Project #3: Parametric surfaces and animation

1. The project

Target due date: Wednesday, March 13. As usual, the project may be submitted earlier or later as long as credits balance out in the end.

Points: This project is worth 4 points.

Description. Using Maple, (a) make plots of three interesting parametric surfaces; (b) do an animation. No prior knowledge of Maple is assumed.

Of your three examples, one should be an example from the handout on parametric surfaces, among those that allow no choice (namely, Examples 1, 3, 6, 7, 9, 10). For the two others, either do some examples from that handout among those that allow you some choice of functions, or else invent new kinds of examples. When inventing examples, don't just use the first thing that comes to mind; plan what you are doing and work to get interesting surfaces.

You may work with a partner. As before, one person should be Project Leader and the other Project Staff. Tell me by email who is who. If you work with the same partner as before, switch Leader/Staff roles.

If you encounter problems with Maple or with the specifications in this handout, please let me know.

Submission: In Maple, use `SaveAs` under the `File` menu to save your work as a Maple worksheet file, with the name `proj3.mws` . Either save into your `submit` directory or copy the resulting file into that directory. First, you can delete any items on the worksheet that you don't want to include; see below for how to delete. If you have problems submitting the one worksheet, do something else and let me know.

2. Parametric surfaces in Maple

This description assumes that you have written your surfaces as $P(t,u) = (x(t,u), y(t,u), z(t,u))$, where you have formulas for $x(t,u)$, $y(t,u)$, and $z(t,u)$.

On the Start menu, find the most current version of Maple. Double-click to start it,

To plot the parametric surface $P(t,u) = (t, u, \sin(t)\sin(u))$ for $-\pi \le t \le \pi$ and $-\pi \le u \le \pi$, type
`plot3d([t,u,sin(t)*sin(u)], t=-Pi..Pi, u=-Pi..Pi);` and then press `Enter`. Notice that it's upper-case `P` in `Pi`.

Wait a moment, and the plot should appear. If you click on the picture, it is replaced by an outline frame. At this point, you can drag with the mouse to change the view. Double-click on the picture to redraw it. You can also change various attributes of the plot by using menu items from the menu bar. (You won't see the plot menu options until you have selected the picture.) The "Constrained" option means uniform scaling in x, y, z.

Notes.

- Be sure to put a semicolon after every command and then press `Enter`. If you do a command and nothing happens after a reasonable time, check to see if you have forgotten the semicolon.
- Maple does a lot more, of course; it will even do derivatives and integrals symbolically. If you have used a variable name for non-plotting purposes, for example `t`, don't use the same one as a parameter in plots.
- You can edit previously typed material and press `Enter` to re-run it.
- To select a whole area of your worksheet, double-click on its vertical bar to the left. You can delete material by selecting it and then pressing the `BACKSPACE` key.
- When in doubt, try looking at menus, consulting the help facility, clicking, and double-clicking.
- Variable names in Maple can be anything reasonable. Be careful, though: If you type `sint` instead of `sin(t)`, Maple will think that's just another name, and since it doesn't have a value for it, it just repeats the name back. It will seem to you as though nothing is happening.

3. Details of plot3d

- You do not need to do any programming or subdividing of parameter intervals; the package does that for you.
- If your plot is too coarse, with curves made of too few line segments, you can use `plot3d(···,numpoints = 900);` (say).
- If your plot involves circles and they are being plotted as ellipses, that means the plotting package is scaling the axes independently; you can use `plot3d(···,scaling = CONSTRAINED);` or select `Constrained` from the plot menu.

- You can get additional information from the help menu.
- If your plot is empty, check the ranges `a..b` used and make sure you didn't have a misprint like `sint` for `sin(t)`, as cautioned above.

4. Animated plots

First give the command
`with(plots);` to load the necessary routines.

Specifying the animated plot is easy: You simply include a third parameter, and it automatically changes with time when you run the animation. Example:

To plot $P(t,u) = (t, u, \sin(t+v)\sin(u))$ for $-\pi \le t \le \pi$ and $-\pi \le u \le \pi$, where v changes with time from $v = 0$ to $v = 2\pi$, type

```
animate3d([t,u,sin(t+v)*sin(u)],t=-Pi..Pi,u=-Pi..Pi,v=0..2*Pi);
```

First, you will see just the first frame, and then you will have to wait while Maple generates additional frames (not yet shown). Then on the `Animate` menu, select `Play`. This will play it once, probably too fast. You can slow it down by selecting another menu item. You can also set it to loop over and over by selecting the `continuous` menu item. Stop it by selecting `Stop`, which will have appeared on the `Animate` menu.

What is really happening is that Maple makes eight frames, while varying the third parameter (v in the example above).

In the example above, the range of the third parameter v has been chosen so that the last value of v gives the same plot as the first value, to have continuous plotting make sense. But then in continuous plotting, there is a resulting jerkiness when the first and last plots are played in succession! In other words, one plot is doubled. To fix that, use this trick:

```
animate3d([t,u,sin(t+v)*sin(u)],t=-Pi..Pi,u=-Pi..Pi,v=0..2*Pi*7/8);
```

Thus the eight plots are fractions $0, 1, \ldots, \frac{7}{8}$ of the full range of v, and the next plot 0 fits exactly into the sequence to make a smooth continuous animation.

If you have an animation where the third parameter does not finish where it starts, for example, a radius r goes from 0 to 1, then there is a trick to make it continuous so you can loop: First, realize that $\cos v$ goes from 1 to -1 and back as v goes from 0 to 2π. Therefore $r = (1 - \cos v)/2$ goes from 0 to 1 and back. Substitute this expression for r in your function to make a plot that can loop smoothly. Or if r is to go from 2 to 5 and back, you could use $r = 2 + (5-2)(1 - \cos v)/2$.

Notice that v rather than t was used for the time-related parameter in the example above. It might be better to use r, s, t for parameters so t *is* time.