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2000 ANNUAL MEETING
OF THE ASSOCIATION FOR SYMBOLIC LOGIC

University of Illinois at Urbana-Champaign, June 3–7, 2000

The Annual Meeting of the Association was held on the campus of the University of Illinois at Urbana-Champaign, June 3–7, 2000. This meeting was organized to have a special impact on the future of logic, in recognition of the new millennium. It was a day longer than usual and, in addition to the usual features of an ASL annual meeting, it included panel discussions on the following topics:

The Development of Logic in the 20th Century (M. Davis, D. A. Martin, W. Tait, and C. Wood).
The Prospects for Mathematical Logic in the 21st Century (S. Buss, A. Kechris, R. Shore, and A. Pillay).
Does Mathematics Need New Axioms? (P. Maddy, H. Friedman, J. Steel, and S. Feferman).

The members of the Program Committee were J. Avigad, M. Davis, Y. Gurevich, M. Hallett, D. Haskell, G. Hjorth, C. Jockusch (Chair), A. Kanamori, H. J. Keisler, V. McGee, T. Pitassi, S. Starchenko, and W. Tait. The members of the Local Organizing Committee were L. van den Dries, C. W. Henson, C. G. Jockusch, and A. Pillay.

There were 200 registered participants at the meeting, including 61 students. This is about 50% higher than typical ASL annual meetings in recent years. Unusually generous outside financial support was provided by the Department of Mathematics of the University of Illinois at Urbana-Champaign, the College of Liberal Arts and Sciences of UIUC, and the Institute for Mathematics and its Applications. The ASL hosted a welcoming reception, and Wolfram Research, makers of *Mathematica*, sponsored an additional reception.

The following invited hour addresses were delivered:

I. Neeman (Harvard), *Mice and scales*.
M. Rathjen (Leeds), *Constructive set theories*.
D. S. Scott (Carnegie Mellon), *Refining realizability*.
W. Sieg (Carnegie Mellon), *Reductive structuralism: Philosophical aspects of proof theory, past and present*.
R. Soare (Chicago), *Computability theory and differential geometry*.
A. Wigderson (Institute for Advanced Study and the Hebrew University), *Some fundamental insights of computational complexity theory*.
A. Wilkie (Oxford), *O-minimal expansions of the reals*: *Techniques and problems*.
D. Marker (UI at Chicago) presented a two-hour tutorial: *O-minimal expansions of the real field*.

Jon Barwise, who was to have given the Eleventh Gödel Lecture at this meeting, passed away on March 5, 2000. In place of the Gödel Lecture there was a session of reflections on the work of Jon Barwise. The speakers in this session were John Etchemendy, Solomon Feferman, and Dana S. Scott.

The meeting included the following special sessions (with organizers given in parentheses):

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Computability Theory (S. Lempp and M. Lerman), Constructivism and Categorical Logic (S. Awodey), Model Theory (D. Haskell and S. Starchenko), Philosophy of Mathematics (M. Hallett and W. Tait), Proof Theory and Complexity (J. Avigad and T. Pitassi), and Set Theory (G. Hjorth and A. Kanamori), with a total of 29 speakers. Also, there were 36 contributed talks on a broad variety of topics, and 10 abstracts were presented by title.

A banquet was held which included a program with remarks on the history of the Association by S. Feferman and J. Shoenfield. Ward Henson was given special commendation for his eighteen years of outstanding service as Secretary-Treasurer of the Association.

Abstracts of talks given in person or by title by members of the Association for Symbolic Logic follow.

For the Program Committee
CARL G. JOCKUSCH, JR.

Abstracts of invited tutorials and plenary talks

► DAVID MARKER, *O-minimal expansions of the real field*.
Department of Mathematics, Statistics, and Computer Science (M/C 249), University of Illinois, 851 S. Morgan St., Chicago, Illinois 60607-7045, USA.
E-mail: marker@math.uic.edu.

I will survey the basic results on o-minimal expansions of the real field and the geometry and topology of definable sets.

► ITAY NEEMAN, *Mice and scales*.
Department of Mathematics, Harvard University, One Oxford Street, Cambridge, Massachusetts 02138, USA.
E-mail: ineeman@math.harvard.edu.

After reviewing the relevant notions from Descriptive Set Theory and Inner Model Theory, I present a general method of associating levels of the Wadge hierarchy to levels of the mouse hierarchy. This is used to construct scales uniformly and directly from large cardinals.

I introduce a *lightening construction* which given a mouse $\mathcal{M}$ built over a real $x^{\mathcal{M}}$, and a (lighter) real $z \leq_{\mathrm{T}} x^{\mathcal{M}}$, produces a mouse $\mathcal{M} \wr z$ built over the real z. Intuitively speaking $\mathcal{M} \wr z$ maintains the large cardinal structure of $\mathcal{M}$, but relativized to the real z rather than x.

