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2003 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC LOGIC COLLOQUIM '03

Helsinki, Finland, August 14–20, 2003

Logic Colloquium '03, the 2003 European Summer Meeting of the Association for Symbolic Logic, took place on August 14–20, 2003, at the University of Helsinki in Finland. The colloquium was mainly funded by the University of Helsinki, the City of Helsinki, Nokia, the Rolf Nevanlinna Institute Fund, the Academy of Finland, the Finnish Cultural Foundation, Academia Scientiarum Fennica, the European Commission, the US National Science Foundation, and the Association for Symbolic Logic.

The Program Committee consisted of P. Aczel, Z. Adamowicz, J. Baldwin, E. Bouscaren, M. Detlefsen, M. Gitik, L. Hella, T. Jech, R. Parikh, H. Schwichtenberg, T. Slaman, V. Stoltenberg-Hansen (Chair), and J. Väänänen. The Colloquium was organized and run in an excellent manner by the local organizing committee, whose members included Å. Ekman, A. Halko, A. Hellsten, T. Huuskonen, J. Kennedy, J. Kontinen, K. Luosto, M. Luosto, H. Niemistö, J. Oikkonen, M. Pauna, J. Ruokolainen, J. Väänänen (Chair), and M. Viljanen.

The main topics of the conference were *Model Theory and Finite Model Theory*, *Set Theory*, *Recursion Theory and Models of Arithmetic*, *Proof Theory and Non-classical Logic*, and *Philosophy of Mathematics and History of Logic*. The program included two tutorials, eighteen invited plenary lectures and fifteen invited lectures in four special sessions corresponding to the first four main topics. There were also 111 contributed talks. Logic Colloquium 2003 had 251 registered participants from 31 countries, representing all continents but Antarctica.
The following tutorials were presented:

Michael Benedikt (Bell Labs), *Model theory and complexity theory*.
Stevo Todorčević (Paris), *Set-theoretic methods in Ramsey theory*.

The following invited plenary lectures were given:

Alexandru Baltag (Oxford), *Doing things to models: trans-structural dynamic logics*.
Howard Becker (Columbia, SC), *Generalized model theory and Polish group actions*.
Matthew Foreman (Irvine), *Has the CH been settled?*
Jean-Yves Girard (Marseille), *From operator algebras to logic*.
Martin Grohe (Berlin), *The succinctness of monadic logics on finite trees*.
Peter T. Johnstone (Cambridge), *A survey of realizability toposes*.
Simo Knuuttila (Helsinki), *On the history of modality as alternativeness*.
Menachem Kojman (Be'er Sheva), *On the role of infinite cardinals*.
Michael C. Laskowski (College Park, MD), *Applications of descriptive set theory to uncountable model theory*.
Larisa Maksimova (Novosibirsk), *Decidable properties of logical calculi and of varieties of algebras*.
Per Martin-Löf (Stockholm), *Are the objects of propositional attitudes propositions in the sense of propositional and predicate logic?*

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Ludomir Newelski (Wroclaw), *Lascar strong types and topology*.
Dag Normann (Oslo), *The continuous functionals in the perspective of domain theory*.
Graham Priest (Melbourne), *Inconsistent arithmetics and inconsistent computation*.
Pavel Pudlák (Prague), *Consistency and games*.
Michael Rathjen (Columbus, Leeds), *Realizability, forcing, and independence results for constructive theories*.
Saharon Shelah (Jerusalem), *Good frames, for what are they good?*
Richard A. Shore (Ithaca), *The boundary between decidability and undecidability in degree structures*.

The proceedings of the conference, edited by V. Stoltenberg-Hansen and J. Väänänen, will contain many of the invited talks and will appear in the book series, *Lecture Notes in Logic*.

For more information on the meeting, the reader is referred to the conference webpage, `http://www.helsinki.fi/lc2003/`.

Abstracts of invited and contributed talks given in person or by title by members of the Association for Symbolic Logic follow.

For the Program Committee
VIGGO STOLTENBERG-HANSEN

Abstracts of Invited Tutorials

► MICHAEL BENEDIKT, *Model theory and complexity theory*.
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This tutorial concentrates on links between traditional (infinitary) model theory and complexity theory. We begin with an overview of the 'classical' connection between complexity theory and finite model theory, giving quickly the basic results of descriptive complexity theory. We then discuss several ways of generalizing this to take account a fixed infinite background structure. We will start by giving the basics of complexity theory parameterized by a model (algebraic complexity over an arbitrary structure). We then cover results characterizing first-order theories of models via the complexity of query problems (embedded finite model theory). Finally, time permitting, we will look at abstractions of descriptive complexity theory to take into account a background structure.

► STEVO TODORCEVIC, *Set-theoretic methods in Ramsey theory*.
C.N.R.S., UMR 7056 2, Place Jussieu, Case 7012 Paris 75251 France.
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We shall start with an overview of some of the basic pigeon-hole principles of Ramsey theory such as the Halpern-Lauchli or the Hales-Jewett theorems. We shall present then a general method for getting the higher-dimensional analogues of these principles. We shall finish with some applications to the Banach space theory such as Gowers' c_0-theorem or Rosenthal's l_1-theorem.

Abstracts of invited plenary talks

► ALEXANDRU BALTAG, *Doing things to models: trans-structural dynamic logics*.
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The title is not referring to sub-structural logics, which bear no relation or similarity to my subject. The word "trans-structural" refers to the fact that the semantics of such logics transcends the current structure on which they are defined, their current "model": these logics contain propositional connectives, modalities or quantifiers whose semantics refers to other models than the one on which they are defined. These logical operators can be interpreted dynamically, as talking about "ways to transform/change/update the current model". Such operators provide ways to capture inherent properties of the model in an indirect manner, by referring to properties of the model's possible modifications. These logics can be thus seen as a "dynamification" of model-theoretic semantics, in which the dynamic (relational, Kripke-style) possible-world semantics is applied to the level of models, instead of the level of states in a single model.

I give examples of such operators, all from very recent literature (as the subject is rather new): the "entailment (or implication) along a model-theoretic relation" (due to Barwise/van Benthem), the "bisimulation quantifiers" (due to G. d'Agostino and M. Hollenberg), the modal logic of "relativisation" (or "public announcement", due to J. Gerbrandy and J. Plaza, and formalised as a trans-structural modality by L. Moss and myself) and, more generally, the "epistemic-action" modalities (introduced by myself and L. Moss to model communication in multi-agent systems). I point to the relations between some of these notions and non-standard versions of the standard logical results of interpolation, preservation and definability. I discuss peculiarities of these logics, e.g., their essential lack of "strong compositionality" (in spite of their "weak" compositionality: the meaning of a formula in a model cannot be defined in terms of the meanings of its subformulas in the SAME model), I mention some completeness, decidability and undecidability results for some of these logics, I discuss their relations with other logics and I give a list of open problems.

► HOWARD S. BECKER, *Generalized model theory and Polish group actions*.
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Most of the model theory of the language $L_{\omega_1,\omega}$ and countable structures can be rephrased in group action terminology, with respect to the logic action by the group of permutations of the integers. We generalize these results to other actions by other Polish groups.

► ZOE CHATZIDAKIS, *Asymptotic theories of fields*.
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Families of fields (maybe with additional structure) are often defined in terms of prime numbers. The simplest example is the family of all fields with p elements (denoted $\mathbb{F}_p$), as p ranges over the set of all prime numbers. James Ax studied in 1968 the asymptotic theory of these fields, i.e., the set of sentences in the language of rings which hold in all but finitely many of the fields $\mathbb{F}_p$, gave an axiomatisation of this theory and showed that it was decidable. The famous results of Ax and Kochen (1965, 1966) show that the asymptotic theory of the valued fields of p-adic numbers $\mathbb{Q}_p$ and the asymptotic theory of the valued fields $\mathbb{F}_p((t))$ (formal power series over $\mathbb{F}_p$) coincide; Ax's result then implies that this theory is decidable.

In the past few years, other asymptotic theories were investigated, usually yielding decidability results. Maybe the most striking example is the asymptotic theory of the fields $(\tilde{\mathbb{F}}_p, \sigma_p)$, where $\tilde{\mathbb{F}}_p$ denotes the algebraic closure of $\mathbb{F}_p$, σ_p is an additional unary function predicate, interpreted as the Frobenius map $x \mapsto x^p$ (Recall that in any field of positive characteristic p, the Frobenius map defines an injective, not necessarily surjective, homomorphism). Hrushovski showed that this asymptotic theory is decidable and model complete (and in fact coincides with the model companion of the theory of fields with a distinguished automorphism). This result was announced independently by Macintyre.

In this talk, I will survey the classical results, and list a few recent examples of asymptotic theories of fields with decidable theories. I will conclude with an example of an asymptotic theory, whose existential part is Σ^0_2-complete.

[This abstract is presented by title; the talk could not be given.]

► MATTHEW FOREMAN, *Has the CH been settled?*
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A collection of generalized large cardinal axioms are presented that provide a reasonably complete picture of the set theoretic universe, deciding the CH, the existence of Suslin trees, Kurepa trees, $AD^{L(R)}$ and many other problems. Most of the results have been known for a long time, but new results are presented and the old results are put in context.

► JEAN-YVES GIRARD, *From operator algebras to logic*.
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We discuss the interpretation of logical laws by means of operators acting on Hilbert space. The case of the finite matricial factor is of special interest.

► MARTIN GROHE, *The succinctness of monadic logics on finite trees*.
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A central topic in finite model theory has always been the expressive power of logics on finite relational structures. However, in some situations of practical interest we have to compare logics of the same expressive power. In particular, there is great interest in a variety of logics on finite trees that all characterise precisely the regular tree languages and thus have the same expressive power as monadic second-order logic.

Succinctness is a natural refined measure for the strength of a logic. For a class F of functions on the natural numbers, we call a logic L F-succinct in a logic L' if for every L-formula A there is an equivalent L'-formula A' and a function $f \in F$ such that $\mathrm{size}(A') \leq f(\mathrm{size}(A))$.

In this talk I will present recent results on the succinctness of first-order logic, monadic second-order logic, and a number of fixed-point logics on finite words and trees.

This is joint work with Nicole Schweikardt.

► PETER T. JOHNSTONE, *A survey of realizability toposes*.
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The notion of realizability topos has been around for over twenty years now: numerous variants of the basic construction of a topos from a Schönfinkel algebra (or PCA) have been studied, and many different examples of Schönfinkel algebras have been considered. However, until recently (with the notable exception of John Longley's Ph.D. thesis) there has been relatively little work on how these different examples fit together: we do not have a well-understood 2-category of realizability toposes in the same way that we have a well-understood 2-category of Grothendieck toposes. I believe that the time is now ripe for the development of such an understanding: in this talk I shall attempt to give some indications of what we already know, and what we might hope to learn, about morphisms between realizability toposes.

► SIMO KNUUTTILA, *On the history of modality as alternativeness*.
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There are four modal paradigms in ancient philosophy: the 'statistical' or 'temporal frequency' interpretation of modality, the model of possibility as a potency, the model of antecedent necessities and possibilities with respect to a certain moment of time (diachronic modalities) and the model of possibility as non-contradictoriness. None of these conceptions, which were well known to early medieval thinkers through the works of Boethius, was associated with the idea of modality as involving reference to simultaneous alternatives. This new paradigm was included in Augustine's theological conception of God as acting by choice between alternative histories and similarly in some Arabic theological theories. Ancient habits of thinking continued to play an important role in scholasticism, and the theoretical significance of the new conception was not fully realized before the works of John Duns Scotus and other early fourteenth century thinkers, such as William Ockham and John Buridan.

Many scholars have recently remarked that the notions of necessity and possibility were treated in fourteenth century modal logic and modal theory in a way which is analogous to the contemporary possible worlds semantics. I shall discuss the principles of the new fourteenth-century modal logic and how this differed from thirteenth century interpretations of Aristotle's modal syllogistic. Second, I shall deal with John Buridan's discussion of Aristotle's indirect proofs and with some other fourteenth century thinkers' theories of counterfactual reasoning based on the new theory of modality as alternativeness. These authors contrasted their approaches to counterfactual reasoning with those of Averroes and Thomas Aquinas which did not operate with the idea of alternative domains but instead with abstract essentialist modalities. I conclude with remarks on this controversy.

► MENACHEM KOJMAN, *On the role of infinite cardinals*.
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The talk will address the role infinite cardinals and infinite cardinal arithmetic have in mathematics and survey several recent results from topology, combinatorics and geometry in which infinite cardinals play a a role, to demostrate various aspects of the topic.

► MICHAEL C. LASKOWSKI, *Applications of descriptive set theory to uncountable model theory*.
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Fix a complete theory T in a countable language. All of Shelah's stability-theoretic technology for analyzing the UNCOUNTABLE models of T can be described in terms of which COUNTABLE configurations of elements embed into models of T.

Recent work has highlighted the utility of examining the descriptive set-theoretic complexity of these notions, either in general or in specific instances.

This talk will survey the role played by such methods in computing the uncountable spectrum of such a theory (which was joint work with Hart and Hrushovski) and discuss recent work with Shelah on the Main Gap for aleph-one saturated models, which also employs descriptive set-theoretic technology.

► LARISA MAKSIMOVA, *Decidable properties of logical calculi and of varieties of algebras*.
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We give an overview of decidable and strongly decidable properties over the propositional modal logics K, GL, S4, S5 and Grz, and also over the intuitionistic logic Int and the positive logic Pos. A property P is said to be decidable over a calculus L if there is an algorithm which, for any finite set Ax of additional axiom schemes, decides if the calculus L+Ax possesses the property P; P is said to be strongly decidable over L if there is an algorithm which, for any finite set Rul of additional axiom schemes and rules of inference, decides if the calculus L+Rul has the property P.

We consider a number of important properties of logical calculi: consistency, tabularity, pretabularity, local tabularity, various forms of interpolation and of the Beth property. For instance, consistency is decidable over K and strongly decidable over S4 and Int; tabularity and pretabularity are decidable over S4, Int and Pos; interpolation is decidable over S4 and Pos and strongly decidable over S5, Grz and Int; the projective Beth property is decidable over Int, Pos and Grz, etc. Some complexity bounds are found.

In addition, we state that tabularity and many variants of amalgamation and of surjectivity of epimorphisms are base-decidable in varieties of closure algebras, of Heyting algebras and of relatively pseudocomplemented lattices.

► PER MARTIN-LÖF, *Are the objects of propositional attitudes propositions in the sense of propositional and predicate logic?*
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On the one hand, propositions serve as the objects of propositional attitudes. On the other hand, propositions and propositional functions are the objects on which the logical operators, the connectives and the quantifiers, operate. The question to be discussed is whether we are really dealing with one and the same concept of proposition in the two cases.

► LUDOMIR NEWELSKI, *Lascar strong types and topology*.
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Assume M is a sufficiently saturated and homogeneous model (a monster model). We define on M some canonical equivalence relations:

E_S is the intersection of all 0-definable equivalence relations on M with finitely many classes;
E_{KP} is the finest 0-type-definable equivalence relation on M with bounded (that is, $\leq 2^{|T|}$) number of classes;
E_L is the finest equivalence relation on M invariant under $\mathrm{Aut}(M)$ (as a subset of $M \times M$) and with bounded number of classes.

So E_L refines E_{KP} and E_{KP} refines E_S. The classes of E_S are called (Shelah) strong types, the classes of E_{KP} Kim-Pillay strong types and the classes of E_L Lascar strong types. In stable theories all these relations and strong type notions coincide. In simple theories E_{KP} equals E_L, while it is open if they equal also E_S there. There are only a few examples where E_L and E_{KP} differ (the first was found by Ziegler).

In 2002 I answered most questions from [1] on the structure of Lascar strong types, particularly referring to their type-definability and to the structure of the Lascar group $\mathrm{Gal}_L(M) = \mathrm{Aut}(M)/\mathrm{Aut}\, f_L(M)$, where $\mathrm{Aut}\, f_L(M)$ is the poinwise stabilizer in $\mathrm{Aut}(M)$ of M/E_L. I obtained also some general results on elimination of hyperimaginaries and on invariant equivalence relations, extending some ideas on genericity from stable theories to arbitrary ones (see also my contributed talk join with M. Petrykowski).

There are still some questions on Lascar strong types motivated by the classification of Borel equivalence relations, since E_L regarded on the level of types is Borel.

The main new tool in the proofs is the notion of an open analysis of a compact space with respect to its covering by a family of subsets. This yields a new way of proving some finiteness results in model theory (not referring to the compactness theorem) and generalizes the Cantor-Bendixson analysis and the Baire category theorem. Everything here is elementary and does not require any knowledge of advanced model theory.

[1] E. CASANOVAS, D. LASCAR, A. PILLAY, and M. ZIEGLER, *Galois groups of first order theories*, ***Journal of Mathematical Logic***, vol. 1 (2001), pp. 305–319.

[2] L. NEWELSKI, *The diameter of a Lascar strong type*, ***Fundamenta Mathematicae***, vol. 176 (2003), pp. 157–170.

► DAG NORMANN, *The continuous functionals in the perspective of domain theory*.
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In the 70's and early 80's the continuous functionals were primarily studied as a hierarchy of total functionals, and the prime notion of computability was based on Kleene's S1–S9. Domain theory originated from Scott and Ershov. Scott's original motivation was to construct a typed hierarchy of partial continuous functionals with a Platek-inspired notion of computability. The domain-theoretical approach leads to an equivalent hierarchy of hereditarily total functionals, but to alternative notions of computability.

One advantage of the domain-theoretical approach is that we may use domain-representations of the reals or other spaces, and then consider additional base types. This has e.g., been used for the semantics of typed languages for exact real-valued computations. We will discuss some advances that have been made over the recent years on the continuous functionals of higher types over the natural and real numbers and corresponding notions of computability.

► GRAHAM PRIEST, *Inconsistent arithmetics and inconsistent computation*.
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The structure of inconsistent arithmetics is well known. (See [2].) Some of these arithmetics are axiomatisable, but can prove their own "Gödel sentences". This fact, in turn, can be used as an argument to the effect that such an arithmetic is the true arithmetic. (See [1], ch. 3.) Shapiro [3], points out that the inconsistencies of such an arithmetic must occur "even in its most elementary" part—essentially the computational part—and argues that this is unacceptable.

