Assignment #6

Quiz 6 in discussion section **Tuesday, November 6**: Be able to prove that every linear transformation on $F^n \to F^m$ is of the form τ_A, for some matrix A. (You will need to be very clear about what the issues are.)

Assignment due nominally in lecture on **Wednesday, November 7** but you can hand it in **Friday, November 9**.

where	Do but don't hand in	Hand in
p. 21	Ex's 3, 4	Ex. 6 (any method OK)
p. 26	Ex's 1, 5, 6	
p. 74	Ex's 11, 12	
p. 84	Ex. 7	
p. 85	Ex 1, 4	Ex. 6
Q	Q-5	Q-1, Q-2, Q-3, Q-4, Q-6, Q-7
R		R-1, R-2, R-3, R-4

Problem R-1. For a square matrix A, an *eigenvector* is a *nonzero* vector v for which $Av = rv$ for some r, so Av lies along the same line as v. In other words, Av will point in the same direction as v or in the opposite direction (except in the case $r = 0$, when $Av = \mathbf{0}$).

Traditionally, we use λ (lambda) for r, so the equation is $Av = \lambda v$. If A is $n \times n$, there are no more than n possible values for λ, it turns out. These are the *eigenvalues* of A. (The "eigen", pronounced "eye-ghen", is German for "own", meaning that eigenvectors and eigenvalues are special vectors and scalars that "belong" to A.)

Following the link on the class home page, try the "Eigenvector Demo". The directions for finding eigenvalues are on the initial page before you click on "Eigen Program". (There is something not quite right about the "Find Eigen" tab; it may be necessary to click on its right-hand edge. The same applies going back to "Select Matrix A".) The idea is first to choose a matrix and then to move v around until Av and v line up—if possible!

The problem: For each of a 90° rotation, a reflection, and a shear, how many different eigenvector lines ("eigenspaces") can you find? (Answers must be among 0, 1, and 2. Two vectors along the same line count as one eigenspace.) Also try some transformations you invent yourself, by dragging $T(e_1)$ and $T(e_2)$ on the "Select Matrix A" panel, but you don't need to turn those in.

Problem R-2. $n \times n$ diagonal matrices behave very neatly when they are added and multiplied: The diagonal entries are added and multiplied without affecting each other. For example, $\begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} \begin{bmatrix} e_1 & 0 \\ 0 & e_2 \end{bmatrix} = \begin{bmatrix} d_1e_1 & 0 \\ 0 & d_2e_2 \end{bmatrix}$. A a result, if you have some polynomial expression such as $p(x) = 3+2x+5x^2$ and apply it to a diagonal matrix such as $D = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}$, the value $p(D)$, meaning $3I + 2D + 5D^2$, is just $\begin{bmatrix} p(d_1) & 0 \\ 0 & p(d_2) \end{bmatrix}$.

(Notice that the 3 becomes $3I$; we can't have 3 + a matrix!)

(a) For the matrix $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, find a polynomial $p(x)$ of degree 2 with coefficients in $\mathbb{R}$ so that $p(D) = 0$. (Use a "monic" polynomial—a polynomial for which the coefficent of the highest power of x is 1.)

(b) Same problem if $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $p(x)$ is of degree 3.

Problem R-3. Read §1.5 and §1.6. Notice that any elementary row operation can be described as multiplying on the left by an elementary matrix. Also notice that when you do any matrix multiplication MA, the columns of A do not affect each other in finding the product; it's like finding M times each column of A separately. In other words, if $A = [c_1|\ldots|c_n]$ then $MA = [Mc_1|\ldots|Mc_n]$, where the c_i are the columns of A.

(a) Show that when an $m \times n$ matrix A is row-reduced to a matrix B, there is an isomorphism of F^m with itself that takes each column of A to a column of B.

(It doesn't matter whether B is in row-reduced echelon form or not.)

(b) Use (a) to re-explain why if certain columns of B are a basis for the column space of B, then the same columns of A are a basis for the column space of A.

Problem R-4. On separate paper from the rest of this assignment, choose one problem from the midterm that you wish you had answered better, and write out a better answer. I'll look at it and comment on it so you can see whether it would be satisfactory. For the most meaningful feedback, say it in your own words instead of copying the solution handed out, although you're welcome to look at the solution beforehand.