DEFINITION. A *smallness condition* is a Π_1 formula θ such that $\forall$ mouse $\mathcal{M}$, $\forall z \leq_{\mathrm{T}} x^{\mathcal{M}}$: $\mathcal{M} \wr z \models \theta \to \mathcal{M} \models \theta$. $\mathcal{M}$ is θ-*small* if and only if $\mathcal{M} \models \theta$.

A smallness condition is a limitation on the large cardinal strength of $\mathcal{M}$. Such limitations correspond to levels of the mouse hierarchy. The formula θ (and below also ψ) may refer to the predicates of $\mathcal{M}$, and in particular may refer to $x^{\mathcal{M}}$.

DEFINITION. For a Σ_1 formula ψ and a smallness condition θ, define a set of reals

$$A(\psi, \theta) = \{\, x \mid \exists \text{ mouse } \mathcal{M} \text{ so that } x^{\mathcal{M}} = x,\ \mathcal{M} \text{ is } \theta\text{-small, and } \mathcal{M} \models \psi \,\}.$$

Given a smallness condition θ define $\Gamma(\theta)$, the *associated pointclass*, to be the pointclass generated by the sets $A(\psi, \theta)$, where ψ ranges over Σ_1 formulae.

DEFINITION. A smallness condition θ is *cofinal* if $\forall$ θ-small mouse $\mathcal{M}$, $\forall x^* \geq_{\mathrm{T}} x^{\mathcal{M}}$, $\exists$ a θ-small mouse $\mathcal{M}^*$ so that $x^{\mathcal{M}^*} = x^*$, and $\mathcal{M}^* \wr x \trianglerighteq \mathcal{M}$ (where $\trianglelefteq$ is the usual "initial segment of an iterate" order on mice).

THEOREM. *Let θ be a cofinal smallness condition. Then $\Gamma(\theta)$ has the scale property.*

Letting θ vary over specific large cardinal axioms one can use the Theorem to recover most known scaled pointclasses, including Σ^0_1, Π^1_1, Σ^1_2, Π^1_3, etc.

► MICHAEL RATHJEN, *Constructive set theory*.
School of Mathematics, University of Leeds, Leeds LS2 9JT, England.
E-mail: rathjen@leeds.ac.uk.

Constructive set theory grew out of Myhill's endeavours to discover a simple formalism that relates to Bishop's constructive mathematics as ZFC relates to classical Cantorian mathematics. The novel ideas were to replace Powerset by the (classically equivalent) Exponentiation Axiom and to discard full Comprehension while strengthening Collection to Strong Collection. Later on Aczel modified Myhill's set theory to a system which he called Constructive Zermelo-Fraenkel set theory, CZF, and showed its constructiveness by interpreting it in Martin-Löf type theory. The latter interpretation is in many ways canonical and can be seen as providing CZF with a standard model in type theory. Aczel then began a broad investigation of further set-theoretic principles that hold in this model.

The first part of the talk will introduce CZF and its extensions and will gauge the proof-theoretic strengths of various systems.

The second part of the talk will be concerned with large set axioms on the basis of CZF. Classically these large set axioms are equivalent to large cardinal axioms. CZF allows for a sensible theory of large sets in that many of their classically known properties can be proved on the basis of CZF. On the other hand, extensions of CZF by large set axioms have comparatively little consistency strength and bear interesting relations to (classical) recursively large ordinals via realizability interpretations.

The third part will focus on recent work on constructive set theories without $\in$-induction and constructive set theories with the anti-foundation axiom.

► DANA S. SCOTT, *Refining realizability*.
Computer Science Department, Wean Hall, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213-3890, USA.
E-mail: dana.scott@cs.cmu.edu.

It was noticed long ago that, in generalizing Kleene's realizability from numbers as realizing objects to more abstract entities, the objects needed to validate intuitionistic principles needed only to be "computable". This allows for a strict interpretation of logic but with applications to a wide selection of structures not containing only "computable" elements. After a review of the historical background, the talk will discuss and contrast several examples.

► WILFRIED SIEG, *Reductive structuralism: Philosophical aspects of proof theory, past and present*.
Department of Philosophy, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA.
E-mail: sieg@cmu.edu.

In the current discussion on philosophy of mathematics some do as if systematic foundational work supported an exclusive alternative between Platonism and Constructivism; others do as if such mathematical and logical research were deeply misguided and had no bearing on our understanding of mathematics. Both attitudes prevent us from grasping insights that underlie such work and from appreciating significant results that have been obtained. In consequence, they keep us from turning attention to the task of understanding the role of "accessible domains" for foundational theories and that of "abstract structures" for mathematical practice.

This twofold task derives from a probing perspective that takes seriously traditional epistemological concerns, but that does not respect time-honored boundaries drawn for philosophical convenience. It will be approached mainly through work that has been done during the last eighty years on versions of Hilbert's Program. Such an avenue may be surprising, because the stand that was taken in the foundational discussion by Hilbert, Bernays, and their collaborators is widely perceived as extremely narrow and technical. So I will first give a revisionary description of the goals of Hilbert's Program, discuss some important results that have been obtained within a general reductive program, and finally formulate the reductive structuralist standpoint (with its philosophical and mathematical challenges).