This paper examines Shapiro's arguments. It argues that the problems arise only if one marries an inconsistent arithmetic with a consistent theory of computation. If one marries an inconsistent arithmetic with an inconsistent theory of computation (which the arithmetic itself shows us how to construct), the problems Shapiro envisages do not arise.

[1] G. PRIEST, ***In Contradiction***, Kluwer Academic Publishers, Dordrecht, 1987.

[2] ———, *Inconsistent models of arithmetic II; the general case*, ***The Journal of Symbolic Logic***, vol. 65 (2000), pp. 1519–1529.

[3] S. SHAPIRO, *Inconsistency and incompleteness*, ***Mind***, vol. 111 (2002), pp. 817–832.

► PAVEL PUDLÁK, *Consistency and games*.
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The statement that a weak formal system is consistent should be equivalent to a simple combinatorial principle. I will report on some results about this problem. The formal systems that we studied are proof systems for propositional calculus and subtheories of Bounded Arithmetic. We found formal statements about strategies for playing several finite games simultaneously which are closely related to the consistency of these systems. We found also interesting computational problems about such strategies.

► MICHAEL RATHJEN, *Realizability, forcing, and independence results for constructive theories*.
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An important technique for achieving independence and consistency results for intuitionistic theories is *realizability*. While there exist detailed accounts of realizability interpretations for arithmetic and analysis, a synopsis of different notions of realizability for the more general frameworks of type theory and set theory has rarely been undertaken.

In this talk I shall survey several notions of realizability for constructive type and set theories such as Kleene realizability, formulae-as-types interpretation based on E-recursive functions, and realizability in Kripke-Platek set theory. These realizability interpretations are then employed to obtain independence and equiconsistency results.

The most famous method for showing independence of statements from axiom systems in the classical context is Cohen's method of forcing or the closely related technique of Boolean-valued models. Forcing has been developed for intuitionistic set theories as well, in particular for intuitionistic Zermelo-Fraenkel set theory, IZF. In this context, forcing models are closely related to Heyting-valued models as well as sheaf models. However, to obtain specific independence results, I shall focus on particular forcing constructions (due to Lubarsky), where the underlying partial order is a class and one utilizes techniques from classical set-theoretic forcing as well as techniques that are germane to the classical constructible hierarchy L such as Σ_1 Skolem hull and condensation arguments.

► SAHARON SHELAH, *Good frames, for what are they good?*
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The aim of the lecture is to show how we can get good frames starting from an abstract elementary class which is categorical in a large enough cardinal (but in ZFC!). We think this will lead to reasonable understanding of the categoricity spectrum in large enough cardinals.

But probably the reader needs more background. An abstract elementary class is e.g the class of models of a first order theory omitting some types with the usual notion of an elementary submodel.

A good frame $\mathfrak{s}$ is a mini version of a superstable first order theory; we have an abstract elementary class $\mathcal{K}$, a fixed cardinal λ in which $\mathcal{K}$ has amalgamation, a set of basic types over models in λ and for them a notion of non-forking for such types. The restriction to one cardinal makes a difference. Now [5], [1], and [2] did the parallel of [3], [4] (+ justification of "good frames are like superstable first order theory"). Essentially, assuming enough instances of weak GCH, when an abstract elementary class $\mathcal{K}$ is categorical in λ, λ^+ (and has a model in λ^{++}) we get a good frame and assuming not too many models in the λ^{+n} we arrive at a parallel to excellent classes. A consequence is: If $\mathcal{K}$ is categorical in a large enough cardinal then it is categorical in every large enough cardinal, when $\mathcal{K}$ is: (a) an abstract elementary class with amalgamation or (b) $(Mod(T), \prec)$, $T \subseteq \mathbb{L}_{\kappa,\omega}$, κ measurable.

[1] Saharon Shelah, *Categoricity in abstract elementary classes: going up inductive step*, preprint.

[2] ———, *Toward classification theory of good λ frames and abstract elementary classes*.

[3] ———, *Classification theory for nonelementary classes, I. The number of uncountable models of $\psi \in l_{\omega_1,\omega}$. Part A*, ***Israel Journal of Mathematics***, vol. 46 (1983), pp. 212–240.

[4] ———, *Classification theory for nonelementary classes, I. The number of uncountable models of $\psi \in l_{\omega_1,\omega}$. Part B*, ***Israel Journal of Mathematics***, vol. 46 (1983), pp. 241–273.

[5] ———, *Categoricity of an abstract elementary class in two successive cardinals*, ***Israel Journal of Mathematics***, vol. 126 (2001), pp. 29–128.

► RICHARD A. SHORE, *The boundary between decidability and undecidability in degree structures*.
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Very early research suggested and sought decidability results for many degree structures. Although the lowest levels (in quantifier complexity) were shown decidable, most structures turned out to be highly undecidable. Finding the precise dividing line between decidability and undecidability provides a setting for formulating the extent of our knowledge of, and control over, the notion of relative computability in different settings. Older results typically showed the two quantifier level decidable and the third undecidable. We examine the situation for the r.e. degrees, the degrees below $0'$ and the structure of all the Turing degrees when the language is expanded to include join and infimum or (for all the degrees) the Turing jump. The border now typically moves to between the first and second quantifier level with new theorems needed for both types of results and new problems and questions coming to light.

Abstracts of invited talks in the Special Session on Model Theory

► TUNA ALTINEL, *Simple groups of finite Morley rank*.
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The Cherlin-Zil'ber conjecture states that an infinite simple group of finite Morley rank is a linear algebraic group over an algebraically closed field. Various approaches to this conjecture have been developed over the years from both model theoretic and group theoretic sides.

On the group theory side, the presence of a well-developed Sylow 2-theory and various finiteness conditions permitted the use of finite group theoretic ideas, and it has been possible to adapt techniques from the classification of the finite simple groups. The context where these techniques have been the most efficient has been the context of simple *K^*-groups*. The notion of K^*-group, a group of finite Morley rank all of whose infinite definable simple and proper sections are algebraic groups over algebraically closed fields, was introduced to set firm grounds for an analogue of the *revisionist approach* to the classification of the finite simple groups, and has made it possible to carry out a systematic analysis of various classes of simple groups of finite Morley rank. As a result, the algebraicity conjecture has been verified for the simple K^*-groups that have infinite 2-subgroups of bounded exponent.

A weakness of the notion of K^*-group is the strength of its definition. In order to arrive at a classification statement free of an inductive hypothesis one needs to verify that the Cherlin-Zil'ber conjecture holds for *all* the simple K^*-groups of finite Morley rank. In my talk, I will discuss a broader inductive notion which is most relevant in the context of simple groups that have infinite 2-subgroups of bounded exponent. Indeed, if this approach succeeds then the Cherlin-Zil'ber conjecture will be verified for all simple groups of finite Morley rank that have infinite 2-subgroups of bounded exponent. As important as this potential success are the new ideas and techniques which have been introduced to tackle various technical difficulties in the new context and bring closer the model theoretic and group theoretic sides in the area. Part of my talk will be devoted to exposing these new methods.

► TAPANI HYTTINEN, *An example of a homogeneous class of structures*.
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We give an example of a non-elementary class of structures, which is homogeneous for non-trivial reasons and which arises from classical mathematics. In addition, a model theoretic classification of the class is given.

► WAI YAN PONG, *The differential order*.
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Differential Order is a model theoretic rank that generalizes the concept of the order of a differential polynomial. We will discuss its application in differential algebra and its relation with other ranks.

This is joint work with Matthias Aschenbrenner.

Abstracts of invited talks in the Special Session on Proof Theory and Non-classical Logic

► RALPH MATTHES, *Inductive constructions for classical natural deduction*.
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Termination for classical natural deduction is difficult in the presence of commuting/permutative conversions for disjunction. We present an approach based on reducibility candidates that involve non-strictly positive inductive definitions.

It also covers the extension of the logic by least and greatest fixed-points of monotone operators, which appears to be a new result.

An informal realizability interpretation yields an embedding of Parigot's second-order lambda-mu-calculus into second-order lambda-calculus (published at TLCA 2001). We discuss how this could be extended to disjunctions.

Finally, the relation to Parigot's strictly-positive inductive definition of his set of reducibility candidates is explained.

► ERIK PALMGREN, *Problems of predicativity in constructive topology*.
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The theory of locales provides a framework for constructive general topology, albeit in impredicative settings such as toposes or higher order logics. It has been recognised that much of the theory can be carried out in constructive type theory or set theory, predicatively, without using full separation principles. The main idea of this theory, formal topology, is to generate the open cover relation inductively. It appeared in the doctoral thesis of Per Martin-Löf 1970, and has been investigated by Sambin, Aczel, Coquand and their associates. Formal topology avoids some known artificialities of the definition of continuous function present in the metric theory advocated by Bishop.

Lacking the separation principle the distinction of small versus large type (set versus class) enters in unexpected and unwelcome places. For instance neither the points of a formal topological space, nor the continuous functions between such spaces, need to form a set. We survey results of G. Curi and Aczel and present some new methods of proving smallness, which use topological properties as well as predicative type-theoretic principles: a general dependent choice principle and type universe operators.

An important problem is to find a predicatively well-defined category with sufficiently good closure properties for carrying out standard constructions of algebraic and differential topology.

► JEFF PARIS, *Uncertain reasoning*.
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In the real world we can rarely enjoy the luxury of drawing definitive conclusions from our knowledge. Indeed even our knowledge itself is usually tainted with some degree of uncertainty. Nevertheless the real world demands action, we usually do not have the option of saying that we cannot decide and will therefore simply refuse to do anything.

This has led to logics, or patterns of reasoning, with such knowledge that allow rules of proof which go beyond simple 'truth preservation' and take instead their justification from considerations of 'rationality'. I shall describe one such approach along with some of its surprising, and for the most part, welcome, consequences.

► TARMO UUSTALU, *Type-based termination of recursive definitions*.
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We describe an approach to supplementing a type assignment system with inductive and coinductive constructors where the recursors and corecursors compute almost as general recursion operators, but their typing rules ensure that the system stays strongly normalizing and is sound with respect to the expected initial algebra, final coalgebra semantics. This way of directly supporting sophisticated structured (co)recursive definitions is radically different from that of syntactic guardedness conditions. The simplest instance appeared in the systems of N. P. Mendler's LICS'87/APAL'91 paper, but the underlying principle, once explicated, gives the approach a much wider scope.

Besides considering (co)inductive types in a simply typed system, we discuss (co)inductive constructors of higher kinds and a setting of dependent typing.

Abstracts of invited talks in the Special Session on Recursion Theory and Arithmetic

► KLAUS AMBOS-SPIES, *Generators of the computably enumerable degrees*.
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We will survey some older results and present some new results on generators of the computably enumerable (c.e.) degrees. We will focus on the role played by the meet operator in the process of generating (subclasses of the) c.e. degrees under join and meet. In particular we will address the question whether every set of degrees which generates the c.e. degrees under join and meet will generate the nonzero c.e. degrees under join. We have raised this question in a paper of 1985 where we had shown that every generator of the c.e. degrees intersects all nonzero initial segments. Our main results are that every generator of the c.e. degrees also generates the high c.e. degrees under join (joint work with D. Ding and P. A. Fejer) but that there is a generator of the c.e. degrees which does not generate the nonzero c.e. degrees under join (joint work with S. Lempp and T. A. Slaman).

► O. V. KUDINOV, *Computable listings and autodimensions of rings and fields*.
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There are several methods to code computable families of computably enumerable sets or computable graphs in computable integral domains—one of them was proposed by the author in 1997 to prove the following:

THEOREM 1. *There exists a computable integral domain with exactly two nonequivalent computable presentations.*

Since for fields the similar problem is still open, it would be natural to describe the index sets of the classes of computably categorical (autostable) fields and fields of infinite autodimension. The reasonable conjecture states that the index set of the first class is Π^1_1-complete.

The next result deals with fields of algebraic numbers.

THEOREM 2. *The index set of autostable fields of algebraic numbers of zero characteristic is* Π^0_4*-complete.*

The general case is very hard since we need a lot of autostable fields F with infinite $rank_{tr}(F)$. However, for the class of integral domains the description of corresponding index set is closely related to the following.

CONJECTURE 3. *The index set of autostable graphs is* Π^1_1*-complete.*

It is worth to note that index sets of the classes of fields are defined via classical computable enumeration of the class of semicomputable commutative rings. We can not define them via suitable computable enumeration of computable fields as the following result states.

THEOREM 4. *There does not exist a computable listing of all the class of computable fields of zero characteristic up to isomorphism.*

This research is partially supported by RFBR grant 02-01-00593.

[1] M. KUMMER, *Some applications of computable one-one numberings*, ***Archive for Mathematical Logic***, vol. 30 (1990), pp. 219–230.

[2] M. KUMMER, S. WEHNER, and XIAO-DING YI, *Discrete families of recursive functions and index sets*, ***Algebra and Logic***, vol. 33 (1994), pp. 85–94.

► CHRIS POLLETT, *Weak arithmetics and unrelativised independence results*.
214 MacQuarrie Hall, Department of Computer Science, San Jose State University, One Washington Square, San Jose, CA 95192-0249, USA.
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In this talk I will survey some techniques which can be used to show unrelativized independence results for questions like NP vs coNP, the collapse of the polynomial hierarchy, or Hilbert's Tenth Problem in weak systems of arithmetic. I will also present some new results concerning the limits of formalizing padding arguments in commonly studied weak arithmetics.

► JOHN V. TUCKER, *Specification and computation on topological algebras*
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We consider computability theory for infinite data, such as real numbers, signals and wave forms, streams, spatial objects, and functions. Usually, such data is collected into topological spaces and so we consider models of computation for functions and sets on topological spaces.

The many models of computation fall into two types. First, there are concrete models of computation which depend on building a representation of the space of data, usually from the natural numbers, and defining computability on the space in terms of computability on the representation. This is the standard approach of Computable Analysis starting in the 1950s; it is used in current work based on Pour El and Richards' computation structures, Klaus Weihrauch's TTE, Stoltenberg-Hansen, Tucker and Edalat's domain representations, and Spreen's effective topology.

Secondly, there are abstract models that define computability independently of all representations. The spaces are given algebraic structure (e.g., that of a Banach space) and programs and machines are based on the structure. Thus, computation is abstract in precisely the way algebra is abstract: computation is invariant under isomorphism. This is the approach of many generalisations of recursion theory starting in the 1950s; it is used in some current work by S. Smale and his collaborators (on rings and fields) and J. I. Zucker and J. V. Tucker (on many sorted universal algebras).

The lecture will discuss some current results and problems, especially on the equivalence of the concrete and abstract models. Recent results of Tucker and Zucker show that equivalence requires programmable approximations of partial multivalued functions on topological algebras.

We will also mention applications to

(i) specifying and programming in the growing practical field of exact real number computation, and

(ii) long standing problems to do with machines and the analogue-digital interface.

Some background can be found in Stoltenberg-Hansen and Tucker [1], [2], and Tucker and Zucker [3]. Some new results can be found in Tucker and Zucker [4], [5].

[1] V. Stoltenberg-Hansen and J. V. Tucker, *Effective algebras*, ***Handbook of Logic in Computer Science***, vol. IV, (S. Abramsky, D. Gabbay and T. Maibaum, editors), Oxford University Press, 1995, pp. 357–526.

[2] ———, *Computable rings and fields*, (E. Griffor, editor), ***Handbook of Computability Theory***, Elsevier, 1999, pp. 363–447.

[3] J. V. Tucker and J. I. Zucker, *Computable functions and semicomputable sets on many sorted algebras*, ***Handbook of Logic for Computer Science***, vol. V, (S. Abramsky, D. Gabbay and T. Maibaum, editors), Oxford University Press, 2000, pp. 317–523.

[4] ———, *Abstract versus concrete computation on metric partial algebras*, ***ACM Transactions on Computational Logic***, in press.

[5] ———, *Computable total functions, algebraic specifications and dynamical systems*, ***The Journal of Logic and Algebraic Programming***, in press.

Abstracts of invited talks in the Special Session on Set Theory

► ARTHUR W. APTER, *Indestructibility and strong compactness*.
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I will discuss forcing indestructibility for non-supercompact strongly compact cardinals, emphasizing the construction of a model in which the first two strongly compact cardinals aren't supercompact yet satisfy significant indestructibility properties. I will also mention the problems that seem to be inherent in trying to extend both this result and results of a similar nature.

► STEVE JACKSON, *Some applications of cubical markers*.
Department of Mathematics, University of North Texas, Denton, TX 76203, USA.
E-mail: `jackson@unt.edu`.

We use the existence of cubical markers for equivalence relations generated by actions of $\mathbb{Z}^n$ to obtain new results and new proofs for these equivalence relations and others.

[This abstract is presented by title; the talk could not be given.]

► HEIKE MILDENBERGER, *New canonization theorems and dense free subsets in* $\aleph_\omega$.
University of Vienna, Vienna, Austria.
E-mail: `heike@logic.univie.ac.at`.

In the talk I will sketch the forcing construction, starting from ω compact cardinals, of a model of ZFC where every structure with countable signature and domain $\aleph_\omega$ has a free subset which hits every infinite cardinal interval. Combinatorics in ZFC shows that a denser distribution of members of free subsets cannot be obtained.

► MAURICE C. STANLEY, *Outer models and genericity*.
San Jose State University, 436 W. Highland Ave., Tracy, CA 95376, USA.
E-mail: `stanley@math.sjsu.edu`.

If V is a standard transitive model of ZFC, say that a transitive $W \supseteq V$ is an *outer model* of V if W contains the same ordinals as V and if (W, V) satisfies ZFC in a language with a predicate symbol for V. Why is forcing the only known method for constructing outer models? How strong is generic absoluteness? We approach these questions by characterizing those outer models of a given model that are set generic and those that are class generic in each of two senses.

