Comments for 0-W

$\mathbb{Z}$ is the set of integers (positive, negative, and 0).

To mathematicians, "integers" and "whole numbers" mean the same thing. (Sometimes in K-12 math people say "whole numbers" to mean just integers > 0.)

$\mathbb{Q}$ is the set of rational numbers, such as $\frac{3}{2}$, $-\frac{7}{4}$, 1.252, and 7.

$\mathbb{R}$ is the set of real numbers, which includes the rational numbers and also other numbers such as $\sqrt{2}$, π, and e.

$\mathbb{C}$ is the set of complex numbers $a + bi$, $a, b \in R$. We'll look at complex numbers more closely later.

$f : S \to T$ means a function that assigns to each $s \in S$ some value $f(s) \in T$. Notice that this means f is defined for *all* $s \in S$. When specifying a function such as $1/x$ we should write "define $g : \{x \in \mathbb{R} : x \neq 0\} \to \mathbb{R}$" by $g(x) = 1/x$", since $g : \mathbb{R} \to \mathbb{R}$ would not be correct.

In calculus, we talk about a function $f(x, y)$ of two variables, or a vector-valued function $P(t)$ of one variable, etc. But using functions on sets we can describe these all in the same general way: $f : \mathbb{R}^2 \to \mathbb{R}$, $P : \mathbb{R} \to \mathbb{R}^2$.

Notation: We're using $\mathcal{C}(\mathbb{R} \to \mathbb{R})$ for the vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$.

To repeat the proof given in class:

Theorem. $\sqrt{2} \notin \mathbb{Q}$.

Proof. Suppose not; in other words, suppose $\sqrt{2} \in \mathbb{Q}$. Then we can write $\sqrt{2} = \frac{a}{b}$, in lowest terms, where $a, b \in \mathbb{Z}$. Squaring we get $2 = \frac{a^2}{b^2}$, so $2b^2 = a^2$. Then a^2 is even, so a is even and we can write $a = 2r$ for some $r \in \mathbb{Z}$. In other words, $2b^2 = (2r)^2 = 4r^2$, so $b^2 = 2r^2$. Then b^2 is even, so b is even. But since both a and b are even, $\frac{a}{b}$ is not a fraction in lowest terms, which is a contradiction.

Therefore we were incorrect in supposing $\sqrt{2} \in \mathbb{Q}$, and the theorem is proved. $\square$

Notice that saying a is even is the same as saying that 2 divides a.

In doing the homework, which asks for a generalization to any prime p in place of 2, it might be helpful to try $p = 3$ privately first. Then write the same thing for a prime p in general and hand it in.

By way of background, you can use the fact that every integer > 1 is a product of primes, and in only one way.

Your proof doesn't have to be phrased like mine; you can even use implication arrows as long as it's clear. But you *do* need to start by saying you are supposing the opposite and you do need to mention near the end that there is a contradiction.

The box $\square$ is optional; it's a way of saying that's the end of the proof, so anyone reading the proof doesn't get confused and keep reading. An older way is to say "Q.E.D." (initials in Latin for "which was to be proved").