[1] W. SIEG, *Relative consistency and accessible domains*, ***Synthese***, vol. 84 (1990), pp. 259–297.

[2] ———, *Aspects of mathematical experience*, in ***Philosophy of Mathematics Today*** (E. Agazzi and G. Darvas, editors), Kluwer Academic Publishers, 1997, pp. 197–217.

► ROBERT I. SOARE, *Computability theory and differential geometry*.
Department of Mathematics, University of Chicago, Chicago, Illinois 60637-1546, USA.
E-mail: soare@math.uchicago.edu, soare@cs.uchicago.edu.

In their preprint, *The Fractal Nature of Reim/Diff*, the authors Alex Nabutovsky and Shmuel Weinberger (NW) begin, "In this paper we would like to expose some of the astonishing richness of the space of Riemannian metrics on a smooth manifold, up to reparametrization." Part of this richness is due to the connection of computability, and in particular computably enumerable (c. e.) sets, to the geometry, as we will explain in this talk. It is particularly interesting that these are not merely undecidability results, but results about the complexity of the geometry using the complexity of the c. e. sets.

The authors write "there are large 'basins' that have topology, and are repeated infinitely often within the space," and "there seem to be infinitely many different sorts of basins with different geometries from each other." They show that, under certain circumstances, deciding whether a configuration is a basin is equivalent to an element being enumerated in a certain c. e. set, $x \in W_e$, and even more surprising, that the depth of that basin is roughly the same as the least stage s such that $x \in W_{e,s}$. To help demonstrate this geometric complexity the authors NW asked Soare to prove a result that there exists a decreasing sequence $\{A_n\}_{n\in\omega}$ of c. e. sets with a certain property ensuring that for almost every x, the initial segment $A_{n+1}\restriction x$ settles much more quickly than $A_n\restriction x$ even when the former halting stages $\{s_x^n\}_{x\in\omega}$ are composed with any computable function f, namely $f(s_x^{n+1}) < s_x^n$ for almost every x. NW use this to show that there is an infinite sequence of levels in the geometry $\{L_n\}_{n\in\omega}$ such that almost all basins at level L_{n+1} have depth less than those of L_n, even if the former depths are composed with any computable function. This is an unexpected geometric complexity.

► AVI WIGDERSON, *Some fundamental insights of computational complexity theory*.
Institute for Advanced Study, School of Mathematics, Olden Lane, Princeton, New Jersey 08540, USA, and The Hebrew University, Jerusalem, Israel.
E-mail: avi@cs.huji.ac.il.

Computational Complexity Theory has been one of the most exciting and successful scientific disciplines in the last few decades, with dramatic achievements and fundamental understandings appearing at high rate.

One obvious explanation is that this research is guided by a few clear and focused questions, deeply motivated on scientific, practical and philosophical grounds. The most central of these are

1. P = NP, or more generally, are there natural computationally difficult problems?
2. P = BPP, or more generally, does randomization help computation?
3. NP = coNP, or more generally, what constitutes a difficult theorem to prove?
4. BPP = QP, or more generally, can quantum mechanics be efficiently simulated classically?

Resolving any of these questions is clearly a very long term goal, but each has stimulated the development of concepts, problems, proof techniques and results which start paving a path towards a possible resolution.

But what really characterized the progress, and explained much of the successes so far, was the unveiling of many rich and beautiful connections between the sets of concepts and subproblems each of these major questions gave rise to.

In this talk, I would like to probe some of the unique features and insights of the complexity theory viewpoint which afforded these connections. This will be done by considering how (and why) notions which intrigued humanity for centuries or even millenia (like Knowledge, Randomness, Cryptography, Learning, Proof, and naturally, Computation), reveal new dimensions, and are surprisingly linked together, when viewed from our special Computational Complexity glasses.

► ALEX J. WILKIE, *O-minimal expansions of the reals: techniques and problems*.
Mathematical Institute, 24-29 St Giles, Oxford OX1 3LB, UK.
E-mail: `wilkie@maths.ox.ac.uk`.

I survey the methods that have been used to establish that various natural expansions of the real ordered field are o-minimal. I also report on the humbling difficulties that seem to have arisen in pushing these methods to what seemed like their natural limits.

Abstracts of invited panel discussions

► PANELISTS: MARTIN DAVIS, DONALD A. MARTIN, WILLIAM TAIT, AND CAROL WOOD, *The development of logic in the 20th century*.
New York University and University of California, Berkeley; University of California, Los Angeles; University of Chicago; Wesleyan University.

This panel will address some topics in the history of logic in the twentieth century. In addition to individual presentations there will be general discussion. Carol Wood will speak on "Defining Moments in Model Theory" giving a selection of results as well as indicating their impact on the maturation of model theory during the century. William Tait will speak about the extended Hilbert program in the foundations of mathematics. Martin Davis will speak on "Gödel's Legacy" surveying his major discoveries, how his philosophical views changed over time, and what all of this suggests for the future. Donald A. Martin will speak about the development of set theory and about its changing relationships to other parts of logic and mathematics and to philosophy.