Abstracts of contributed talks

► YOSHIHIRO ABE, *Ineffability of $P_\kappa\lambda$ when $cf(\lambda) < \kappa$.*
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We say $X \subset P_\kappa\lambda$ is (almost) ineffable if for every $\{S_x \subset x : x \in X\}$ there exists $S \subset \lambda$ such that $\{x \in X : S_x = S \cap x\}$ is stationary (unbounded) in $P_\kappa\lambda$. Let $NIn_{\kappa,\lambda}(NAIn_{\kappa,\lambda}) = \{X \subset P_\kappa\lambda : X \text{ is not (almost) ineffable}\}$, which is a normal ideal on $P_\kappa\lambda$. By Magidor [3] and others $P_\kappa\lambda$ is ineffable if κ is λ supercompact if $P_\kappa(2^{\lambda^{<\kappa}})$ is almost ineffable.

We know fairly well in case $cf(\lambda) \geq \kappa$; (i) $\lambda^{<\kappa} = \lambda$ (Johnson [1]), (ii) $\{x \in P_\kappa\lambda : P_{x\cap\kappa}x$ is not almost ineffable$\} \in NIn_{\kappa,\lambda}$ (Kamo [2]).

While the case $cf(\lambda) < \kappa$ is less known, we show:

(1) $NAIn_{\kappa,\lambda} = NIn_{\kappa,\lambda}$ if $\lambda^{<\kappa} = 2^\lambda$.

(2) λ ineffability is not always equivalent to $\lambda^{<\kappa}$ ineffability.

Note that λ (super)compactness is equivalent to $\lambda^{<\kappa}$ (super)compactness.

[1] C. A. JOHNSON, *Some partition relations for ideals on $P_\kappa\lambda$*, ***Acta Mathematica Hungarica***, vol. 56 (1990), pp. 269–282.

[2] S. KAMO, *Remarks on $P_\kappa\lambda$-combinatorics*, ***Fundamenta Mathematicae***, vol. 145 (1994), pp. 141–151.

[3] M. MAGIDOR, *Combinatorial characterization of supercompact cardinals*, ***Proceedings of the American Mathematical Society*** vol. 42 (1974), pp. 279–285.

► YUUKI ANDOU, *A cut-elimination for LK corresponding with a normalization for NK.*
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Normalization procedures for systems of natural deduction in the style of Prawitz [3], [4] are the representations of Gentzen's Hauptsatz [2], and they possess some properties as strong nomalization and confluence which do not hold for Gentzen's original cut-elimination procedure for sequent calculi. We define an interpretation of the sequent calculus LK to the system of classical natural deduction NK in the style of our previous work [1], and reorganize ordinary cut-elimination procedure according to the normalization procedure for NK.

[1] Y. ANDOU, *Church-Rosser property of a simple reduction for full first-order classical natural deduction*, ***Annals of Pure and Applied Logic*** vol. 119 (2003), pp. 225–237.

[2] G. GENTZEN, *Untersuchungen über das logische Schliessen*, ***Mathematische Zeitschrift*** vol. 39 (1935), pp. 176–210, 405–431.

[3] D. PRAWITZ, ***Natural deduction—A proof theoretical study***, Almqvist & Wiksell, Stockholm, 1965.

[4] ———, *Ideas and results in proof theory*, ***Proceedings of the Second Scandinavian Logic Symposium***, pp. 235–307, North-Holland, Amsterdam, 1971.

► PETR ANDREYEV AND EVGENII GORDON, *A theory of hyperfinite sets.*
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The presented theory implements the idea, naturally arising from the practice of non-standard analysis, that mathematics can be developed within the world of hyperfinite sets, i.e., finite sets some of them being so big that not all their elements and subsets can be well described. This leads to a failure of the induction principle (the well known paradox of a pile of sand). Continuous objects are considered as quotients of sets with respect to some indiscernibility relation. In many aspects the theory THS is close to P. Vopěnka's AST [2].

The primary model for THS is the collection of all subclasses of the set of hereditarily finite sets in the nonstandard class theory NCT [1].

THS is a theory of classes in the ϵ-language, sets being defined as elements of classes. The universe of sets satisfies all the axioms of $\mathrm{ZF}^{\mathrm{fin}}$. The central notion of *thin class* is defined by a formulation equivalent in NCT to the definition of class of standard size. Principles customary in nonstandard analysis such as saturation, standard size choice and others are naturally expressed using the notion of thin class.

All results of classical mathematics that can be obtained in ZC (the Zermelo set theory with the axiom of choice) can be proved for thin structures in THS, which is a substantial difference between THS and AST.

Many results on hyperfinte approximations of topological groups due to the second author can be naturally formalized in THS.

Partially supported by RFBR, grant N 03-01-00757.

[1] P. V. Andreyev and E. I. Gordon, *An axiomatics for nonstandard set theory, based on von Neumann-Bernays-Gödel theory*, ***The Journal of Symbolic Logic***, vol. 66 (2001), pp. 1321–1341.

[2] P. Vopěnka, ***Mathematics in the alternative set theory***, Teubner, Leipzig, 1979.

► DAVID ASPERÓ, *Forcing notions in inner models*.
Institut für Formale Logik, Universität Wien, Währingerstr., 25, A-1090 Wien, Austria.
E-mail: aspero@logic.univie.ac.at.

Theorem 0.1. *Given any stationary subset S of ω_1 there is a forcing notion $\mathbb{P}$ which preserves ω_1 and there is an ordinal α such that, given any $\mathbb{P}$–generic G over V and any forcing notion $\mathbb{Q}$ in V, $V[G] \models \alpha < \omega_2$ and, if forcing with $\mathbb{Q}$ over V collapses α to ω_1, then*

(a) *either forcing with $\mathbb{Q}$ over V makes S nonstationary, or*
(b) *forcing with $\mathbb{Q}$ over $V[G]$ collapses ω_1.*

Theorem 0.1 was motivated by the question (which it answers negatively) whether it is true in general that if $\omega_1 = \omega_1^L$ and $\alpha < \omega_2$, then there is a constructible partial order preserving stationary subsets of ω_1 both over the universe and over L and collapsing α to ω_1 over L.

Theorem 0.2. *The following are equivalent*:

(1) $0^\sharp$ *exists.*
(2) *Given any constructible partial order $\mathbb{P}$, if $\mathbb{P}$ has size ω_1^V in L and forcing with $\mathbb{P}$ over the universe does not collapse ω_1, then there is a filter $G \subseteq \mathbb{P}$ in the universe which is generic over L.*

I proved Theorem 0.1. Most of Theorem 0.2 was proved by Sy D. Friedman.

► JOHN BALDWIN, *Local homogeneity, benign sets and expansions of models*.
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We study the expansion of stable structures by adding predicates for *arbitrary* subsets. Generalizing work of Poizat-Bouscaren on the one hand and Baldwin-Benedikt-Casanovas-

Ziegler on the other we provide a sufficient condition for such an expansion to be stable. This generalization weakens the original definitions in two ways: dealing with arbitrary sets instead of models (Poizat-Bouscaren) indiscernible sets (Baldwin-Benedikt) or sets without fcp (Casanovas-Ziegler) and removing the 'small' or 'belles paires' hypothesis. We use this generalization to characterize in terms of pairs, the 'triviality' of the geometry on a strongly minimal set. Call a set A *benign* if any type over A in the expanded language is determined by its restriction to the base language. We characterize the notion of benign as a kind of local homogeneity. Answering a question of Casanovas-Ziegler we characterize the property that M has the finite cover property over A. This is joint work with Bektur Baizhanov.

► ANATOLY BELTIUKOV, *Polynomially decidable theories of polynomial constructive arithmetic with recurring induction*.
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An approach of building weak arithmetic intuitionistic theories with recurring induction is proposed. The built theories possess the following properties:

— The proof search of any given formula can be fulfilled within the time bounded by a polynomial of the derived formula length.

— The time complexity of any automatically extracted derived formulas witness functions algorithm is also polynomial (in the binary representation length of the input data).

— The theories under consideration are complete with respect to some syntactically defined class of algorithms for witness functions.

This work is supported by INTAS-2000-447 grant.

► ALESSANDRO BERARDUCCI AND TAMARA SERVI, *An effective version of Wilkie's theorem of the complement and some effective o-minimality results*.
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Wilkie [1] proved a "theorem of the complement" which implies that in order to establish the o-minimality of an expansion of $\mathbb{R}$ with C^∞-functions it suffices to obtain uniform (in the parameters) bounds on the number of connected components of quantifier free definable sets. He deduced that any expansion of the real ordered field $\mathbb{R}$ with a family of Pfaffian functions is o-minimal. We prove an effective version of Wilkie's theorem of the complement, so in particular given an expansion of $\mathbb{R}$ with finitely many C^∞ functions, if there are uniform and computable upper bounds on the number of connected components of quantifier free definable sets, then there are uniform and computable bounds for all definable sets. In such a case the theory of the structure is effectively o-minimal: there is a recursively axiomatized subtheory such that each of its models is o-minimal. This implies the effective o-minimality of any expansion of $\mathbb{R}$ with Pfaffian functions. We apply our results to the open problem of the decidability of the theory of the real field with the exponential function. We show that the decidability is implied by a positive answer to the following problem (raised by van den Dries [2]): given a language L expanding the language of ordered rings, if an L-sentence is true in every L-structure expanding the ordered field of real numbers, then it is true in every o-minimal L-structure expanding any real closed field.

[1] A. J. Wilkie, *A theorem of the complement and some new o-minimal structures*, ***Selecta Mathematica. New Series***, vol. 5 (1999).

[2] L. van den Dries, *o-minimal structures*, ***Logic: from foundations to applications (Staffordshire, 1993)***, Oxford Science Publications, Oxford University Press, New York, 1996.

► HOVHANNES R. BOLIBEKYAN AND ANAHIT A. CHUBARYAN, *On some proof systems for I. Johansson's minimal logic of predicates*.
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We introduce some proof systems for I. Johansson's minimal logic of predicates: a sequent system with the cut rule (GM) and a cut-free system $(GM^{\sim})$, a natural system (NM) and a resolution system (RM). GM system is constructed on the basis of $G1$ [1] by dropping right structuring rules and with empty succedent of the conclusion in the left negation rule. $GM^{\sim}$ system is constructed on the basis of intuitionistic $G3$ [1] by claiming that the succedent of the conclusion in the left negation rule must be empty. NM system is introduced on the basis of the natural system for intuitionistic logic of predicates [2] by dropping $\frac{\Gamma\to\perp}{\Gamma\to A}$ rule of inference. RM system is constructed on the basis of resolution system for intuitionistic logic of predicates [2] by dropping $\frac{\Gamma\to\perp}{\Gamma\to A}$ rule of inference and transforming $(\supset^{\sim})$ rule to the following form:

$$\frac{(A\supset B^*)\to C;\Sigma, A^0\to B^*}{\Sigma\to C}\quad (\text{if } A^\circ \text{ is empty then } B^* \text{ is } B).$$

The following theorem is proved.

Theorem. The formula A is deducible in the Hilbert style system M for I. Johansson's minimal logic of predicates [1] if and only if the sequent $\to A$ is deducible in any of the systems GM, $GM^{\sim}$, NM, or RM.

[1] S. C. KLEENE, ***Introduction to metamathematics***, D. Van Nostrand Company, Inc., New York-Toronto, 1952.

[2] G. E. MINTS, *Resolution calculi for nonclassical logics*, ***Semiotika i Informatika***, vol. 25, pp. 120–135, 1985 (in Russian).

► IZABELA BONDECKA-KRZYKOWSKA, *Is structuralism an alternative to platonism in the philosophy of mathematics?*
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Mathematical structuralism is a view that the subject of any branch of mathematics is structure or structures. The slogan connected with this view is that mathematics is the science of structures. For example, we can define a natural numbers system to be a countably infinite collection of objects with one designated initial object and the successor relation that satisfies the principle of mathematical induction. According to structuralism, arithmetic is a science about the form or structure common to natural number systems. Structuralism is often treated as an alternative to platonism in the philosophy of mathematics. The chief motivation of structuralism is to avoid some ontological and epistemological problems of platonism. On the one hand structuralism allows to eliminate platonic problem about the nature of numbers and other mathematical objects and to explain in an easy way how we can get the knowledge of mathematical objects. On the other hand structuralistic theories are not free of problems (usage of second order logic or modal logic). Advantages and disadvantages of structuralism will be considered in the paper.

I. BONDECKA-KRZYKOWSKA, *Structuralistic views in the contemporary philosophy of mathematics. A critical-comparative analysis*, PhD dissertation, Adam Mickiewicz Uniwersity, Poznań, 2002.

G. HELLMAN, ***Mathematics without numbers. Towards a modal-structural interpretation***, Clarendon Press, Oxford, 1989.

M. D. RESNIK, ***Mathematics as a science of patterns***, Clarendon Press, Oxford, 1997.
S. SHAPIRO, ***Foundations without foundationalism: A case for second-order logic***, Oxford University Press, Oxford, 1991.

► ANDREY I. BOVYKIN, *Some $\Sigma^1_1(0')$ characterizations of chronic resplendency*.
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We prove that chronic resplenency is implied by a $\Sigma^1_1(0')$-sentence and that in the class of ω_1-saturated models of any unstable theory, chronic resplendency is equivalent to a $\Sigma^1_1(0')$-sentence. The main tool of our proofs is internal constructions: the analogues of classical theorems and constructions regarding resplendency and recursive saturation conducted inside a model of first-order Peano Arithmetic.

► JEFFREY BURDGES, *A signalizer functor theorem for groups of finite Morley rank*.
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There is a longstanding conjecture, due to Gregory Cherlin and Boris Zilber, that all simple groups of finite Morley rank are simple algebraic groups. One of the major theorems in the area is Borovik's trichotomy theorem. The "trichotomy" here is a case division of the minimal counterexamples within odd type, i.e., groups whose Sylow 2-subgroup is divisible-by-finite, We introduce a charateristic zero notion of unipotence which can be used to obtain a connected nilpotent signalizer functor from any sufficiently non-trivial solvable signalizer functor. This result plugs seamlessly into Borovik's work to eliminate the assumption of tameness from his original trichotomy theorem for odd type groups. This work also provides us with a form of Borovik's theorem for degenerate type groups.

This article has been submitted to the ***Journal of Algebra***. A DVI of the article is online at http://www.math.rutgers.edu/~burdges/sigfunc.dvi

► WESLEY CALVERT, *The isomorphism problem for classes of computable groups and fields*.
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In some classes of structures which are non-classifiable in the Shelah sense, the complexity is in some way so intrinsic to the theory that it shows up even among the countable members, in the form of high Borel complexity. Study of the recursion-theoretic complexity of the isomorphism problem demonstrates cases in which this structural complexity sometimes shows up even among the computable members of the class.

For some classes (linear orders, for instance) the isomorphism problem is Σ^1_1 complete (that is, even the class of computable models is maximally complicated), and for others it is of relatively low complexity. Classes which in this sense are not maximally complicated include the class of models of any strongly minimal theory, Archimedean real closed fields, reduced Abelian p-groups of bounded Ulm length, and finite rank torsion-free Abelian groups.

In this talk I will discuss the calculation of the complexity of the isomorphism problem. Some cases will be discussed in which it is hard to separate the complexity of the isomorphism relation from the complexity of the index set problem for the class.

► ANUJ DAWAR, LAURI HELLA, AND MIKKO SUONIO, *Fixed-point logics with generalized quantifiers on unordered structures*.
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We explore the possibilities of extending the theorem of Abiteboul and Vianu [2] to fixed-point logics with generalized quantifiers. The theorem states that *inflationary fixed-point logic* (IFP) and *partial fixed-point logic* (PFP) are equivalent on all finite structures if and only if they are equivalent on structures of the class $\mathcal{O}$ where one of the relations is an ordering of the universe. Inspired by the work of Grädel and Otto [3], we allow the logics to access a separate ordering by considering the class $\mathcal{FL}$ of disjoint unions of structures and linear orderings, where the length of the ordering is the size of the original structure. We make the following assumption: for every k there exists an interpretation Γ definable in $\mathrm{IFP}(Q^{<\omega})$ and such that all $\mathbf{A}, \mathbf{B}$ in a class $\mathcal{C} \subseteq \mathcal{FL}$ satisfy: $\Gamma\mathbf{A}$ is an ordered structure, and $\mathbf{A} \equiv \mathbf{B}$ in $\mathrm{FO}^k(Q^{<\omega})$ if $\Gamma\mathbf{A} \cong \Gamma\mathbf{B}$ holds. Assuming this *definable invariant* exists we get

$$\mathrm{IFP}(Q^{<\omega}) \equiv \mathrm{PFP}(Q^{<\omega}) \text{ on } \mathcal{O} \Longrightarrow \mathrm{IFP}(Q^{<\omega}) \equiv \mathrm{PFP}(Q^{<\omega}) \text{ on } \mathcal{C},$$

where $Q^{<\omega}$ denotes all vectorizations of the quantifier Q. The left side is equivalent to $\mathrm{PTIME}^Q = \mathrm{PSPACE}^Q$, which refer to computational complexity classes for machines with an oracle for Q.

The result of Abiteboul and Vianu was extended to other fixed-point logics in [1]. We consider the extension of the theorem to (inflationary) *nondeterministic fixed-point logic* (NFP) and PFP extended with a vectorized generalized quantifier. We show that

$$\mathrm{NFP}(Q^{<\omega}) \equiv \mathrm{PFP}(Q^{<\omega}) \text{ on } O \Longleftrightarrow \mathrm{NFP}(Q^{<\omega}) \equiv \mathrm{PFP}(Q^{<\omega}) \text{ on } \mathcal{FL}$$

for any quantifier Q. These are equivalent to the equality $\mathrm{PH}^Q = \mathrm{PSPACE}^Q$, where PH is the polynomial hierarchy, i.e., the classes definable in SO.

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[2] S. Abiteboul and V. Vianu, *Generic computation and its complexity*, ***Proceedings of the 23rd ACM Symposium on the Theory of Computing***, 1991, pp. 209–219.

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► NATASHA DOBRINEN, *A very weak distributive law and a related game in Boolean algebras*.
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Infinitary distributive laws in Boolean algebras are of interest in their own right and also for their equivalences to some useful forcing properties. Jech pioneered the connections between distributive laws involving countable families of maximal incompatible subsets and related infinitary games between two players. Foreman, Kamburelis, and others have added to this body of knowledge.

