Notes

Here are some comments/suggestions on Assignment #2.

p. 5, Ex. 8: As mentioned on p. 3, a field F is said to have characteristic 0 if in F you never get $1 + 1 + \cdots + 1 = 0$.

In this problem, a "copy" of the rationals is a subfield that is isomorphic to the field of rationals. Here "isomorphic" means isomorphic as fields rather than as vector spaces—there is a one-to-one correspondence that preserves plus, unary minus, multiplication, 0, and 1.

It is up to you to say what the correspondence is. You could start like this: "Given a field F of characteristic 0, define $f : \mathbb{Q} \to F$ by $f(0) = 0$, $f(1) = 1$, ... You'll need to say what f is on positive integers, negative integers, and fractions.

In addition to defining f, there are several issues to think about—give the best explanation you can.

First, why is f one-to-one? For example, why can't $f(7) = f(3)$?

Second, is f "well defined"? In other words, if you say what $f(\frac{3}{2})$ should be and what $f(\frac{6}{4})$ should be, are these two values the same? (They had better be.)

Third, why does f preserve the field operations? (Don't go into too much detail on this.)

p. 48, Ex's. 2, 3: Try the method in the on-line notes for 1-F.

p. 48, Ex. 7: For a basis, think about what you get when you set one letter to 1 and the others to 0. The dimension of a vector space means the number of vectors in a basis. (We'll be discussing dimension more soon.)

p. 149, Ex. 9: These letters are alpha, beta, gamma. For a method, see comments below on E-1.

E-1: For linear independence, use an implication: Suppose that there is a linear relation $ru + s(u + v) + t(u + v + w) = 0$. Does this imply that r, s, t are all 0?

E-2: Notice the coefficients are in the 2-element field $GF(2) = \{0, 1\}$.

E-3: For (a), row-reduce and take the nonzero rows of the matrix in row-reduced form—we'll be discussing why this works. For (b), see the on-line notes for 1-F. For (c), take the general solution to the homogeneous equations with this matrix of coefficients and write it in vector form.

E-8: (b) asks you to explain why if you know the linear relations between columns in a matrix that is in row-reduced echelon form, you can determine the entries of the matrix. Notice that linear relations can be used to say how one column is or is not a linear combination of other columns.

For (c), your answer can be short, just quoting (b).