► PANELISTS: SOLOMON FEFERMAN, HARVEY FRIEDMAN, PENELOPE MADDY, AND JOHN STEEL, *Does mathematics need new axioms?*.
Stanford University; Ohio State University; University of California, Irvine; University of California, Berkeley.

This panel will discuss a range of issues surrounding contemporary axiomatics: e.g., precisely which axioms are needed for which theorems?, when is it appropriate to add new axioms?, what sort of justification is needed for adding new axioms? Particular attention will focus on the search for new axioms to settle open questions of set theory like the Continuum Hypothesis, and the search for applications of large cardinal axioms to discrete and finite mathematics.

► PANELISTS: SAM BUSS, ALEKOS KECHRIS, ANAND PILLAY, AND RICHARD SHORE, *The prospects for mathematical logic in the 21st century*.
University of California, San Diego; Caltech; University of Illinois at Urbana-Champaign; Cornell University.

This panel will discuss prospects and expectations for future developments in mathematical logic. Like the rest of mathematics, mathematical logic underwent a truly revolutionary development in the past century. Also like the rest of mathematics, it faces the challenges of, firstly, coping with increasing technical depth and specialization of core research, and, secondly, responding to ever-growing applications arising from the increasing mathematization and computerization of human society.

For the first part of the panel, the panel members will discuss the future prospects of their respective subfields of mathematical logic, concentrating particularly on areas that may become, or should become, more important.

In the second part, the panel discussion will turn to broader issues, including topics such as:

- How will logic interact with computer science?
- How will logic interact with other fields of mathematics?
- Can mathematical logic remain unified or is it fragmenting into disjoint fields?
- What should be the role of mathematical logic in graduate level mathematics education, in computer science education, in general college level education?

It is hoped that the panel discussions will include substantial audience participation in the form of questions and comments.

Abstracts of invited talks in the Special Session in Philosophy of Mathematics

► PATRICIA BLANCHETTE, *Universalism: A Fregean perspective*.
Department of Philosophy, University of Notre Dame, Notre Dame, Indiana 46556, USA.
E-mail: `blanchette.1@nd.edu`.

This paper explores what Sheffer calls the "logocentric predicament"—i.e., that "in order to give an account of logic, we must presuppose and employ logic"—especially with respect to the difficulties (purportedly) arising in the metatheory of formal systems intended to be "universal" in scope. Of particular concern here is the question of whether Frege's brand of universalism entails the incoherence of metatheory, either for reasons that were or should have been clear to Frege, or for reasons made clear only in light of later (e.g., incompleteness) results. It is argued, contrary to a variety of recent interpretations, that universalism of a Fregean stripe does not entail the incoherence of metatheory.

► GILA SHER, *Tarski's thesis: Philosophical grounding and defense*.
Department of Philosophy, University of California, San Diego, La Jolla, California 92093, USA.
E-mail: `gsher@ucsd.edu`.

In his 1966 lecture (first published in 1986), "What Are Logical Notions?", Tarski proposed a criterion for logical constants generalizing Klein's Erlanger program of classifying geometrical notions based on their invariance properties. Lindström proposed a similar criterion (in the same year) generalizing Mostowski's 1957 notion of logical quantifier. In my book, ***The Bounds of Logic: A Generalized Viewpoint*** (1991), I arrived at essentially the same criterion by asking: what notion of logical constant will allow us to capture the intuitive notion of logical consequence in full generality? In the present lecture I further develop the philosophical motivation for this criterion and respond to three criticisms due to Feferman in a recent paper, "Logic, Logics and Logicism".

► JAMIE TAPPENDEN, *Transformation invariance in Frege's foundations of geometry*.
Department of Philosophy, 2228 Angell Hall, University of Michigan, Ann Arbor, Michigan 48109, USA.
E-mail: `tappen@umich.edu`.

This talk will develop one surprising respect in which an aspect of Frege's foundations of geometry arose from his "mainstream" mathematical activity. In his teaching and "mainstream" geometrical research, Frege worked on transformations in projective geometry, and in particular Lie's theory of contact transformations. I'll indicate some places in Frege's philosophical and logical work that reflect this geometrical research, and then relate Frege's stance to the treatment in Artin's Geometric Algebra and to contemporary discussions of logic that specify logical constants in the way that the Klein Erlangen program specifies geometries.

Abstracts of invited talks in the Special Session in Model Theory

► STEVEN BUECHLER, *Dividing in nonelementary classes*.
Mathematics Department, University of Notre Dame, Notre Dame, Indiana 46556, USA.
E-mail: Steven.A.Buechler.1@nd.edu.

Stability and simplicity are properties of elementary classes that lead to deep results about these classes. Researchers can now formulate these properties for some nonelementary classes in a way that results in a dependence relation with the basic properties of forking. These basic foundational theorems and several examples will be discussed.