We will review some results of Jech, Foreman, and Kamburelis along with some of our own regarding generalized distributive laws and related games. We will then present a generalized notion of weak distributivity due to Prikry, namely the hyper-weak (κ, λ)-distributive law, give a forcing equivalence, define a related game, and show that for certain pairs of cardinal numbers, the hyper-weak (κ, λ)-distributive law is equivalent to the non-existence of a winning strategy for the first player in the game. Under GCH, this equivalence holds for all pairs $\kappa > \lambda$. It is consistent with ZFC that, for all $\kappa \geq \lambda$, this game is undetermined, via κ^+-Suslin trees.

► MIRNA DŽAMONJA, *A characterisation of homogeneity in epsilon dense families under MA*.
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We present some recent results on Fremlin's problem of $1/2$-density. Calling a family F of finite subsets of ω_1 $1/2$-dense if every finite subset of ω_1 has a subset in F of size at least $1/2$ the size of the original set, then the question asks if under $\mathrm{MA}(\omega_1)$ it is true that for every $1/2$-dense family F that is closed under subsets, there is an uncountable subset H of ω_1 whose all finite subsets are in F. Such a set H is called homogeneous for F.

We shall give some background and motivation and then present a characterisation under $\mathrm{MA}(\omega_1)$ of those $1/2$-dense families for which there is such an H. The characterisation is given in terms of a covering property.

The results presented are obtained jointly with Grzegorz Plebanek.

► PHILIP EHRLICH, *Surreal numbers and the unification of all numbers great and small*.
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In his monograph *On Numbers and Games* [1], J. H. Conway introduced a real-closed field of *surreal numbers* containing the reals and the ordinals as well as a great many other numbers including $-\omega$, $\omega/2$, $1/\omega$, $\sqrt{\omega}$ and $\omega - \pi$ to name only a few. Indeed, this particular real-closed field, which Conway calls *No*, is so remarkably inclusive that, subject to the proviso that numbers—construed here as members of ordered "number" fields—be individually definable in terms of sets of von Neumann-Bernays-Gödel set theory with Global Choice, it may be said to contain "All Numbers Great and Small." In this paper attention is drawn to the fact that in addition to providing a unifying framework for the reals and the ordinals, the surreals provides a natural setting for Abraham Robinson's infinitesimal approach to analysis as well as for the late nineteenth- and early twentieth-century non-Cantorian theories of the infinite (and infinitesimal) pioneered by Giuseppe Veronese (1891), Tullio Levi-Civita (1892; 1898) David Hilbert (1899) and Hans Hahn (1907) in connection with their work on non-Archimedean ordered algebraic and geometric systems and by Paul du Bois-Reymond (1870/71–82) and Otto Stolz (1883) in connection with their work on the rate of growth of real functions.

[1] J. H. CONWAY, ***On numbers and games***, Academic Press, 1976.

► FREDRIK S. G. ENGSTRÖM, *Omitting types in expansions and related strong saturation properties*.
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For recursive languages and countable structures the two notions recursive saturation and resplendency coincide. We prove that given a countable model $\mathfrak{M}$ in a recursive language which satisfies a rather strong saturation property (still, any recursive theory has countable models with this saturation property), T a recursive theory in some recursive extension of the language of $\mathfrak{M}$ and $p(x)$ a set of formulas (with finitely many parameters from $\mathfrak{M}$). Then, if there exists a model satisfying $T + \mathrm{Th}(\mathfrak{M})$ and omitting $p(x)$ (i.e., satisfying $\forall x \bigvee_{\varphi(x)\in p(x)} \neg\varphi(x)$) whose reduct to the language of $\mathfrak{M}$ is ω-saturated, there is an expansion of $\mathfrak{M}$ satisfying T and omitting $p(x)$.

We also prove that if $\mathfrak{M} \models \mathrm{PA}$ then this expandability property implies that $\mathfrak{M}$ is $\mathfrak{X}$-saturated for some $\mathfrak{X}$ such that the second-order theory (with parameters) of $\mathfrak{X}$ is true in the standard second-order model of arithmetic (we say that $\mathfrak{M}$ is analytically saturated in this case).

Finally we give a characterisation of those countable models of PA which are $\mathfrak{X}$-saturated for a β-model $\mathfrak{X}$ in terms of expandability and omitting types.

► GILDA FERREIRA, *Weak theories of arithmetic and the counting hierarchy*.
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For the past two decades several subsystems of Bounded Arithmetic have been introduced and studied because of their close connection to computational complexity. Starting with a computational complexity class of functions, the idea is to develop a theory which has this class as its provably total functions.

We are specially interested in *FCH*, the *Hierarchy of Counting Functions*.

After presenting an inductive characterization of this class, we define a second order theory, *TCA* (an acronym for *Theory for Counting Arithmetic*) in binary notation, whose provably total functions (having appropriate graphs) are the functions in *FCH* [1]. To establish this connection between *TCA* and *FCH*, we formalize our theory in a modified version of Gentzen's sequent calculus and we use the free-cut elimination theorem.

Our next step is to enrich *TCA* with schemes of bounded collection and recursive comprehension, and express it in a more powerful second order language which enables us to consider infinite sets. To prove that the class of provably total functions in this enlarged system is still *FCH*, we prove some conservation results over *TCA*.

We finish presenting a weak theory for analysis, $TCA^2 + s\Pi^1_1$-Ref, obtained from the previous system by adding the strict-Π^1_1 reflection scheme. The forcing method makes it possible to show that $TCA^2 + s\Pi^1_1$-Ref is first-order conservative over *TCA*.

Our interest in TCA^2 and related theories is based in the conviction that TCA^2 is just enough to develop the Riemann integral.

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► SY D. FRIEDMAN, *Constructible forcing and 0-sharp*.
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We provide a characterisation of the existence of 0-sharp in terms of the existence of generics for certain constructible forcings.

► MICHELE FRIEND, *Some philosophical concerns about the 'confidence' in 'confident learning'*.
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There is a realist assumption made, by those working in learning theory, that, often, there is a function which underlies a given set of data. We shall not question the assumption here. In learning theory, the computer (henceforth: learner) is asked to find the underlying function. Usually there is a lot of data, and it is not obvious whether or not the learner will ever find a function which accords with the data, or how long it will take to do so, if it will. More precisely, when a learner is presented with an initial sample of data, learners are asked to find a programme which will indicate how to generate a hypothesis which will generate the remaining data. This might then be confirmed or denied by the data being fed into the computer. The hope is that, after a while, the computer will learn the underlying function. That is, its data will accord with all the data presented to it. This is called 'convergence' (between the data from the outside and the hypothesized data generated by the learner).

There are some remarkable recent results. Learning theorists have the learner work at two levels. One level is finding the (hypothesis) function which, hopefully, models the data. The other level concerns the 'quality' of the programme: so assessing how 'good' is the hypothesis. That is, some hypotheses might concur with all the data which has so far been given. The hypothesis might concur with some of the data, and consider the rest to be 'white noise' or 'disruptions'. Furthermore, the hypothesis might stabilize (concur with parts of the data) for a while and then diverge from the data again. Some will eventually stabilize: confirm every part of the data. This is called "confident learning". This notion is explored in papers by Frank Stephan and Valentina Harizanov. We say that the learner converges "in the limit".

To the philosopher, eventual stability sounds like something rather close to truth. That is, the philosopher might be tempted to ask in what sense the learner has learned the true function which underpins the data.

There is a problem with 'confident learning' which is recognized by the learning theorists. This is that while the learner will eventually develop a function which generates hypotheses which always converge with the data, the data (input from outside) might be from a function which is not written into the programme (for finding a function). So the function is incorrect, in some sense. (This will happen more especially in BC-learning).

The problem is philosophically quite deep. It is related to "the Skolem permutation argument": which makes plain the misplaced confidence we have in the convergence between hypotheses and data in "confident learning". According to the permutation argument, there are an infinite number of functions which will accord with any finite set of data. The reasoning in the Skolem argument depends on the parameters set for generating a function, just as the programme of a learner can only generate certain types of function to yield hypotheses. This Skolem problem makes explicit some of the limitations of learning theory.

► ANDREY N. FROLOV, *Computable copies of Boolean and Ershov algebras*.
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Feiner showed [4] that a Δ^0_2 Boolean algebra need not have a computable copy. Downey and Jockusch [3] showed that every *low* Boolean algebra does have a computable copy. Thurber [5] showed that every low_2 Boolean algebra has a computable copy.

In my talk I will show that every low_2 Ershov algebra is isomorphic to a computable one. (Distributive lattice with a first element called Ershov algebra if for all $a \leq b$ there exists a' such that $a \wedge a' = 0$ and $a \vee a' = b$. See [1] and [2].).

Besides, in my talk I study computable copies of Boolean algebras and Ershov algebras with several predicates.

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[5] J. J. THURBER, *Every low_2 Boolean algebra has a recursive copy*, ***Proceedings of the American Mathematical Society***, vol. 123 (1995), pp. 3859–3866.

► DAVID GABELAIA AND MICHAEL ZAKHARYASCHEV, *Modal products with* ***GL.3***.
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The *Cartesian product* $L_1 \times L_2$ of two unimodal Kripke complete logics L_1 and L_2 is the bimodal logic determined by the class of all product frames $\mathfrak{F}_1 \times \mathfrak{F}_2$, where $\mathfrak{F}_i$ is a Kripke frame for L_i, $i = 1, 2$. The *commutator* $[L_1, L_2] \subseteq L_1 \times L_2$ of L_1 and L_2 is the independent join of L_1 and L_2 extended with the Church-Rosser and commutativity axioms. (For more details consult [2] and [1].) In this paper, we present some new results on products with the well-known logic **GL.3** (determined by the class of Noetherian strict linear orders).

THEOREM 1. *For every unimodal L, the logics $L \times$ **GL.3** and $[L, \mathbf{GL.3}]$ have precisely the same Kripke frames (but do not necessarily coincide).*

THEOREM 2. *Let L be a modal logic such that either* (i) $L \supseteq \mathbf{K4}$ *and L has an infinite rooted frame, or* (ii) *L has a frame with infinite branching. Then neither $L \times$ **GL.3** nor $[L, \mathbf{GL.3}]$ has the finite model property.*

THEOREM 3. *Both* $[\mathbf{GL.3}, \mathbf{GL.3}]$ *and* $[\mathbf{Grz.3}, \mathbf{GL.3}]$ *are Kripke incomplete.*

THEOREM 4. *Let* $L \in \{\mathbf{K4}, \mathbf{K4.3}, \mathbf{S4}, \mathbf{S4.3}, \mathbf{GL}, \mathbf{Grz}\}$. *Then $L \times$ **GL.3** is undecidable.*

It should be noted that the incompleteness proof of Theorem 3 is indirect. It is based on Theorem 1 and the fact that the logics $[\mathbf{GL.3}, \mathbf{GL.3}]$ and $[\mathbf{Grz.3}, \mathbf{GL.3}]$ are not recursively enumerable (see Theorem 7.16 in [1]).

This work was partially supported by UK. EPSRC grant no. GR/R42474/01.

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► ÇIĞDEM GENCER, *Description of modal logics inheriting admissible rules for $K4$ and tabularity*.

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Investigation of inference rules for non-standard logics has different aspects. One of them is the exploration of admissible inference rules. These rules do not depend on the choice of axiomatic systems for any logic λ. At the beginning the attention was focused on the algorithms and tools for recognizing admissibility of inference rules. It turns out that the majority of superintuitionistic logics, as well as intuitionistic logics, and most of transitive modal logics with finite model property (fmp) have algorithms determining admissibility [2]. Next it was natural to study the preservation of admissible rules. In 1993, Rybakov addressed this problem for superintuitionistic logics [2]. He gave a complete description of all superintuitionistic logics with fmp which preserve all rules admissible in intuitionistic logics [2]. Then similiar description for modal logics inheriting admissible rules of $S4$ was given in [2] and [3]. In [1], a necessary and suffcient condition for any modal logic with fmp to inherit all inference rules admissible in $K4$ was established. In this paper, in the same spirit, we give the definition of co-covers poster for tabular logics extending $K4$ and co-cover property for these logics.

This work supported by Boğaziçi University.

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► NORMA B. GOETHE, *Some considerations concerning the mathematical study of induction and its riddles*.
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Putnam's early methodological work shows some prima facie puzzling sounding features. In his critique of Carnap's inductive logic, a critique guided by reflections on the role of simplicity in inductive inference, Putnam argues that one can show that no definition of degree of confirmation "can be adequate or can attain what any reasonably good inductive judge might attain without using such a concept". The arguments rely on recursive-theoretic considerations.

Yet, Putnam's conclusions contain some strong negative philosophical morals about 'mechanically' following computable prediction methods in science. Much against the grain of such conclusions, learning theorists regard Putnam's 1963 article today as a pioneer piece of work for having made a lasting contribution of this new field of research. What was the point? In his arguments against Carnap, Putnam exploits an important, thus far ignored analogy between the concepts of computability and induction. Ironically, to formal learning theorists these results showed, just how promising the study of effective rules of inductive inference could be, when approached from a recursion-theoretic perspective without making any recourse to probability or degrees of belief.

The analogy between induction and computation points to the mathematical study of induction, and as in the mathematical study of deduction, the objects of inductive reasoning become problems and relations of relative difficulty between mathematical structures. No space seems left for philosophical concerns. Yet there are some issues to be raised here. Some of the results of the mathematical study of induction are very different from what we would expect in classical formal inquiry, which excluded any uncertainty in formal results. In this regard, the formal learning theorist's proposal to mathematize induction points to some puzzling questions, which more often than not seem to rehearse the classical riddles of induction.

► MARTIN GOLDSTERN, *The Galois connection between relations and automorphism*.
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Fix a base set A. Consider the Galois connection induced by the relation "p is a permutation of A, R is a relation on A, and p is an automorphism for the R" (i.e., preserves R and its complement).

The Galois-closed sets on the permutation side are easily described. On the relational side, it is clear that any Galois-closed set is closed under intersections, and is a Krasner algebra (closed under all logical operations).

For finite or countable base sets A, also the converse holds.

I will mainly discuss the case of uncountable sets. I will give a plausible conjecture that attempts to characterize all Galois-closed sets of relations, sketch a counterexample to the conjecture, and formulate the correct characterization.

This is joint work with Ferdinand Börner and Saharon Shelah.

► EVAN J. GRIFFITHS, *Characterising algorithmic randomness*.
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There are several notions of algorithmic randomness for real numbers, including the influential concepts defined by P. Martin-Löf and by C.-P. Schnorr. We identify reals with their infinite binary expansion, so consider them as infinite strings. Randomness is usually defined either in terms of measure theory (random strings should not be in effective null sets), compression (initial segments of random strings should be hard to describe) or martingales (the bits of a random string should be hard to predict).

We will consider briefly the motivation for the definitions of Martin-Löf and Schnorr, and the intermediate notion of *computable randomness*. We present characterisations of Schnorr-random reals and computably-random reals in terms of Turing machines (the compression approach) and effective null sets (the measure theory or 'effective statistical test' approach). We look at the connections between the various characterisations, and give related results on *computably enumerable* random reals. We will look at related work in Kolmogorov complexity, halting probabilities, Turing degrees, and 'Kurtz-randomness', as time permits.

This is joint work with Rod Downey (Victoria University of Wellington, New Zealand) and Geoff LaForte (University of West Florida, USA).

► MILAN Z. GRULOVIĆ, *A word on infinite forcing*.
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We introduce two new notions in the theory of infinite forcing: n-infinite forcing and the reduced products of infinite forcing systems.

The first one is a natural generalization of Robinson's infinite forcing which preserves all its crucial properties in the sense of their "translation" for n. It holds as well that in obtaining the n-infinite forcing companion of a given theory, from a purely technical point of view, the "standard" infinite forcing relation is quite appropriate tool. However, the examination of n-infinite forcing companions of a given theory for different natural numbers n and their mutual relations certainly can be of use in the research of the theory and its models.

The notion of reduced products of (n-)infinite forcing systems is patterned on the previously defined notion of reduced products of forcing systems. Though, because of its semantical nature, we deal with quite different type of questions, there are some "common" results. Anyhow, the Łos theorem is here (always) preserved and under some (natural) conditions we are given the propositions considering the infinitely generic models of ultraproducts of infinite forcing systems which correspond to the known ones from the theory of (n-)infinite forcing.

► EDWARD HERMANN HAEUSLER AND LUIZ CARLOS PEREIRA, *Multiplicative quantifiers*.
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First Order Classical Linear Logic works with "additive" quantifiers. In section 7 of [1], Andreas Blass admits that a multiplicative version of the universal quantifier could be formulated, but explicitly calls into question the plausibility of a multiplicative version of the existential quantifier. The aim of this paper is to define an extension of Linear Logic that contains multiplicative versions of both the universal and the existential quantifiers. We formulate a Sequent Calculus for these multiplicative quantifiers and prove soundness and completeness of the system with respect to ∞-phase structures. Such structures have an infinitary phase space (defined in [2] as the underlying semantics for an infinitary extension of CLL) as the truth set. The paper is organized as follows: in section 1 we present the language and the semantics for Classical Linear Logic with Multiplicative Quantifiers (MQCLL); in section 2 we formulate a Sequent Calculus for MQCLL; in section 3 we prove soundness and completeness; and, finally, in section 4 we carry out the proof theory for MQCLL.

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► ALEX HELLSTEN, *Killing a weakly compact set*.
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Associated with weakly compact cardinals is an ideal over the cardinal in question. The sets that have positive measure with respect to this ideal are said to be *weakly compact*. The ideal can be characterised through Π^1_1-indescribability in the same way as the weak compactness of the cardinal itself. Namely, a set $X \subseteq \kappa$ is weakly compact iff every Π^1_1-sentence true at κ, reflects down to some $\alpha \in X$. The weakly compact ideal is normal and thus it extends the ideal of non-stationary sets.

A basic idea used in many forcing arguments is the fact first presented in [1], that any bistationary subset of $\aleph_1$ can be "killed" by adding a closed unbounded set to its complement. This can not be generalised to $\aleph_2$ or other regular cardinals as such, as various side conditions on the set to be killed become necessary.

But the idea generalises to the weakly compact ideal. A set that will be in the weakly compact filter can be added into any weakly compact set, using a forcing notion that preserves weak compactness of the cardinal in question. No assumptions besides the weak compactness of the original set are needed. Thus any weakly compact set that has a weakly compact complement may be killed by forcing.

