems

#### Affine transformations

- Affine transformations
- Extended matrices
- Preservation properties
- Mapping triangles and parallelograms
- Idea of mapping a standard object of any sort
- "Hidden explanation" (see under homogeneous too)
- Applications to windowing, area of triangle, volumes

Homogeneous coordinates and the real projective plane

- Homogeneous coordinates
- Projective transformations
- Points at infinity
- The real projective plane

- Geometry of the real projective plane
- Making projective transformations
- How much freedom?
- How to make a specific quadrangle go to a specific quadrangle.
- Three-dimensional projective space
- Affine transformations as projective transformations
- The "hidden" or "secret" explanation using hlt's in one dimension higher

### Projections from three dimensions to two

- The setup and main classification
- Characteristics of the main types
- How to calculate projections
- Projections on a slanted viewplane
- Kinds of perspective projections—how to recognize
- Subclassifications and how to recognize them, whether given as a picture or given by an algebraic description of the object, viewpoint, and viewplane.
- Rotating given latitude/longitude to north pole. Be clear on which is latitude and which is longitude!

#### Convex sets and convex polyhedra

- Convexity
- The convex hull of a set
- Convex combinations
- Convex polyhedra
- Hidden-surface removal for convex polyhedra

#### Interpolation for polynomial parametric curves

- Parametric curves in general; polynomial curves
- Polynomial curves
- Lagrange interpolation (concepts; basis functions)
- Properties of polynomials

#### Cubic Bézier curves

- Definitions of Bernstein and Bézier
- Properties of the cubic Bernstein polynomials and Bézier curves. Know those that were used in homework or mentioned in lecture.
- Applications: loops, arcs, arrows, S-curve, Hermite
- Derivative theorem for Bézier curves (of any degree).

### Cubic spline curves

- Bézier curves with zero second derivative at one end
- Gluing two Bézier curves; A-frames
- B-spline curves and their construction
- Interpolation by splines
- Basis functions, how to calculate
- (Skip other possible end conditions and non-uniform spline curves)
- Applications to animation

## Parametric Surfaces

- Parametric curves and surfaces
- Ruled surfaces; bilinear patch (know)
- (Coons patch-don't need to know)
- Tensor bases
  - for interpolated spline surfaces
  - for Lagrange surfaces
  - (not for Bézier surfaces, even though that was on homework)
- How to invent parametric surfaces, as in lab assignment
- Isoparametric curves
- Normal vector at a point

## Ray-tracing

- The setup and its ingredients
- Tracing a ray for a given pixel
- Phong shading
- (not mirror reflection)

## Linear geometry

- Ways of expressing lines in  $\mathbb{R}^2$
- Relational form and applications
- Relational form
- $f_{AB}(x,y)$
- How to tell whether two line segments cross , and where
  - $-\Delta(P,Q,R)$
  - Two-point relational formula
  - Crossing of lines, line segments
  - Points inside/outside triangles, convex polygons
  - Points inside non-convex polygons—if discussed

Labs — Look them over.