► THOMAS SCANLON, *Euler characteristics and Grothendieck rings of first-order structures*.
Department of Mathematics, University of California, Berkeley, Evans Hall, Berkeley, California 94720-3840, USA.
E-mail: scanlon@math.berkeley.edu.

For a first-order structure $\mathfrak{M}$ denote by $\mathrm{Def}(\mathfrak{M})$ the set of all definable subsets of any power of $\mathfrak{M}$. Two definable sets X and Y are definably isomorphic if there is a definable bijection between X and Y. For $X \in \mathrm{Def}(\mathfrak{M})$ denote by $[X]$ the equivalence class of definable sets definably isomorphic to X. The set of definable sets up to definable isomorphism, $\widetilde{\mathrm{Def}}(\mathfrak{M})$, is an $\mathcal{L}(+, \cdot, 0, 1)$-structure with $[X] + [Y] := [X' \cup Y']$ for $[X'] = [X]$, $[Y'] = [Y]$, and $X' \cap Y' = \emptyset$; $[X] \cdot [Y] := [X \times Y]$; $0 := [\emptyset]$, and $1 := [\{*\}]$ for any element $* \in \mathfrak{M}$.

A (weak) Euler characteristic for $\mathfrak{M}$ is an $\mathcal{L}(+, \cdot, 0, 1)$-homomorphism $\chi\colon \widetilde{\mathrm{Def}}(\mathfrak{M}) \to R$ where R is a ring. A strong Euler characteristic is a weak Euler characteristic satisfying the fiber condition: if $f : A \to B$ is a definable function between definable sets and $c \in R$ so that $c = \chi(f^{-1}\{b\})$ for all $b \in B$, then $\chi(A) = c \cdot \chi(B)$.

We show that $\mathfrak{M}$ admits a universal weak Euler characteristic $\chi_0\colon \widetilde{\mathrm{Def}}(\mathfrak{M}) \to K_0(\mathfrak{M})$ where $K_0(\mathfrak{M})$ is called the Grothendieck ring of $\mathfrak{M}$. Moreover, $\mathfrak{M}$ also admits a universal strong Euler characteristic. We investigate the dependence of Grothendieck groups on the theory of a structure showing that $\mathfrak{M} \equiv \mathfrak{N} \Longrightarrow K_0(\mathfrak{M}) \equiv_{\exists_1} K_0(\mathfrak{N})$. We discuss $K_0(F)$ for F a field showing among other things that $K_0(\mathbb{C})$ contains a subring isomorphic to the polynomial ring in continuum many variables over $\mathbb{Z}$. We close with some questions about general dimension functions introduced by Schanuel.

[This is joint work with Jan Krajíček.]

[1] J. Krajíček and T. Scanlon, *Combinatorics with definable sets*: *Euler characteristics and Grothendieck rings*, this Bulletin, to appear.

[2] S. H. Schanuel, *Negative sets have Euler characteristic and dimension*, in ***Category theory***, Como '90 (A. Carboni, M. Pedicchio, and G. Rosolini, editors), Lecture Notes in Mathematics, vol. 1488, Springer-Verlag, 1991, pp. 379–385.

► CHARLES STEINHORN, *On classes of finite structures with dimension and measure*.
Mathematics Department, Vassar College, Poughkeepsie, New York 12604-0257, USA.
E-mail: steinhorn@vassar.edu.

This is joint work (in progress) with Dugald Macpherson, the point of which is to develop model theory for classes of finite structures having dimension and measure. The definition of dimension and measure is directly inspired by the work of Chatzidakis, van den Dries, and Macintyre on the asymptotics of definable sets in finite fields (*Definable sets over finite fields*, ***Journal für die Reine und Angewandte Mathematik***, 1992).

► KATRIN TENT, *BN-pairs of finite Morley rank*.
Mathematisches Institut, Universität Würzburg, 97074 Würzburg, Germany.
E-mail: tent@mathematik.uni-wuerzburg.de.

We report on recent progress on the Cherlin-Zil'ber Conjecture, viz. that a simple group of finite Morley rank is an algebraic group over an algebraically closed field. The conjecture implies in particular that any such group must have a definable 'split' BN-pair. Using the connections between BN-pairs and generalized polygons we show the following.

THEOREM 1 ([2]). *If G is a simple group of finite Morley rank with a definable irreducible BN-pair of (Tits-) rank 2 such that* $\mathrm{RM}(P_i/B) = 1$ *for both proper parabolic subgroups* P_1, $P_2 \geq B$, *then* $G \cong PSL_3(K)$, $PSp_4(K)$ *or* $G_2(K)$ *for some algebraically closed field* K.

This implies the Cherlin-Zil'ber Conjecture for all groups whose associated building has panels of Morley rank 1.