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► MARKUS JUNKER, *Reducts of* $(\mathbb{Q}, <)$.
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I present a concise and in many details new proof classifying the reducts of the structure $(\mathbb{Q}, <)$ and possibly some generalisations. (In collaboration with Martin Ziegler).

► VLADIMIR KANOVEI, *On rod-hyperfinite equivalence relations*.
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It is known since [3] that classical theorems on Borel and analytic sets tend to generalize to all projective, generally, all real-ordinal definable (ROD) sets in the Solovay model. In particular, as one of the authors demonstrated in [1], the fundamental Harrington-Kechris-Louveau theorem on Glimm-Effros classification of Borel equivalence relations admits such a generalization (although not straightforward). The prefix "ROD-" below means that ROD maps are meant instead of Borel maps, as in the ordinary definitions, for instance, an equivalence relation E is called ROD-hypersmooth if there exists a ROD reduction of E to the equatity on the reals.

THEOREM. *In the Solovay model, if* E *is a ROD-hypersmooth equivalence relation then either* $\mathsf{E} \leq_{\mathrm{ROD}} \mathsf{E}_0$ *or* $\mathsf{E} \sim_{\mathrm{ROD}} \mathsf{E}_1$. *The two cases are incompatible.*

This is a partial generalization of a fundamental result on the Borel reducibility, saying that any Borel hypersmooth equivalence relation E satisfies either $\mathsf{E} \leq_{\mathrm{B}} \mathsf{E}_0$ or $\mathsf{E} \approx_{\mathrm{B}} \mathsf{E}_1$ (Theorem 2.1 in [2], also known as "the third dichotomy theorem"). The generalization is not complete: due to a simple counterexample, E is not necessarily ROD-hyperfinite in the "either" case.

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► BART KASTERMANS AND YI ZHANG, *Very MAD families*.
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In this talk we will present a class of maximal almost disjoint (mad) families in the Baire space $\mathbb{N}^{\mathbb{N}}$.

DEFINITION. $f_1, f_2 \in \mathbb{N}^{\mathbb{N}}$ are *almost disjoint* iff

$$|f_1 \cap f_2| = |\{n \in \mathbb{N} \mid f_1(n) = f_2(n)\}| = \aleph_0.$$

A family $\mathcal{A} \subseteq \mathbb{N}^{\mathbb{N}}$ is *almost disjoint* iff every two functions in $\mathcal{A}$ are almost disjoint.

An $f \in \mathbb{N}^{\mathbb{N}}$ is *not finitely covered* by an $\mathcal{A} \subseteq \mathbb{N}^{\mathbb{N}}$ iff for every finite set of functions $g_1, \ldots, g_n \in A$ the set $f \setminus \bigcup_{1 \leq i \leq n} g_i$ is infinite. A family $F \subseteq \mathbb{N}^{\mathbb{N}}$ is *not finitely covered* by an $\mathcal{A} \subseteq \mathbb{N}^{\mathbb{N}}$ iff all $f \in F$ are not finitely covered by $\mathcal{A}$.

A family $\mathcal{A} \subseteq \mathbb{N}^{\mathbb{N}}$ is *strongly mad* iff for every countable $F \subseteq \mathbb{N}^{\mathbb{N}}$ that is not finitely covered by $\mathcal{A}$, there is a $g \in \mathcal{A}$ such that for all $f \in F$ the set $f \cap g$ is infinite.

A family $\mathcal{A} \subseteq \mathbb{N}^{\mathbb{N}}$ is *very mad* iff for every $F \subseteq \mathbb{N}^{\mathbb{N}}$ satisfying $|F| < |\mathcal{A}|$ and F is not finitely covered by $\mathcal{A}$, there is a $g \in \mathcal{A}$ such that for all $f \in F$ the set $f \cap g$ is infinite.

NOTE. The notion of strongly mad family was introduced by Juris Steprāns to study the question (asked by Yi Zhang) whether there are analytic mad families (the question is still open).

Obviously the notion of very mad family is a strengthening of the notion of strongly mad family. Also obviously, strongly mad families are mad.

We will present several basic results about the cardinality of very mad families, in particular we will outline the proof of the following theorem.

THEOREM. *If M is a model of* ZFC, *and in M we have a regular cardinal κ such that $\aleph_1 \leq \kappa \leq 2^{\aleph_0} = \lambda$, then there is a c.c.c. forcing $\mathbb{P}$ such that the model $M^{\mathbb{P}}$ of* ZFC *satisfies*:

- *There exists a very mad family $\mathcal{A}$ of cardinality κ and $\lambda = 2^{\aleph_0}$.*

We will also ask several open questions concerning these two types of families.

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In one of his works A. Dragalin [1] suggested a new very general approach to construct models for non-standard logics (first for intuitionistic logic), which are called "functional pseudoboolean algebraic models". Such models agree very well with intuitionistic logic and with intuitionistic arithmetic. Many realizability models are functional pseudoboolean algebraic models (for example, the numerical realizability of Kleene-Nelson, the realizability of Beeson, the realizability of Lifschitz and others). Also, many other models of HA are functional pseudoboolean algebraic models (such models are not pseudoboolean-valued models of Grayson for a set theory). But |-realizability of Kleene [2] is not a functional pseudoboolean algebraic model and we proved this fact.

In my report I am going to give many examples of functional pseudo boolean algebraic models for HA and generalization such models for a set theory.

[1] A. DRAGALIN, *Functional algebraic models*, ***Semiotics and Informatics*** vol. 13, (1979) pp. 184–195 (in Russian).

[2] S. C. KLEENE, *Realizability: a retrospective survey*, ***Lecture Notes in Mathematics***, vol. 337 (1973), p. 96

► ASYLKHAN KHISAMIEV, *On quasiresolvable models*.
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In [1], Yu. L. Ershov has introduced the definition of a quasiresolvable admissible set and has obtained the following results on such sets: if an admissible set $\mathbb{A}$ is quasiresolvable, then there exists a binary Σ-function which is universal for the class of unary Σ-functions; if $\mathfrak{M}$ is a model of a decidable model complete theory, then the admissible set $\mathbb{HF}(\mathfrak{M})$ is quasiresolvable. The definition of a quasiresolvable model was introduced by the author in [2], where the following result was obtained: a model $\mathfrak{M}$ is quasiresolvable if and only if the admissible set $\mathbb{HF}(\mathfrak{M})$ is quasiresolvable. Thus, the study of quasiresolvable models seems to us to be of a some interest. Here, we would like to present the following two parts of our investigation:

1. We introduce the notion of a 1-quasiresolvable model and the notion of a quasiregular model and we establish some relations between them;
2. We describe the quasiresolvable models in the class of countable Boolean algebras and in the class of countable Abelian p-groups.

Partially supported by grants: RFBR 02-01-00593, INTAS 00-499, Russian Ministry of Education PD02-1.1–201.

[1] Yu. L. Ershov, ***Definability and computability***, Plenum, New York, 1996.
[2] A. N. Khisamiev, *On quasiresolvable models and B-models*, ***Algebra and Logic***, vol. 40 (2001), no. 4, pp. 322–334.

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Explaining biological functions is one of the most important problems in philosophy of biology. It is not easy to define functions of biological organs because function is closely related to our recognition or intention. The notion of function can be considered as a sort of modality. That is, *F is a behavior of a* and *F is a function of a* can be represented by $F(a)$ and $\Box F(a)$ respectively. Several important philosophical definitions of function have been proposed in these decades. Millikan's proper function [3] is one of them. The concept of proper function does not intend to be a philosophical analysis of our intuition about the notion of function, but it is proposed as a formal framework with which we can use the notion of function. We analyze the concept of proper function from the viewpoint that it is a definition of modality. We discuss also its relation to Kripke's causal theory of reference [2] and to the provability predicate in arithmetic [1].

[1] J. Boolos, ***The logic of provability***, Cambridge University Press, 1995.
[2] S. A. Kripke, ***Naming and necessity***, Harvard University Press, 1980.
[3] R. G. Millikan, ***Language, thought, and other biological categories: new foundations for realism***, MIT Press, 1984.

► LESZEK ALEKSANDER KOŁODZIEJCZYK, *Truth Definitions in finite models*.
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The concept of *finite model truth definitions* (or *FM-truth definitions*) was introduced by M. Mostowski in an attempt to transfer Tarski's "truth-definition" method of comparing the expressive power of logics to finite model theory. We discuss what it means for a formula to be an "FM-truth definition for a logic L over vocabulary σ", and try to compare the notion of FM-truth definitions for a logic with Vardi's notion of the *combined complexity* of a logic. One important difference is that whether a given language defines FM-truth for (i.e., has an FM-truth definition for) a logic L depends only on the expressive power of L and not on its syntax.

We also characterize the logics for which some natural fragments of second and higher order logic can define FM-truth. For example, Σ^1_1 defines FM-truth for a logic L over vocabulary σ if and only if there exists a fixed k such that model checking for all L-sentences over σ can be done in nondeterministic time n^k.

As a side result, we show that if there is any non-decreasing recursive function f growing to infinity such that $DSPACE(O(log(n)f(n)))$ is contained in NP, then Σ^1_1 defines FM-truth for first order logic over every vocabulary (and hence, for any vocabulary there exists a k such that model checking for any FO sentence over this vocabulary requires nondeterministic time at most n^k).

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J. Kearns [3] and Yu. Ivlev [2] in different ways suggested an idea of modal semantics which is different from Kripke-style semantics in that it does not use the idea of *possible worlds*. Instead of the alternative worlds [2] one deals with alternative interpretation quasi-functions formed by the given interpretation function. We present the two systems of quasi-matrix monadic deontic logic, S_{3qd} and S_{5qd} and discuss some informal questions concerning these systems. To avoid *deontic paradoxes*, which are presented by the formulas $(OA \wedge O\neg A) \supset OB$, $O(A \wedge \neg A) \supset OB$, we construct the systems S_{3qrd} and S_{5qrd} on the basis of the *semantics of truth value gluts*.

Consider the model $M = \langle W, \rho \rangle$, where W is some non-empty set of possible worlds, and ρ is an assignment function assigning to each variable p a tuple of subsets of W, $\langle \rho_t(p), \rho_t(p) \rangle$. Standard semantics of classical two valued logic includes the two requirements: (1) $\rho_t(p) \cap \rho_f(p) = \emptyset$ and (2) $\rho_t(p) \cup \rho_f(p) = W$. Acceptance of the condition (1) and refusal of (2) leads to the semantics with *truth value gaps* (**TGp**), acceptance of (2) and refusal of (1) gives the semantics with *truth value gluts* (**TGl**) (*impossible possible worlds*, in terms of J. Hintikka [1]), refusal of both conditions (1) and (2) gives the semantics with *relevance* (**SR**). Given a model M and a world $w \in W$, the conditions for the formula A to be true or false are defined as: (1) $(M, w) \models_t A$ iff $w \in \rho_t(A)$; (2) $(M, w) \models_f A$ iff $w \in \rho_f(A)$. The formula of the type $A \rightarrow B$ is *valid* in (**TGp**, **TGl**, **SR**) if and only if, for each world w, if $w \in \rho_t(A)$ then $w \in \rho_t(B)$. Formalization of these semantics gives the systems **HW,DHW,M**.

We construct the deontic systems S_{3qrd} and S_{5qrd}, which semantics represents combination of quasi-matrix semantics and the semantics of truth value gluts and which are free of the above paradoxes.

[1] J. Hintikka, *Impossible possible worlds vindicated*, ***Journal of Philosophical Logic***, vol. 44 (1975), pp. 475–484.

[2] Yu. V. Ivlev, *Modal logic*, Moscow University Publ., Moscow, 1991 (in Russian).

[3] J. Kearns, *Modal semantics without possible worlds*, ***The Journal of Symbolic Logic***, vol. 46 (1981), no. 1, pp. 77–86.

[4] A. M. Kouznetsov, *Quasi-matrix deontic logic*, PhD-thesis, Moscow State University, 1998 (in Russian).

► GYESIK LEE AND ANDREAS WEIERMANN, *Worms, tree-ordinals and the Schütte-Simpson ordinal notation system for ϵ_0*.
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Using results by A. Weiermann and T. Arai we obatain *PA* independence results about Worm principles, Hydra battle and the Schütte-Simpson ordinal notation system for ϵ_0. The Worm principle was introduced by L. Beklemishev. It will turn out that the three principles are identical modulo some isomorphisms. The isomorphisms developed independently by Schütte-Simpson and Beklemishev are indeed the same. Via these isomorphisms we achieve the same kind of independence results as the ones by Weiermann and Arai.

[1] T. ARAI *On the slowly orderedness of ϵ_0* ***Mathematical Logic Quarterly***, vol. 48(2002) no. 1, pp. 125–130.

[2] L. D. BEKLEMISHEV, *The Worm principle*, to appear, 2003.

[3] K. SCHÜTTE AND S. G. SIMPSON, *Ein in der reinen Zahlentheorie unbeweisbarer Satz über endliche Folgen von natürlichen Zahlen*, ***Archiv für mathematische Logik und Grundlagenforschung***, vol. 25 (1985), nos. 1–2, pp. 75-89.

[4] A. WEIERMANN, *An application of graphical enumeration to PA*, ***The Journal of Symbolic Logic***, vol. 68 (2003), no. 1 pp. 5–16.

► STEFFEN LEMPP, *Extensions of embeddings in the Σ^0_2-enumeration degrees*.
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I will present a complete solution to the so-called extension of embeddings problem for the Σ^0_2-enumeration degrees, i.e., the problem of deciding for which finite partial orders $P \subset Q$, any embedding of P into the Σ^0_2-enumeration degrees can be extended to an embedding of Q into the Σ^0_2-enumeration degrees. There are three conditions on P and Q, the first two of which are the same as the two conditions of Slaman and Soare for the computably enumerable Turing degrees, and the last of which is a generalization of a nonsplitting phenomenon first discovered by Ahmad.

Joint work with Theodore A. Slaman (University of California-Berkeley) and Andrea Sorbi (University of Siena).

► GIACOMO LENZI AND ERICH MONTELEONE, *On the analytic hierarchy, μ-Calculus and Infinite Time Turing Machines*.
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In this talk we deal with the analytic hierarchy, μ-Calculus and Infinite Time Turing Machines. First we show that every set of reals definable in the level Π^1_1 of the analytic hierarchy is definable by a formula of Arithmetic μ-Calculus as well. Then we show that every set of reals definable by a formula of Arithmetic μ-Calculus is decidable by an Infinite Time Turing Machine. As a corollary, the latter result holds also for the Modal μ-Calculus.

It is known that every set decidable by Infinite Time Turing Machines is Δ^1_2. So we have a chain of inclusions from Π^1_1 to μ-Calculus to Infinite Time Turing Machines to Δ^1_2; we will see that all these inclusions are proper.

[1] J. D. HAMKINS and A. LEWIS, *Infinite Time Turing Machines*, ***The Journal of Symbolic Logic***, vol. 65 (2000), no. 2, pp. 567–604.

[2] R. LUBARSKY, *μ-Definable sets of integers*, ***The Journal of Symbolic Logic***, vol. 58 (1993), pp. 291–313.

[3] E. MONTELEONE, *La Super Macchina di Turing e la gerarchia analitica*, Tesi di Laurea, Università di Pisa, 1999 (in Italian).

▶ THIERRY LIBERT, *A note on positive set theory*.

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As far as we know the consistency problem for *positive* comprehension principles originates in the pioneering work by Skolem in Łukiasiewicz logics, e.g., [1]. Without any references to Skolem, this problem was reinvestigated much later in [2] where very strong topological models called *hyperuniverses* were constructed by using a technique similar to Scott's one for the untyped λ-calculus. The aim of the talk is to review and compare such *classical* attempts at providing a universe of type-free sets, in order to point out and distinguish, at least in the context of a positive logic, two ways of formalizing set theory, namely *abstraction* and *comprehension*. As shown in [3], the consistency of positive abstraction with *extensionality* can only be achieved by dropping the equality out of the formulae defining sets, while hyperuniverses really appear as the most natural extensional models of positive comprehension with equality in the language. The limit of combining positive abstraction and/or comprehension with extensionality is so established.

[1] T. SKOLEM, *Studies on the axiom of comprehension*, ***Notre Dame Journal of Formal Logic***, vol. 4 (1963), pp. 162–170.

[2] M. FORTI and R. HINNION, *The consistency problem for positive comprehension principles*, ***The Journal of Symbolic Logic***, vol. 54 (1989), pp. 1401–1418.

[3] R. HINNION and T. LIBERT, *Positive abstraction and extensionality*, ***The Journal of Symbolic Logic*** (to appear).

▶ DOMINIKA MAJSAK AND MARCIN MOSTOWSKI, *Logics with semilinear generalized spectra*.

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Let σ_k be a vocabulary with k unary predicates. For a model $M = (U, P_1, \dots, P_k)$ of σ_k we define the vector $v_M \in \omega^{2^k} - \{(0, \dots, 0)\}$, $v_M = (v_1, \dots, v_{2^k})$ such that the i-th coordinate is the cardinality of the i-th constituent of M. Since such vectors identify σ_k-models uniquely up to isomorphisms, we identify a model M with the vector v_M. Then the generalized spectrum of φ is the set $\{v \in \omega^{2^k} - \{(0, \dots, 0)\} : v \vDash \varphi\}$.

Let $MSO1F$ be the fragment of second order logic with infinitely many unary second order variables and one unary function variable which can be quantified in an arbitrary way allowing iterating quantification.

THEOREM 1. *Generalized spectra of $MSO1F$ in vocabulary σ_k are exactly semilinear subsets of $\omega^{2^k} - \{(0, \dots, 0)\}$.*

As a corollary we obtain:

THEOREM 2. *MSO enriched by Krynicki quantifiers $(MSO(F_\omega))$ in vocabulary σ_k has the generalized spectra that are exactly semilinear subsets of $\omega^{2^k} - \{(0, \dots, 0)\}$.*

This theorem generalizes one of the main theorems from [2]. As other corollaries one can obtain the main theorems in [3] and [1].

[1] Y. Gurevich and S. Shelah, *Spectra of monadic second-order formulas with one unary function*, http://research.microsoft.com/gurevich/Opera/162.ps, 2002.