The following is a partial analog of the famous paper by Fong and Seitz on finite groups with split BN-pairs:

THEOREM 2 ([1]). *Let G be a simple group with a definable BN-pair of rank* 2 *where* $B = U.T$ *for* $T = B \cap N$ *and a normal subgroup U of B with* $Z(U) \neq 1$. *It was shown in* [3] *that the Weyl group* $W = N/B \cap N$ *has cardinality* $2n$ *with* $n = 3, 4, 6, 8$ *or* 12. *If G has finite Morley rank then furthermore the following holds*:

(i) *If* $n = 3$, *then G is definably isomorphic to* $PSL_3(K)$ *for some algebraically closed field* K.
(ii) *If U is nilpotent, then* $n \neq 12$.
(iii) *If* $Z(U)$ *contains a B-minimal subgroup A with* $\mathrm{RM}(A) \geq \mathrm{RM}(P_i/B)$ *for both parabolic subgroups* P_1 *and* P_2, *then* $n = 3$, 4 *or* 6 *and G is definably isomorphic to* $PSL_3(K)$, $PSp_4(K)$ *or* $G_2(K)$ *for some algebraically closed field* K.

[1] K. TENT, *Split BN-pairs of finite Morley rank*, preprint.
[2] K. TENT and H. VAN MALDEGHEM, *Cherlin's conjecture for buildings with panels of Morley rank* 1, preprint.
[3] ———, *On irreducible* (B, N)*-pairs of rank* 2, preprint.

► SIMON THOMAS, *Asymptotic cones of finitely generated groups*.
Mathematics Department, Rutgers University, New Brunswick, New Jersey 08903, USA.
E-mail: sthomas@math.rutgers.edu.

If an observer moves steadily away from the Cayley graph of a finitely generated group, then any finite configuration will eventually become indistinguishable from a single point; but he may observe certain finite configurations which resemble earlier configurations. The asymptotic cone is a topological space which encodes all of these recurring finite configurations. Unfortunately the construction of an asymptotic cone involves the choice of a nonprincipal ultrafilter on the set of natural numbers, and it was not clear whether the resulting asymptotic cone depended on the choice of the ultrafilter. In this talk, answering a question of Gromov, I shall present an example of a finitely generated group which has two non-homeomorphic asymptotic cones. This is joint work with Boban Velickovic.

Abstracts of invited talks in the Special Session in Computability Theory

► PETER CHOLAK, *Maximal contiguous degrees*.
Department of Mathematics, University of Notre Dame, Notre Dame, Indiana 46556-5659, USA.
E-mail: Peter.Cholak.1@nd.edu.

A computably enumerable (c. e.) degree is contiguous if it does not split into two or more weak truth table degrees. By Downey and Lempp [2], the property of a c. e. degree being contiguous is definable in the c. e. degrees. There is a maximal contiguous degree; a contiguous degree above which there are no contiguous degrees. Hence the class of maximal contiguous degrees is non-empty; in fact there are infinitely many maximal contiguous degrees. So the class of maximal contiguous degrees provides the first example of a definable infinite anti-chain in c. e. degrees. In addition, the class of maximal contiguous degrees forms an automorphism base for the c. e. degrees. It is easy to modify this result to show that there is an a. c. (array computable) degree above which there are no contiguous degrees. This refutes the conjecture of Walk [3] and Downey that the array computable degrees are naturally defined as the degrees above which there is a contiguous degree. This work is joint with Rod Downey and Steve Walk and will appear in [1].

[1] P. Cholak, R. Downey, and S. Walk, *Maximal contiguous degrees*, in preparation.

[2] R. G. Downey and S. Lempp, *Contiguity and distributivity in the enumerable Turing degrees*, ***Journal of Symbolic Logic***, vol. 62 (1997), no. 4, pp. 1215–1240.

[3] S. M. Walk, *Toward the definability of the array noncomputable degrees*, ***Ph. D. thesis***, University of Notre Dame.

► R. G. DOWNEY, GEOFFREY LAFORTE*, AND R. A. SHORE, *Decomposition and infima in the c. e. degrees*.
*Institute for Human and Machine Cognition, 40 S. Alcaniz, Pensacola, Florda 32501, USA.
E-mail: glaforte@ai.uwf.edu.

Although the partial order of the Turing degrees forms an upper semilattice, there is no natural notion of the greatest information that two sets share, a fact first demonstrated explicitly for the degrees in general by Spector, [6]. The existence of pairs of computably enumerable degrees with no infimum was established independently by Lachlan, [1], and Yates, [7]. By the Sacks splitting theorem, [3], every nontrivial c. e. degree can be decomposed as the join of a pair of incomparable degrees below it. We show that given any two incomparable computably enumerable sets A and B there exists a c. e. set C computable from their join such that C is the infimum of its joins with A and B respectively. This result complements and extends well-known theorems of Lachlan, [2], and Slaman, [5], concerning the structure of the usl of c. e. Turing degrees. The proof of our main theorem has an added technical interest because it involves a nontrivial extension of the transfinitely-branching-tree method introduced originally by Shore, [4], to prove the noninversion theorem for the jump operator.

[1] A. Lachlan, *Lower bounds for pairs of recursively enumerable degrees*, ***Proceedings of the London Mathematical Society***, vol. 3 (1966), no. 16, pp. 537–569.

[2] ———, *Decomposition of recursively enumerable degrees*, ***Proceedings of the American Mathematical Society***, vol. 79 (1980), pp. 629–634.

[3] G. Sacks, *On the degrees less than* $\mathbf{0}'$, ***Annals of Mathematics***, vol. 2 (1963), no. 77.

[4] R. Shore, *A non-inversion theorem for the jump operator*, ***Annals of Pure and Applied Logic***, vol. 40 (1988), pp. 277–303.

[5] T. A. Slaman, *The density of infima in the recursively enumerable degrees*, ***Annals of Pure and Applied Logic***, vol. 52 (1991), pp. 155–179.

[6] C. SPECTOR, *On degrees of recursive unsolvability*, ***Annals of Mathematics***, vol. 64 (1956), pp. 581–592.

[7] C. E. M. YATES, *A minimal pair of recursively enumerable degrees*, ***Journal of Symbolic Logic***, vol. 31 (1966), pp. 159–168.

► VALENTINA HARIZANOV, *Immune relations on computable structures*.
Department of Mathematics, The George Washington University, Washington, D. C., 20052, USA.
E-mail: harizanv@gwis2.circ.gwu.edu.

We study Post type computability theoretic properties of substructures of computable structures or, in general, of relations on the domains of computable structures. Examples of such structures include $\mathcal{V}_\infty$, an infinite-dimensional computable vector space over a computable field, which has a computable basis, and also include computable linear orderings and computable Boolean algebras. This study imitates classical computability theory, that is, the study of the algorithmic properties of the structure $(\omega, =)$. However, classical results do not always carry over to other computable structures. In the case of $\mathcal{V}_\infty$ (a Steinitz closure system with dependence algorithm) there is an interesting interplay between the computability-theoretic properties of a subset of a computable basis and computable model theoretic properties of the space (closed set) that it generates. A nontrivial relation on the domain of a computable structure is immune if it does not contain any formally infinite c. e. relation, where a formally infinite c. e. relation is defined syntactically using existential effective formulae. We investigate various kinds of immune relations on computable structures and their Turing degrees.

► JEFFRY L. HIRST, *Reverse mathematics and Brouwer's fixed point theorem*.
Department of Mathematical Sciences, Appalachian State University, Boone, North Carolina 28608, USA.
E-mail: jlh@math.appstate.edu.

In [1], Arnold gives a short proof of the fundamental theorem of algebra using Brouwer's fixed point theorem. From a reverse mathematics viewpoint, the fundamental theorem of algebra is provable in $\mathbf{RCA}_0$ [3], while Brouwer's theorem is equivalent to the stronger system $\mathbf{WKL}_0$ [2]. In this talk, a computable restriction of Brouwer's theorem will be presented, yielding a corresponding formalization of Arnold's proof using only $\mathbf{RCA}_0$.

[1] B. H. ARNOLD, *A topological proof of the fundamental theorem of algebra*, ***American Mathematical Monthly***, vol. 56 (1949), pp. 465–466.

[2] N. SHIOJI and K. TANAKA, *Fixed point theory in weak second-order arithmetic*, ***Annals of Pure and Applied Logic***, vol. 47 (1990), pp. 167–188.

[3] S. G. SIMPSON, ***Subsystems of second order arithmetic***, Springer-Verlag, Berlin and Heidelberg, 1999.

► BAKHADYR KHOUSSAINOV, *On positive algebras*.
Department of Computer Science, University of Auckland, Private Bag 92019, Auckland, New Zealand.
E-mail: bmk@cs.auckland.ac.nz.

An algebra is *positive* if its positive diagram (the set of all atomic formulas without negation) is computably enumerable (c. e.). A canonical way of obtaining examples of positive algebras is the following. Let E be a c. e. equivalence relation on ω. Let ω_E be the set of all E-equivalence classes. A computable n-ary function f on ω *respects* E if f induces (in a natural way) a function, denoted by f, on ω_E. Then any algebra $(\omega_E, f_1, \dots, f_n)$, where each f_i respects E, is called an E-*algebra*. Clearly every E-algebra is positive. On the other hand, each positive algebra is an E-algebra for some E. Let P_E be the class of all E-algebras for a given E.

In the setting above two topics of study arise naturally. One is related to understanding the relationship between algebraic (or model-theoretic) properties of E-algebras and computable-theoretic properties of E. The other is related to understanding the relationship between different P_E classes.

In this talk we present several examples of results concerning these two topics. Some results and definitions are obtained jointly with N. Kassymov.

► REED SOLOMON, *The Lindenbaum algebra of the class of all finite models*.
Department of Mathematics, University of Wisconsin, 480 Lincoln Drive, Madison, Wisconsin 53706-1388, USA.
E-mail: rsolomon@math.wisc.edu.

One method of studying a theory T is to examine its Lindenbaum algebra $L(T)$. In this talk, we let T be the theory of the class of all finite models of a sufficiently rich signature. We classify $L(T)$ up to isomorphism by showing that $L(T)$ modulo its Fréchet ideal is universal over the class of Σ^0_2-Boolean algebras. This material is joint work with Steffen Lempp and Mikhail Peretyat'kin. Time permitting, we will discuss similar work by Selivanov on other semantic classes of models.