[2] M. Krynicki and M. Mostowski, *Decidability problems in languages with Henkin quantifiers*, ***Annals of Pure and Applied Logic***, vol. 58 (1992), pp. 149–172.

[3] A. Durand, R. Fagin, and B. Loescher, *Spectra with only unary function symbols*, (M. Nielsen and W. Thomas, editors), Lecture Notes in Computer Science, vol. 1414, Springer-Verlag, 1998, pp. 189–202.

► GIAN ARTURO MARCO, *Ideals of closed sets and automorphism group actions*.
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This paper concerns a research on the characterization of uncountable cardinals from the Scott sentence (Ss) of the orbits generated by the automorphism group of a countable structure. The paradigmatic strain "countable–uncountable", unraveled by the complexity of the automorphism group acting on L-countable structures, for L a countable language, is located in the crossroads of the topological Vaught conjecture (TVC), the analysis of non closed left invariant (non c.l.i.) subgroups of $\omega!$ (the symmetric group of permutations on ω equipped with the topology of pointwise convergence), and the characterization of the size of saturated models for Ss-completions.

This research is in the streamline of works on non c.l.i. group actions; in particular its release point is a question raised by G. Hjorth [1] on the necessary existence of models of cardinality the continuum for the Ss of non c.l.i. automorphism groups whose corresponding spaces of transformations violate the TVC. Recently G. Hjorth answered negatively his question by showing that for each countable α, there is a countable model M "characterizing" α; thus, in these cases, the Ss.ϕ of $\mathrm{Aut}(M)$ has no "perfect" model (i.e., a model of cardinality the continuum).

My contribution to the field consists in finding a sufficient condition for $\phi(\mathrm{Aut}[M])$ to have a "perfect" model, where $\mathrm{Aut}(M)$ is any automorphism group whose action is not closed in the Baire space.

Our approach links results on the ideals of closed sets in compact spaces with the analysis of non c.l.i. group actions; in particular we focus on the strength of the notion of calibration for ideals of closed sets in compact spaces to the "closed approximation property" for coanalytic (in the codes of Borel sets) ideals of closed sets with a Borel basis. These concepts from the theory of ideals on Polish spaces turn out to be fruitful in the analysis of groups which don't contain all there limit points and at the same time, they widen the usual perspective merely applying to the ideal of meager sets.

Let G be a non c.l.i. group acting on a Polish space X, A an analytic G-invariant subset of X, and I a coanalytic ideal on $|k(A)/G$. We prove that if I has not the closed approximation property, than A embeds a perfect collection of G-closed nowhere dense sets outside of I.

As a corollary it follows that, whenever $G = \mathrm{Aut}(M)$ for some countable structure M, $\phi(\mathrm{Aut}[M])$ has unboundedly many realizations in ω_1, thus avoiding all the possible α cardinal characterizations. We then prove the relative consistency with ZFC + GCH of the following statement: if $\phi(\mathrm{Aut}[M])$ has unboundedly many realizations in ω_1, then $\phi(\mathrm{Aut}[M])$ has a "perfect model".

The above results can be partly interpreted as particular instances of the Hurewicz type separation theorem for ideals of closed sets.

[1] G. Hjorth, *Orbit cardinals: on the effective cardinalities arising as quotient spaces of the form X/G where G acts on a Polish space X*, ***Israel Journal of Mathematics***, vol. 111 (1999), pp. 221–261.

► MARIA DA PAZ NUNES DE MEDEIROS, *Modal logic systems in natural deduction*.
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One of the first formulations in natural deduction for modal logic was developed by Prawitz [1]. He presented three different formulations and showed that just one satisfied the Normalization Theorem. However, we noted, through an example, that the reductions involving the classical absurd rule did not work very well. We aim to present such example as well as a new S4 classical modal logic system in natural deduction logically equivalent to Prawitz's system and to show that every derivation in this new system can be transformed into normal derivation.

[1] DAG PRAWITZ, ***Natural Deduction***, Almqvist & Wiksel, Stockholm, 1965.

► ANDERS MOEN, *A normal form for Gödel Löb sequent calculus*.
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Propositional sequent calculus augmented with modal rules for proving theorems in provability logic was investigated in [1] and [2]. With the Gödel Löb Rule (GLR), Löb's derivability conditions follows, hence Gödel Löb sequent calculus (GLS) captures provability logic.

Algorithms for eliminating cuts in GLS rely on the possibility of transforming modal cuts into Sambin Normal Form (SNF). A cut is on SNF if the cut formula is a modal formula and the final inference rule in each subtree is an application of GLR. But no argument is given in the literature whether it is possible to rewrite an arbitrary proof in GLS to a proof where every cut is on SNF.

The paper presents an algorithm for transforming a GLS proof to one where every modal cut is on SNF. The concept of 'region' is introduced to capture blocks of inferences within proofs. The most interesting regions are the modal-free closures of proofs, i.e., the regions with no applications of the GLR rule. Moreover, we define algebraic operations on regions within proofs, in order to argue that every proof actually can be mapped to a proof where every modal cut is on SNF. The most significant operator, region multiplication, forms an abelian monoid. In the end we consider arbitrary modal sequent calculi, based on propositional logic, and observe that our algorithm applies to all of them. Our algorithm introduces a number of new cuts, depending on the number of branching rules in the modal-free closures of each subtree of the original cuts.

[1] DANIEL LEIVANT *On the proof theory of the modal logic for arithmetic*, ***The Journal of Symbolic Logic***, vol. 46 (1981), pp. 531–538.

[2] GIOVANNI SAMBIN AND SILVIO VALENTINI *The modal logic of provability: The sequential approach*, ***Journal of Philosophical Logic***, vol. 11 (1982), pp. 311–342.

► MARCIN MOSTOWSKI AND ANNA WASILEWSKA, *Elementary properties of divisibility in finite models*.
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We consider finite arithmetic with divisibility relation $|$ only. It is determined by the family of models $M_n = (\{0,\ldots,n-1\}, |_n)$, for $n > 0$. The relation $|_n$ is obtained by restricting $|$ to the universe $\{0,\ldots,n-1\}$. We prove that

THEOREM 1. *Finite arithmetic with addition and multiplication is interpretable in finite arithmetic with divisibility relation only.*

As corollaries we obtain the following

THEOREM 2. *The set of formulae true in all models M_n, $n > 0$, is Π^0_1-complete.*

THEOREM 3. *The set of formulae true in all sufficiently large models M_n is Σ^0_2-complete.*

We follow the papers [1], [2], [3].

[1] M. KRYNICKI, and K. ZDANOWSKI, *Theories of arithmetics in finite models*, in preparation.

[2] M. MOSTOWSKI, *On representing concepts in finite models*, ***Mathematical Logic Quarterly***, vol. 47 (2001), pp. 513–523.

[3] M. MOSTOWSKI and K. ZDANOWSKI, *Beyond FM–representability*, in preparation.

► VETA MURZINA, *The completeness theorem for temporal logic based on strictly ordered f-spaces*.
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The temporal logic based on f-spaces is considered. The concept of f-space was introduced by Ershov [1] with the purpose to develop the theory of computable functionals. It is known that the triple $\langle X, X_0, \leq\rangle$, where $\leq$ is a partial order on X and $X_0 \subseteq X$, is defined by f-space if and only if some special conditions are possessed. If the least element exists in X then f-space is called f_0-space [1].

Frames $\langle X, X_0, R\rangle$ are considered being similar to Kripke's frames. If the frame $\langle X, X_0, \leq\rangle$ is a linearly ordered f-space or a linearly ordered f_0-space then the frame $\langle X, X_0, R\rangle$, where for any $x, y \in X$ $(xRy \Longleftrightarrow x \leq y$ and $x \neq y)$, is called a strictly ordered f-frame or a strictly ordered f_0-frame respectively. In [2] an axiomatization of modal logics associated with strictly ordered f-frames or strictly ordered f_0-frames was obtained.

Now the temporal modalities $\Diamond$, $\Box$ associated with the relation R and $\Diamond^*$, $\Box^*$ associated with the relation R^{-1} are introduced. The set X_0 is represented as a constant β. We define a calculus L' obtained by adding the following axioms to the minimal temporal calculus K_t:

(1) $\Diamond\Diamond A \longrightarrow \Diamond A$; (1*) $\Diamond^*\Diamond^* A \longrightarrow \Diamond^* A$;
(2) $\Box(\Box A_1 \longrightarrow A_2) \vee \Box(A_2 \& \Box A_2 \longrightarrow A_1)$;
(2*) $\Box^*(\Box^* A_1 \longrightarrow A_2) \vee \Box^*(A_2 \& \Box^* A_2 \longrightarrow A_1)$;
(3) $\Diamond A_1 \& \Diamond A_2 \longrightarrow \Diamond(\beta \& (\Diamond A_1 \vee A_1) \& (\Diamond A_2 \vee A_2))$;
(4) $\Diamond^* \beta \vee \beta$.

A calculus L_0' is obtained from K_t by adding the axioms (1)–(3), (1*), (2*) of L' and the following axiom: $\Diamond^*(\beta \& \Box^* \bot) \vee (\beta \& \Box^* \bot)$.

Completeness theorem (a) The calculus L' is complete with respect to the class of all strictly ordered f-frames. (b) The calculus L_0' is complete with respect to the class of all strictly ordered f_0-frames.

Also we prove that L' and L_0' have the finite model property, and therefore, they are decidable.

This work is sponsored by the Russian Foundation for Basic Research, grant 03-06-80178.

[1] YU. L. ERSHOV, *Theory of enumerations*, 1973, Novosibirsk University, vol. 2, p. 169.

[2] V. MURZINA, *The completeness theorem for modal logic based on linearly ordered f_0-spaces*, ***Collegium Logicum Annals of the Kurt-Gödel-Society***, vol. 4, Vienna 2001, p. 137.

► NICK PEATFIELD, *Analytic Zariski structures and the Hrushovski construction*.
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A set of axioms is presented defining an 'analytic Zariski structure', as a generalisation of Hrushovski and Zilber's Zariski structures. Some consequences of the axioms are explored. A simple example of a structure constructed using Hrushovski's method of free amalgamation is shown to be a non-trivial example of an analytic Zariski structure. A number of 'quasi-analytic' results are derived for this example e.g. analogues of Chow's theorem and the proper mapping theorem.

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§0. The theory of difference-fields is known to have a model-companion, ACFA; the theory of differential fields (in unspecified characteristic) has the model-companion DCF. Let OF (for 'operator-field') be the theory whose models are (K, D, σ), where K is a field of which D is *either* a difference-operator *or* a derivation, and σ is respectively either $x \mapsto Dx+x$ or the identity; equivalently, σ is a field-endomorphism, D is an additive endomorphism, and the equations

$$D(x \cdot y) = Dx \cdot y + Dy \cdot \sigma x \wedge (\sigma x - x) \cdot (Dy - \sigma y + y) = 0$$

are identities. Then OF has the model-companion OF*, axiomatized by

$$OF \cup \{\neg\theta \to \phi \colon \phi \in \text{ACFA}\} \cup \{\theta \to \psi \colon \ \to \psi \in \text{DCF}\},$$

where θ is $\forall x\, \sigma x = x$. By considering the derivation $x \mapsto x + Dx - \sigma x$, we can axiomatize OF* geometrically and *uniformly*, that is, without considering whether σ is trivial.

§1. The theory of a field with m derivations whose linear span is closed under the bracket has a model-companion, DCF^m, whose completions are ω-stable. If such derivations over a field K span an m-dimensional space E, then E has a basis $(\partial_i \colon i < m)$ such that $\partial_i t^j = \delta_i^j$ for certain t^i in K. As an abelian group with the bracket, E is a Lie-ring, of which the t^i are additive endomorphisms. The structure $(E, t^0, \dots, t^{m-1})$ is an example of what I propose to call an *m-dimensional modular Lie-ring*; the theory of these has a model-companion, bi-interpretable with DOF^m.

► V. PUZARENKO, *On some aspects of theory of admissible sets*.
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§1. On computability over models On hereditarily finite superstructures it can be defined notions of computable (c.e.) sets and computable functions as Δ-(Σ-)subsets and Σ-functions respectively. Now this approach to computability is developed by Yu. L. Ershov. The properties of Σ- and Δ-subsets of ω on $\mathbb{HF}(\mathfrak{M})$ was described by V. Rudnev, A. Morozov and V. Puzarenko.

THEOREM 1.

1. *Given any* T-(e-)*ideal* I *there exists a model* $\mathfrak{M}_I$ *such that* $A \subseteq \omega$ *is* Δ-(Σ-)*subset on* $\mathbb{HF}(\mathfrak{M}_I)$ *iff* $d_T(A) \in I$ $(d_e(A) \in I)$.
2. *Given any model* $\mathfrak{M}$ Δ-(Σ-)*subsets* ω *on* $\mathbb{HF}(\mathfrak{M})$ *are closed under* T-(e-)*reducibility and the operation* $\oplus$.

It was proved that the class $\mathcal{K}_I$ of elementary equivalent models constructed in the proof of previous theorem for e-ideal I satisfies the following condition: If a model $\mathfrak{M}$ of finite language realizes e-ideal I (in sense of previous theorem) then there exists a model $\mathfrak{M}'$ from this class such that $\mathbb{HF}(\mathfrak{M}') \leqslant_\Sigma \mathbb{HF}(\mathfrak{M})$.

Furthermore, the least classes of computable families of subsets of ω was described for any e-ideal I.

For some special class of models $\mathfrak{M}$ of countable categorical theories a Friedberg numbering of the family of all Σ-subsets on $\mathbb{HF}(\mathfrak{M})$ was constructed. It is used finite injury priority method for getting this result.

§2. On definability of models

THEOREM 2.

1. $\mathbb{R}$ *is definable in* $\mathbb{HF}(\mathfrak{M})$ *iff there exists a definable* $\mathbb{HF}(\mathfrak{M})$*-numberings of the family of all the subsets of* ω.

2. *If* $\mathbb{R}$ *is* Σ*-definable in* $\mathbb{HF}(\mathfrak{M})$ *then there exists a decidable computable* $\mathbb{HF}(\mathfrak{M})$*-numbering* ν *such that* $\delta\nu \times \omega \setminus \{\langle x, n\rangle \mid n \in \nu(x)\}$ *is* Σ*-definable.*

COROLLARY 3.

- *Let* $\mathfrak{M}$ *be any model of an arbitrary categorical in some power theory of a finite language. Then* $\mathbb{R}$ *is not definable in* $\mathbb{HF}(\mathfrak{M})$.
- *Let* $\mathfrak{M}$ *and* $\mathfrak{M}'$ *be models of a finite language such that* $\mathbb{HF}(\mathfrak{M}) \preccurlyeq \mathbb{HF}(\mathfrak{M}')$. *If* $\mathbb{R} \leqslant \mathbb{HF}(\mathfrak{M})$ *then* $\mathbb{R} \leqslant \mathbb{HF}(\mathfrak{M}')$.

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We consider the extension of first-order logic with an inductive operator **nio** due to Arvind and Biswas [1]. At each stage in forming the fixed point of a formula, a single tuple is nondeterministically chosen and added to the relation which has been constructed so far.

Arvind and Biswas gave only informal semantics for their logic; we provide the first formal semantics. A difficulty that arises is that nondeterministic formulae may denote a set of relations, each one depending on a sequence of choices. Conventionally, satisfaction is defined for formulae denoting a single relation but, given a set of relations, there are many possible definitions of satisfaction. We separate the semantics of a logic into its denotational element, which assigns to each formula a set of relations, and a satisfaction relation, which provides a means of combining the relations in a formula's denotation into a single one.

By using an appropriate satisfaction relation, essentially any computational complexity class defined in terms of nondeterministic Turing machines operating within polynomial time bounds can be expressed in terms of nondeterministic fixed-point formulae.

[1] V. ARVIND and S. BISWAS, *Expressibility of first order logic with a nondeterministic inductive operator*, ***Proceedings of 4th Annual Symposium on Theoretical Aspects of Computer Science*** (Franz-Josef Brandenburg, Guy Vidal-Naquet, and Martin Wirsing, editors), Lecture Notes in Computer Science, vol. 247, Springer-Verlag, 1987, pp. 323–335.

► WIM RUITENBURG, *Intuitionistic theories with quantifier elimination*.
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We present intuitionistic theories T which satisfy quantifier elimination: For each formula A there exist quantifier-free formulas B such that $T \vdash A \leftrightarrow B$. Examples include theories outside the categories indicated by the 1973 paper of Craig Smoryński, and theories which become inconsistent in the presence of the principle of excluded middle.

C. SMORYNSKI, *Elementary intuitionistic theories*, ***The Journal of Symbolic Logic***, vol. 38 (1973), no. 1, pp. 102–134.

► PAVEL SCHREINER, *Automatic recognition of the interpolation property in some propositional superintuitionistic logics*.
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L. Maksimova has proved in 1977 that there exists only eight propositional superintuitionistic logics that have interpolation property.

By virtue of the given result to check up when some superintuitionistic propositional logic have interpolation property it is enough to find out whether it will coincide with one of above mentioned eight logics.

However problem of establishment of the identity of two logics is undecidable in the general case, therefore we shall consider only those logics which are received from intuitionistic logic by addition of the new formula.

Thus it is enough for us to check up that the added formula on the one hand is true on the class of Kripke frames, which characterizes logic having interpolation property and on the other hand that this formula is rejected on some specially selected classes of Kripke frames.

The algorithm allowing automatic check of identity of the logic which received from intuitionistic logic by addition of the new formula with one of the six logics with interpolation property has been developed. The program which realized given algorithm has been created.

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► JOHN H. SEREMBUS, *A new perspective on consistency and validity of propositional logic*.
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In [1], [2], and [3] the author offered a novel representation of truth tables in Propositional Logic (PL) to prove the functional (in)completeness of sets of connectives of PL and explored the significance of that notational representation. In [4] the author summarized those results and compared them with the traditional literature concerning functional completeness. In the present paper the author explores the application of his representation of truth tables to the logical properties of consistency and validity of sets and arguments respectively of a PL with negation and conjunction as its only primitive connectives.

[1] JOHN H. SEREMBUS, *Functional completeness revisited*, ***European Meeting of the Association of Symbolic Logic***, Vesprem, Hungary. August 9–15, 1992.

[2] ———, *A study of truth table notation of propositional logic*, ***Conference on Notational Engineering***, George Washington University, Washington, D.C. May 23–24, 1996.

[3] ———, *A generalized functional completeness proof for propositional logic*, ***European Summer Meeting of the Association of Symbolic Logic***, Prague, Czech Republic. August 9–15, 1998.

[4] ———, *Last words on the functional completeness of PL*, ***European Summer Meeting of the Association of Symbolic Logic***, Vienna, Austria. August 6–11, 2001.

► V. YU. SHAVRUKOV, *The ∀∃ theory of the lattice of universal sentences in Peano Arithmetic*.
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We show that the ∀∃ theory of the lattice of Π_1 sentences modulo provability in Peano arithmetic is decidable.

This complements our earlier findings on the undecidability of the full 1st order theory.

As usual, the argument involves a study of extensions of embeddings of finite (distributive) lattices into the target lattice with a special role reserved for separated (or rather, in this case we need the dual notion) lattices, that is, lattices satisfying the reduction principle.

In the talk, we also comment on the similarities and differences of our constructions with the case of the lattice of r.e. sets.

This reports on joint work with Per Lindström.

► SONJA SMETS, *Classical reasoning about quantum systems*.
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Current research in the domain of quantum logic has shown that certain quantum phenomena can be captured within a classical/intuitionistic framework equipped with an additional "quantum operator" [1]. First we analyse the advantages of encompassing quantum logic in such a larger framework, and point out how this offers a way out of the problems from which static (standard) quantum logic suffers. Special attention will go to the implication problem and the failure of a quantum deduction theorem in the static case [5]. Secondly, we will analyse the features of the involved modal quantum operator. We indicate how this operator captures the superposition states/properties of physical systems in the larger framework that we call "modal quantum logic" (MQL). When we place MQL in the landscape of other non-classical logics, it will become clear that our approach differs from Goldblatt's in [3] where the orthoframe-semantics of minimal quantum logic favours a translation into the Brouwerian modal logic **B**. Our modal quantum operator has some characteristics of the known modal possibility operator as well as of the necessity operator. In particular, MQL is related to the so-called "propositional flat logic" which computer scientists introduced in the context of hardware verification [2] and to the structure known as a "local Algebra" in the research on mathematical modalities [4]. The mentioned modal characterization of quantum logic supports our pluralistic view on logic. We favour a perspective that allows for the comparison of different logical systems. In connection to logical pluralism, within MQL we see perfectly in which cases the quantum operator reduces to the identity operator yielding as such a classical logic that describes the classical properties of physical systems.

[1] B. Coecke, *Quantum logic in intuitionistic perspective*, ***Studia Logica***, vol. 70 (2002), pp. 411–440.

[2] M. Fairlough and M. Mendler, *Propositional lax logic*, ***Information and Computation***, vol. 137 (1997), pp. 1–33.

[3] R. I. Goldblatt, *Semantic analysis of orthologic*, ***Journal of Philosophical Logic***, vol. 3 (1974), pp. 19–35.

[4] ———, *Grothendieck topology as geometric modality*, ***Zeitschrift für Mathematische Logik und Grundlagen der Mathematik***, vol. 27, (1981) pp. 495–529.

[5] S. Smets, *From intuitionistic logic to dynamic operational quantum logic*, ***Essays in Logic and Ontology: Poznan Studies in Philosophy and the Humanities***, submitted.

► ANDREA SORBI, *Bounding and nonbounding minimal pairs in the enumeration degrees*.
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In a poset $\langle P, \leq\rangle$ with a least element 0, a pair of elements a, b is said to form a minimal pair if $a, b \neq 0$ and 0 is the greatest lower bound of a, b. We investigate minimal pairs in the poset of the enumeration degrees of Σ^0_2 sets (or equivalently, the enumeration degrees below the least enumeration jump). We show that there exist nonzero Σ^0_2 enumeration degrees that do not bound minimal pairs in the enumeration degrees. On the other hand if a is a nonbounding Σ^0_2 enumeration degree as in the previous result then no nonzero enumeration degree below a can contain Δ^0_2 sets, since we can also prove that every nonzero Δ^0_2 enumeration degree bounds a minimal pair of Δ^0_2 enumeration degrees. These theorems are still true if we replace enumeration reducibility by s-reducibility, that is the restricted version of enumeration reducibility in which we are only allowed enumeration operators Φ satisfying: if $\langle x, u\rangle \in \Phi$ then D_u, the finite set with canonical index u, is either empty or a singleton. Moreover, the proof of the nonbounding result can be adapted to show that there is a nonbounding set 1-generic Σ^0_2 enumeration degree.

Joint work with S. Barry Cooper (University of Leeds, Leeds, UK.), Angsheng Li (Academia Sinica, Beijing, China) and Yue Yang (National University of Singapore, Singapore).

► VLADIMIR A. STEPANOV, *Semantics of self-reference: a dynamical systems approach*.
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Let $\boldsymbol{P}(\boldsymbol{x})$ be a predicate formula of a fragment of the type-free language, in which predicates can take other predicate as arguments. By quantor $\mathsf{S}\boldsymbol{x}$ we designate the fact, that the formula $\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x})$ refers to itself and satisfies the self-reference axiom:

$$\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x}) \leftrightarrow \boldsymbol{P}(\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x})).$$

Let $\boldsymbol{P}(\boldsymbol{x})$ is constructed by $\vee\boldsymbol{\&} \rightarrow\leftrightarrow \neg$ from atomic predicate $\boldsymbol{Tr}(\boldsymbol{x})$, which satisfies Tarsky axiom:

$$\boldsymbol{Tr}(\boldsymbol{x}) \leftrightarrow \boldsymbol{x}.$$

Then by a recurring of replacement of $\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x})$ by $\boldsymbol{P}(\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x}))$ in the right member of the self-reference axiom we obtain the following sequence of the formulas:

$$\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x}) \leftrightarrow \boldsymbol{P}(\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x})) \leftrightarrow \boldsymbol{P}(\boldsymbol{P}(\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x}))) \leftrightarrow \dots$$

This sequence reminds driving on a trajectory of a dynamic system.

Let I be dynamical interpretation of our language [1]. Then $\mathsf{I}(\boldsymbol{x}){=}x \in \{0,1\}$—classical truth values; $\mathsf{I}(\boldsymbol{P}(\boldsymbol{x})){=}p(x)$; it's viewed as a map $p\colon \{0,1\} \to \{0,1\}$; $\mathsf{I}(\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x})){=}\{\langle p^{(n)}(x_0)\rangle, n \in \mathbb{Z}^+ \mid x_0 \in \{0,1\}\}$—the set of motions of dyn. sys. $(\{0,1\}, p)$, associated with $\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x})$; $p^{(n)} = p(p^{(n-1)})$; $\mathsf{I}(\boldsymbol{Q}(\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x}))){=}\{\langle q(p^{(n)}(x_0))\rangle, n \in \mathbb{Z}^+ \mid x_0 \in \{0,1\}\}$; $\mathsf{I}(\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x}) \leftrightarrow (\mathsf{S}\boldsymbol{x}\boldsymbol{Q}(\boldsymbol{x})) = (\mathsf{I}(\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x})) \equiv \mathsf{I}(\mathsf{S}\boldsymbol{x}\boldsymbol{Q}(\boldsymbol{x})))$ where $\equiv$ is **topological equivalence** of two sets of trajectories; $\mathsf{I}(\neg\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x})) = \{\langle \neg p^{(n)}(x_0)\rangle, n \in \mathbb{Z}^+ \mid x_0 \in \{0,1\}\}$.

THEOREM. For everyones self-reference statements of the form $\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x})$ and $\mathsf{S}\boldsymbol{x}\boldsymbol{Q}(\boldsymbol{x})$ the following conditions are fulfilled:

$$(\forall \boldsymbol{P})\ \mathsf{I}(\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x})) \equiv \mathsf{I}(\neg\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x})), \text{ and}$$
$$(\exists \boldsymbol{P}, \boldsymbol{Q})\ \mathsf{I}(\mathsf{S}\boldsymbol{x}\boldsymbol{P}(\boldsymbol{x})) \not\equiv \mathsf{I}(\mathsf{S}\boldsymbol{x}\boldsymbol{Q}(\boldsymbol{x})).$$

Priest's topological (in the terms of Mobius strip) model of self-referential statements is improved and generalized.

[1] V. STEPANOV, ***Semantics of self-referential statements bazed by dynamical systems***, Moscow, CC RAS, 2002, 32 pp. (in Russian)

► ALEKSEY STUKACHEV, *Σ-definability in HF-superstructures and pairs of decidable models*.
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For arbitrary structures $\mathfrak{A}$ and $\mathfrak{B}$ a set $I \subseteq A \cap B$ is called a set of $\mathfrak{A}$-*indiscernibles in* $\mathfrak{B}$ if for any tuples $\bar{i}, \bar{i}' \in I^{<\omega}$ of the same length

$$\langle \mathfrak{A}, \bar{i}\rangle \equiv \langle \mathfrak{A}, \bar{i}'\rangle \text{ implies } \langle \mathfrak{B}, \bar{i}\rangle \equiv \langle \mathfrak{B}, \bar{i}'\rangle.$$

Let T and T' be c-simple theories [1]. If some uncountable model of T' is Σ-definable in HF-superstructure over some model of T, then there are decidable models $\mathfrak{A}$ and $\mathfrak{B}$ of T and T' respectively such that there exists an infinite computable set of $\mathfrak{A}^*$-indiscernibles in $\mathfrak{B}$, where $\mathfrak{A}^*$ is expansion of $\mathfrak{A}$ by finite number of constants.

For some c-simple theories this necessary condition of Σ-definability is also sufficient. We denote by T_{DLO} theory of dense linear order and by T_E theory of infinite models of equality.

THEOREM 1. *Let T be c-simple theory and $\mathfrak{A}$ be any decidable model of T. Theory T has uncountable model which is Σ-definable in $HF(L)$ for some $L \models T_{DLO}$ iff there exists an infinite computable set of order indiscernibles in $\mathfrak{A}$. Theory T has uncountable model which is Σ-definable in $HF(S)$ for some $S \models T_E$ iff there exists an infinite computable set of total indiscernibles in $\mathfrak{A}$.*

By using results of [2] we obtain a (partial) negative answer for conjecture stated in [1].

THEOREM 2. *There exists c-simple theory (of infinite language) such that none of it's uncountable models is Σ-definable in $HF(L)$ for any $L \models T_{DLO}$.*

[1] YU. L. ERSHOV, *Σ-definability of algebraic structures*, ***Handbook of Recursive Mathematics***, Elsevier, 1998, vol. 1, pp. 235–260.

[2] H. A. KIERSTEAD and J. B. REMMEL, *Degrees of indiscernibles in decidable models*, ***Transactions of the American Mathematical Society***, vol. 289 (1985), no. 1, pp. 41–57.

► CARLO TOFFALORI, *Strongly minimal modules over group rings*.
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A systematic analysis of strongly minimal modules, enlarging the classification of strongly minimal abelian groups given by Reineke [3] and other significant contributions, has been developed in the latest years by Vera Puninskaya in a series of papers summarized in [2]. Our aim here is to study strongly minimal modules over a group ring RG when R is a Dedekind domain and G is a finite multiplicative group. Note that strongly minimal modules over a Dedekind domain (indeed over a Prüfer domain) R are classified in the Puninskaya work; but, of course, the group G changes the basic algebraic framework and often requires a specific ad-hoc approach. Our analysis emphasizes the role of two crucial examples (at least over the integers): the case when G is cyclic (reducing to the Dedekind domain $\mathbf{Z}[\omega_d]$ where ω_d denotes a primitive d-th root of 1 for some d dividing $|G|$) and the case when G is quaternionic. Coming back to arbitrary Dedekind domains, we discuss the role of localization and reduction to p-groups. Finally, we consider in our setting pseudo strongly minimal modules M (a notion proposed in the general framework of modules over commutative rings by Herzog and obviously related to strong minimality). Recall that M is called pseudo strongly minimal if and only if it has infinite length but every proper pp-definable subgroup has finite length (and that pseudo strong minimality is not an elementary property, in other words it is not preserved by elementary equivalence). We give a complete classification of these modules when G is abelian and $R = \mathbf{Z}$.

[1] S. L'INNOCENTE, V. PUNINSKAYA, and C. TOFFALORI, *Strongly minimal modules over group rings*, preprint.

[2] V. PUNINSKAYA, *Strongly minimal modules*, ***Journal of Mathematical Science*** vol. 114 (2003), pp. 1127–1157.

[3] J. REINEKE, *Minimale gruppen*, ***Zeitschr. Math. Logik Grundl. Math.***, vol. 21 (1975), pp. 357–359.

► SERGEI TUPAILO, *On the intuitiomstic strength of monotone inductive definitions*.
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In his 2002 PhD thesis M. Möllerfeld has shown that the second-order μ-calculus, a theory axiomatizing least fixed points of positively defined monotone operators, when based on classical logic, has the strength of Π^1_2 comprehension axiom, which is the current limit of ordinal analysis. His methods are based on Generalized Recursion Theory, and as such are not amenable to intuitionistic reasoning. However, the μ-calculus presented very little problems for the Gödel-Gentzen-Kolmogorov double-negation translation, so we prove that the intuitionistic theory is proof-theoretically equally strong.

Further interpretation of the intuitionistic μ-calculus in the system $\mathbf{T_0}^i + \mathbf{UMID}$ of Explicit Mathematics provides a first breakthrough into intuitionistic strength of monotone inductive definitions in those theories, showing that one should expect that this strength is as big as the classical one. This question was posed by S. Feferman in 1982, but up to now virtually nothing was known in this area. On the classical side, it came as a surprise when M. Rathjen proved in a series of papers of 1996–2002 that the strength is essentially that of $\mathbf{\Pi^1_2}$-**CA**. Our work determines the exact strength of the intuitionistic $\mathbf{T_0}^i \upharpoonright +\mathbf{UMID_N}$.

[1] S. FEFERMAN, *Constructive theories of functions and classes*, ***Logic Colloquium '78***, (J. N. Crossley, editor), pp. 159–224, 1979.

[2] ———, *Monotone inductive definitions*, ***The L. E. J. Brouwer Centenary Symposium***, (A. S. Troelstra and D. van Dalen, editors), North-Holland, 1982, pp. 77–89.

[3] M. MÖLLERFELD, *Generalized inductive definitions. The μ-calculus and Π^1_2-comprehension*, PhD dissertation, Universität Münster, 2002.

[4] M. RATHJEN, *Explicit Mathematics with monotone inductive definitions: a survey*, (W. Sieg et al., editors), ***Reflections on the Foundations of Mathematics: Essays in Honor of Solomon Feferman***, Lecture Notes in Logic, vol. 15, 2002.

[5] S. TUPAILO, *Realization of constructive set theory into explicit mathematics: a lower bound for impredicative Mahlo universe*, ***Annals of Pure and Applied Logic***, vol. 120 (2003), nos. 1–3, pp. 165–196.

► THOMAS VETTERLEIN, *Partial MV-algebras and the logic of quantum effects*.
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Effect algebras, introduced by Foulis and Bennett [2], are modelled upon the quantum effects of Hilbert space. Each such effect corresponds in quantum physics to the statement that a certain yes-no experiment leads to a positive result. It seems natural to ask if there is some logical calculus formalizing exactly these statements.

Dalla Chiara and Giuntini introduced in [1] the propositional logic UPaQL, which is indeed adaquate for effect algebras; namely, with respect to these algebras, completeness of the calculus was shown. It remains, however, apart from certain interpretational difficulties, the problem that the whole class of effect algebras is taken into account, not only those being important in physics.

We are aiming at an improvement of the situation by proceeding in an alternative way than proposed in [1]. Just as Kochen and Specker generalized boolean algebras to partial boolean algebras [3], we may generalize MV-algebras to partial MV-algebras. The partial logics being based on the latter kind of algebras allows an easier interpretation, and it corresponds to a subclass of the effect algebras, including those used in physics.

[1] M. L. DALLA CHIARA and R. GIUNTINI, *The logics of orthoalgebras*, ***Studia Logica***, vol. 55 (1995), pp. 3–22.

[2] D. J. FOULIS and M. K. BENNETT, *Effect algebras and unsharp quantum logics*, ***Foundations of Physics***, vol. 24 (1994), pp. 1325–1346.

[3] S. KOCHEN and E. P. SPECKER, *Logical structures arising in quantum theory*, ***Theory of Models***, (J. Addison, L. Henkin, A. Tarski editors), North-Holland, 1965; pp. 177–189.

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We prove that for each positive integer n there is a finite list of equivalence relations $\{\mathbb{E}_i : 1 \leq i \leq r_n\}$ on $\mathcal{G}^n$, where $\mathcal{G}$ is a random graph, such that for every other equivalence relation $\mathbb{E}$ on $\mathcal{G}^n$ there is an induced random subgraph $\mathcal{H} \subset \mathcal{G}$ and an equivalence relation $\mathbb{E}_i$ from the list such that

$$\mathbb{E} \upharpoonright \mathcal{H}^n = \mathbb{E}_i \upharpoonright \mathcal{H}^n.$$

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The relation $\leq_1$ which was introduced by T. Carlson originally as a tool in confirming Reinhardt's conjecture and then applied by him to develop a new approach to ordinal notations is defined recursively on the ordinals involving the notion of Σ_1-elementary substructure.

The weak version of $\leq_1$ is given by $\alpha \leq_1 \beta$ iff $(\alpha, <, \leq_1)$ is a Σ_1-elementary substructure of $(\beta, <, \leq_1)$. Carlson analyzed this relation in terms of ordinal arithmetic showing that it provides descriptions for ordinals below ϵ_0. The strong version of $\leq_1$ involves ordinal addition as a partial operation and is defined on structures $(\alpha, <, +, \leq_1)$. This is when $\leq_1$ becomes interesting as a new approach to ordinal notations since the inclusion of addition yields a powerful dynamization which allows for a description of ordinals below the proof theoretic ordinal of the fragment Π^1_1-CA_0 of second order number theory as was conjectured by Carlson.

We discuss how relativized versions of ordinal notations well known in ordinal informative proof theory can be used to analyze the strong $\leq_1$ relation in terms of ordinal arithmetic in order to obtain an explicit understanding of the structure raised by this notion of Σ_1 elementarity.

[1] T. J. Carlson, *Ordinal arithmetic and* Σ_1 *elementarity*, ***Archive for Mathematical Logic***, vol. 38 (1999) no. 7, pp. 449–460.

[2] ———, *Knowledge, machines, and the consistency of Reinhardt's strong mechanistic thesis*, ***Annals of Pure and Applied Logic***, vol. 105 (2000), pp. 51–82.

[3] ———, *Elementary patterns of resemblance*, ***Annals of Pure and Applied Logic***, vol. 108 (2001), pp. 19–77.

► DAN E. WILLARD, *On the interface between mathematical logic and computerized database theory*.
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This talk will summarize a 37-page article that we recently published in the *Journal of Computer and System Sciences* that is germane to the Mathematical Logic Community but which is unlikely to be seen by most members of the ASL because of its computer science orientation. The topic of [1] is the question of how one can efficiently find the subset of records R that satisfies a First Order Predicate Calculus Query Q lying in an unindexed base set B of finite size. One language for retrieving data is called the Relational Calculus in Database Theory. It is related to some of the interests in the Logic Community because the Relational Calculus Language contains Universal and Existential quantifiers that are analogous to the Predicate Calculus's quantifiers — except for the always-bounded nature of Relational quantifiers. The always-finite range of the Relational Calculus's quantifiers has led many Computer Scientists to conjecture that the study of how to efficiently perform Relational Calculus queries could have significant pragmatic implications in Database Theory and Applied Finite Model Theory.

There is no known algorithm to perform *all Relational Calculus queries efficiently*. It has been widely conjectured that no such algorithm exists. However, our article [1] shows that there is a subset of Relational Calculus queries that consists of a very broad class of likely queries that occur in a computing environment and which has the property that these queries can be performed efficiently. In particular, let I denotes the size of the input and U the size of the output. Our algorithm can construct the set R from an *unindexed data base* B in time $O(U + I \log^d I)$ for some constant d that depends on the query and which usually equals zero or one.

Our summary of [1] in this talk will focus on the open questions raised by this article and its connections to the Predicate Calculus in Mathematical Logic and to Automated Deduction.

[1] D. WILLARD, *An algorithm for handling many relational calculus queries efficiently*, ***Journal of Computer and Systems Sciences***, vol. 65 (September 2002), pp. 295–331.

Abstracts presented by title

► ADIB BEN JEBARA, *Why accept the well ordering principle?*
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In the middle ages of Europe, people had a philosophy of nothing or everything, they were obsessed by exhaustivity. Their idea was that if you cannot change the whole of Aristotle, no need to try to change any knowledge. That was mainly why they were making little progress. Nowadays, such a philosophy is back among some mathematicians, among others. The theory derived from the well-ordering principle is an Aristotelian type theory : try to provide an alternative but do not try to make a change, is what is thought about it.

The well-ordering principle states that every set can be well-ordered. The well-ordering principle is logically equivalent to the axiom of choice.

In web page: `http://www.logic.at/LC2001/files/ben_jebara.adib.txt` I wrote that we can assume only a restricted axiom of choice which is choice for countable family of sets of n elements, n from 2 to m. This axiom was defined and studied by Andreas Blass, trying to answer one of my questions, from 1999.

See web page: `http://www.math.lsa.umich.edu/~ablass/dpcc.pdf`

I think that only that restricted axiom is true, although the axiom of choice is consistent with the other usual axioms. I think so mainly because of the interesting consequences of the existence of some infinite products and the non existence of others. So, I think also that the well ordering principle is not true.

Anyway, this principle is an impoverishing principle as it enables to set every set the way the set of positive integers is set. Also, it is against mathematical intuition but it is accepted with fatalism. Beside, a powerful principle applying to a general case does not necessarily solve problems, even via the induction principle, which is an extension of recurrence.

► JOHN CORCORAN, *Fertile postulate sets*.
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Mathematical archeology is the name recently given to the activity of applying mathematics to historically important mathematics and logic texts in order to illuminate and sometimes explain historical facts. In this paper we study certain passages in the classic 1932 *Symbolic logic* [1] where various properties of postulate sets are defined and discussed. Some are familiar today, e.g., independence, consistency and categoricity [1, pp. 337–357]. But there is one called fertility that is not mentioned in contemporary literature. Lewis and Langford say that this property is suggested by considering "how the joint force of several conditions may be vastly different from that of the several conditions separately" [1, p. 357]. A non-empty set of postulates is defined to be *fertile* if it has at least one consequence that is not a truth-functional combination of separate consequences of its individual members. A two-postulate set for what is today called dense linear order is cited without proof as an example. However, if a given consequence of a given finite set of postulates is joined by non-exclusive disjunction with each of the postulates, the resulting set of disjunctions is logically equivalent to the given consequence. This yields the Non-fertility Lemma: for first-order and similar logics, every consequence of a set of propositions is logically equivalent to a conjunction of consequences of individual members of the set. E.g., if R is a consequence of $\{Q, P\}$, then R is logically equivalent to $((R \vee Q)\&(R \vee P))$. This might explain why we do not today read about fertile sets of postulates. Three other proofs of the Non-fertility Lemma were found in April 1995 independently by R. Vesley, K. Regan, and the author.

[1] C. I. LEWIS and C. H. LANGFORD, ***Symbolic logic***, Century, NY, 1932, reprinted in 1959, Dover, NY.

► ALBERT HOOGEWIJS AND PIETER AUDENAERT, *Combinatory logic, a bridge to verified programs*.
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To facilitate program development, researchers and practitioners enhanced and invented new programming languages, all with the same goal: striving to balance five objective criteria: user acceptance rate, program correctness, development speed, execution speed, and program compactness. In [1] the authors introduced Alfred ("Another Language for Functional REDuction"), a new programming language based on untyped combinatory logic (CL). The main emphasis lies on program correctness, development speed and program compactness. The whole language is built from the two basic combinators S and K that satisfy the axioms $Sxyz = xz(yz)$ and $Kxy = x$ for all x, y and z in an equational theory as defined in [2]. This lazy functional language is shown to be Turing-complete. An Alfred program can be compiled to binary code and executed, and to a PVS specification and proved to be partial correct using PPHOL (see [3]).

Supported by research grant of UGent, BOF 011D0900.

[1] P. AUDENAERT and A. HOOGEWIJS, *Alfred: Formal lazy functional reduction*.
[2] H. P. BARENDREGT, ***The lambda calculus: its syntax and semantics*** North-Holland, 1984.
[3] A. HOOGEWIJS and P. AUDENAERT, *Implementing undefinedness in a two-valued prooftool through a four-valued Kleene logic*, ***The Bulletin of Symbolic Logic***, vol. 9, (2003) pp. 92–93.

► S. MARDAEV, *Lfp and Ifp in modal logic*.
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1. The least fixed point (Lfp) in p of the temporal $(\Diamond_L, \Diamond_R, \Box_L, \Box_R)$ formula $q \wedge \Box_L \Box_L (p \vee \Diamond_R p)$ isn't definable in the class of all finite strictly linearly ordered temporal models (i.e., there is no defining formula $\omega(q)$). An analogous example exists for the class of all finite linearly ordered models. So Lfp isn't definable even in simple enough finite models in general temporal case. Nevertheless there is a "good" subclass of formulas. If all negations are tight, a temporal Σ-*formula* contains $\Diamond_L, \Diamond_R$ only.

THEOREM. *For each positive in p temporal Σ-formula $\varphi(p, q_1, \ldots, q_n)$ there is a formula $\omega(q_1, \ldots, q_n)$ which defines Lfp in p of φ in the class of all transitive linear temporal models.*

2. The inflationary fixed point (Ifp) in p of the modal $(\Diamond, \Box)$ formula $(q \wedge \Box p) \vee (\Box\Box p \wedge \neg\Box p)$ isn't definable in the class of all finite strictly linearly ordered modal models. An analogous example exists for the class of all finite linearly ordered models.

It is known [2], [3] that Lfp are definable in such models with preservation of positivity of parameters. So μ-operator can be eliminated in these classes of models in the μ-calculus. It gives another proof that MIC [1] (i.e., modal logic + Ifp) is more expressive than the μ-calculus.

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[1] A. Dawar, E. Grädel, and S. Kreutzer, *Inflationary fixed points in modal logic*, ***Computer Science Logic, 2001, Proceedings***, Lecture Notes in Computer Science, vol. 2142, 2001, pp. 277–291.

[2] S. Mardaev, *Least fixed points in Grzegorczyk logic and in the intuitionistic propositional logic*, ***Algebra and Logic***, vol. 32 (1993), no. 5, pp. 279–288.

[3] ———, *Least fixed points in the Gödel-Löb logic*, ***Algebra and Logic***, vol. 32 (1993), no. 6, pp. 372–375.

► S. YU. PODZOROV, *On the definition of Lachlan semilattices*.
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We say that preordered structure $\langle A, \leqslant\rangle$ is a *prelattice* (*presemilattice*) if the associated partially ordered structure is a lattice (semilattice). Let $\mathcal{L}$ be an upper semilattice. We say that $\mathcal{L}$ is *Lachlan semilattice* if there exists an upper presemilattice $\langle \mathbb{N}, \leqslant_\omega\rangle$ such that $\mathcal{L}$ is isomorphic to the upper semilattice associated with $\langle \mathbb{N}, \leqslant_\omega\rangle$ and there exists a sequence of finite distributive prelattices $\{\mathcal{D}_i = \langle D_i, \leqslant_i\rangle\}_{i\in\mathbb{N}}$ for which the next 7 conditions hold:

1. D_i are finite subsets of $\mathbb{N}$ which are strongly computable uniformly for i;
2. For all i $0, 1 \in D_i$, $0 <_i 1$; for all $x \in D_i$ $0 \leqslant_i x \leqslant_i 1$;
3. Π^0_2-indices of $\leqslant_i$ are computable uniformly for i;
4. For all i $D_i \subseteq D_{i+1}$; for $x, y, z \in D_i$ if z represents the least upper bound for x and y in $\mathcal{D}_i$ than the same holds in $\mathcal{D}_{i+1}$;
5. $\bigcup_{i\in\mathbb{N}} D_i = \mathbb{N}$; $x \leqslant_\omega y \Leftrightarrow \exists i(x \leqslant_i y)$;
6. There exists a sequence of functions $\{u_i : D_i^2 \to D_i\}_{i\in\mathbb{N}}$ computable uniformly for i such that for $x, y \in D_i$ $u_i(x, y)$ represents the least upper bound of x, y in $\mathcal{D}_i$;
7. There exists a sequence of functions $\{v_i : D_i^2 \to D_i\}_{i\in\mathbb{N}}$ computable uniformly for i such that for $x, y \in D_i$ $v_i(x, y)$ represents the greatest lower bound of x, y in $\mathcal{D}_i$.

It's easy to see that any Lachlan semilattice is a distributive upper semilattice with the least and greatest elements (it is isomorphic to the direct limit of finite distributive lattices associated with $\mathcal{D}_i$-s). Lachlan proved that these semilattices are exactly the principal ideals in the upper semilattice of c.e. m-degrees. Now Lachlan semilattices play an important role in the theory of numberings.

From the definition we can see that every Lachlan semilattice has a Σ^0_3-representation. Whether converse is true (i.e., whether every distributive upper semilattice with the least and the greatest elements which has Σ^0_3-representation be a Lachlan) is still unknown.

But some advantage has been made. The author proved that in the definition of Lachlan semilattice condition 7 could be omitted.

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On the class of all intermediate logics Ext**Int** we can define an equivalence relation $\equiv_{\text{fin}}$ such that $L_1 \equiv_{\text{fin}} L_2$ iff logics L_1 and L_2 are true exactly on the same finite Heyting algebras. The aim of this paper is to present some results concerning the classification of intermediate logics wrt the relation $\equiv_{\text{fin}}$.

Fact 1. *For any L, only the biggest logic in the class $L/\equiv_{\text{fin}}$ is finitely approximable and only the smallest one is χ-axiomatisable (axiomatisable by Jankov formulas).*

Question. *For a given L what is the cardinality of $L/\equiv_{\text{fin}}$?*

We have the following characterisation:

Fact 2. *L is finitely approximable and χ-axiomatisable iff $|L/\equiv_{\text{fin}}| = 1$.*

Examples of logics with $|L/\equiv_{\text{fin}}| = 1$:

FACT 3. *If L is a tabular or locally tabular logic then it is χ-axiomatisable and consequently* $|L/\equiv_{\text{fin}}| = 1$.

And a special case at the other end:

FACT 4. *There exists a continuum of intermediate logics with the same finite models, i.e., there exists L such that* $|L/\equiv_{\text{fin}}| = 2^{\aleph_0}$.

The proof of this fact, in which ideas from [1] and [2] are used, also gives some support for the following:

CONJECTURE. *For any* $L \in \mathrm{Ext}\mathbf{int}$, $|L/\equiv_{\text{fin}}| = 1$ *or* $|L/\equiv_{\text{fin}}| = 2^{\aleph_0}$.

[1] K. FINE, *An ascending chain of* S4 *logics*, ***Theoria***, vol. 40 (1974), pp. 110–116.

[2] A. WROŃSKI, *Intermediate logics and the disjunction property*, ***Reports on Mathematical Logic***, vol. 1 (1973), pp. 39–51.

► WAGNER WANECK MARTINS, *The e-book: "Boolean arithmetic and its applications"*.
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There is a deep abyss between logic and technology represented by the absence of mathematics in the information science, which is responsible for the emergence of the 4th great crisis in foundations of mathematics. Back in 1976, at the "Logic Colloquium", Oxford, England, it was presented a general numerical method of solution for the Boolean equation system, as belonging to the Boolean arithmetic, a new field of research. It means that was discovered, that the Boolean algebra has a mathematical isomorphic numerical aspect, what was unknown hitherto. Nowadays, mathematics is not yet used in software engineering, that is, their programs are built empirically. There is an absence of mathematics contribution to software engineering and firmware engineering, mainly after the emergence of microelectronics. The three great crises in the foundations of mathematics are well known. The chronic crisis regarding software, the present proliferation of "viruses", the lack of a common nucleus in the different applications of artificial intelligence, the unobtainable software automatically free from failures caused by human factors, allows us to conclude that the 4th great crisis in mathematics is a reality and it is due to the growing need for the mathematicalization of the modern information science in that new numerical aspect. This Boolean numerical logic could be useful to the searching of the solution of problems in the computer operation of any software and firmware engineering programs.

The e-book: "Boolean arithmetic and its applications" is available by download in the site: http://www.poli.usp.br/pea5737.

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Historically speaking the first attempt of the reconstructing of the mathematics on a phenomenal basis was the alternative set theory (AST) created by P. Vopenka. This theory, simlarly to nonstandard analysis, enables to treat the infinity phenomenon emerging on formally finite collections of object. AST admits the existence of semisets (proper semisets) which are not sets ("... we meet proper semisets whenever in considering a property of some objects we emphasize its intension rather than its extension.") Proper semisets rather arise as mathematical counterparts of nonsharp collections without definite boundaries. Second urgent issue with which Cantorian set theory failed to copy is a problem of so-called circular situations. Circular situations are common in our daily life. For example considering the situation in which we utter the statement "This is a very exciting situation." While we are referring to a situation, say s, by saying "this situation," our utterance is also a part of that situation. For another example, one sometimes hears public announcements concluding with "This announcement will not be repeated." If announcements are assumed to be situation, then this one surely contains itself. In order to investigate aftermentioned state of affairs we must work in hyperset theory obtained from ZFC by removing the axiom of foundation and adding the anti-foundation axiom. This theory adopted the graph representation for sets. What is advantageous with the above theory is that since graphs of arbitrary form are allowed, including the ones containing proper cycle, one can represent self-referring sets. In order to investigate logical interrelationship between two above approaches we are obliged to introduce the generalized notions of Hamiltonian graph and Hamiltonian embedding. I will comprehesively discuss the theorem stating that for any finite graph without isolated vertices there is a Hamiltonian embedding into any infinite set connected with respect to some q-equivalance (which is a representation of indiscernibility phenomena in AST) or x-equivalance (which is a representation of accessibility phenomena in AST). In some sense this means that each such an infinite connected set (in particular, each connected set in a complete metrizable topological space), contains each finite graph inside, and even is exhausted by the images of its edges.

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Say that a c.e. degree $\mathbf{a}$ is cappable in the c.e. degrees if there is a c.e. degree $\mathbf{b} > \mathbf{0}$ such that the infimum of $\mathbf{a}$ and $\mathbf{b}$ is $\mathbf{0}$. Ambos-Spies, Jockusch, Shore and Soare [1] gives several characterizations of cappable/noncappable degrees: $\mathbf{a}$ is cappable iff $\mathbf{a}$ contains no promptly simple set iff $\mathbf{a}$ cannot be cupped to $\mathbf{0}'$ by any low c.e. degree. Downey, Li and Wu [2] proved that $\mathbf{a}$ is cappable iff $\mathbf{a}$ has a nonzero d.c.e. degree as its complement. In this paper, we consider the complementation phenomenon in the c.e. degrees, $\mathcal{R}$. According to Lachlan's nondiamond theorem, $\mathbf{a}$ cannot have its complement in $\mathcal{R}$. This leads us to define the *quasi-complement* of c.e. degrees as follows: $\mathbf{0} < \mathbf{b} \in \mathcal{R}$ is a *quasi-complement* of $\mathbf{a}$ if $\mathbf{a} \cap \mathbf{b} = \mathbf{0}$, and $\mathbf{a} \cup \mathbf{b}$ is high. It is easy to see that $\mathbf{a} \in \mathcal{R}$ is cappable iff $\mathbf{a}$ has a high quasi-complement. By the existence of the almost deep degrees, there are cappable c.e. degrees having no low quasi-complements. We will prove that any nonzero cappable degree has a low$_2$ quasi-complement.

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[2] R. DOWNEY, A. LI and G. WU, *Complementing cappable degrees in the difference hierarchy*, submitted.