Abstracts of invited talks in the Special Session in Proof Theory and Complexity

► NORIKO H. ARAI, *Toward automated intellectual reasoning*.
Department of Computer Science, Hiroshima City University, 3-4-1 Ozuka-higashi, Asaminami-ku, Hiroshima 731-31 Japan.
E-mail: narai@cs.hiroshima-cu.ac.jp.

This is joint work with Alasdair Urquhart.

Since A. Haken [3] showed that the pigeonhole principle is exponentially hard for resolution, many other examples were added to the list of tautologies which require superpolynomially long proofs for resolution and analytic tableaux. Actually, most of the interesting combinatorial problems were found hard for these proof systems.

In Krishnamurthy (1985), it is shown that resolution with a new inference rule called *symmetry rule* polynomially proves the pigeonhole principle. The symmetry rule can be described as a restricted version of "without-loss-of-generality" argument, which is frequently used to shorten proofs in combinatorics. When the symmetry rule is added to cut-free LK, we can polynomially prove many combinatorial problems including the pigeonhole principle, the mod-k principle, the nonunique end-node principle and Bondy's theorem [1, 2]. The cutting plane is another propositional system which polynomially proves the pigeonhole principle.

In this talk, we discuss the relative complexity of resolution with symmetry rule, cutting planes and cut-free LK with symmetry rule. We show that resolution with symmetry rule and cutting planes are incomparable in terms of p-simulation.

[1] N. H. ARAI, *Tractability of cut-free Gentzen type propositional calculus with permutation inference*, ***Theoretical Computer Science***, vol. 170 (1996), pp. 129–144.

[2] ———, *Tractability of cut-free Gentzen type propositional calculus with permutation inference II*, ***Theoretical Computer Science***, to appear.

[3] A. HAKEN, *The intractability of resolution*, ***Theoretical Computer Science***, vol. 39 (1985), pp. 297–308.

► TOSHIYASU ARAI, *The epsilon substitution method revisited*.
Hiroshima University, Faculty of Integrated Arts and Sciences, Higashi-Hiroshima 739-8521, Japan.
E-mail: arai@mis.hiroshima-u.ac.jp.

I will discuss termination problems in the epsilon substitution method. The problem of termination was first raised by D. Hilbert, and was attacked by J. von Neumann and W. Ackermann. Recently it has been revived by G. Mints et al.

► ARNOLD BECKMANN, *Can consistency statements separate bounded arithmetic?*.
Department of Mathematics, University of California, San Diego, 9500 Gilman Drive, La Jolla, California 92093-0112, USA.
E-mail: abeckman@math.ucsd.edu.

Takeuti conjectured (cf. [3], p. 5, problem 9): "Let $S_2^{-\infty}$ be the equational theory involving equations $s = t$, where s, t are closed terms in the language of S_2, with natural rules based on recursive definitions of the functions symbols. Show that $S_2 \nvdash \mathrm{Con}(S_2^{-\infty})$. [...]."

If this conjecture would be true then it would be likely that consistency statements cannot be used to negatively answer the finitely axiomatizability question of bounded arithmetic, or, equivalently, the question if the hierarchy of bounded arithmetic theories is proper. In this talk we will disprove this conjecture, thus there is hope that consistency statements can lead to a negative answer.

More generally we will consider equational theories for functions defined via recursion involving equations between closed terms with natural rules based on recursive definitions of the function symbols. These equational theories are connected to term rewriting relations for which we prove a feasible Church-Rosser-property for paths starting from numerals. This way we show that the consistency of such equational theories can be proved in the fragment S_2^1 of bounded arithmetic [1].

[1] A. Beckmann, *Proving consistency of equational theories in bounded arithmetic*, submitted to the ***Journal of Symbolic Logic***, 1999.

[2] Samuel R. Buss, ***Bounded arithmetic***, Studies in Proof Theory, Lecture Notes, 3. Bibliopolis, Naples, 1986.

[3] P. Clote and J. Krajíček, *Open problems*, in ***Arithmetic, proof theory, and computational complexity*** (P. Clote and J. Krajíček, editors), Oxford University Press, Oxford, 1993, pp. 1–9.

► TIMOTHY J. CARLSON, *Applications of ranked partial structures*.
Department of Mathematics, Ohio State University, Columbus, Ohio 43210, USA.
E-mail: carlson@math.ohio-state.edu.

A ranked partial structure can be thought of as a hierarchy approximating some structure where each stage contains the values of the functions of the structure applied to elements of earlier stages. We will discuss a semantics for ranked partial structures and several applications, in particular, model-theoretic versions of cut-elimination.

► OLIVER KULLMANN, *Resolution for conjunctive normal forms derived from first order sentences*.
Department of Computer Science, University of Toronto, Toronto, Ontario, Canada M5S 3G4.
E-mail: kullmann@cs.toronto.edu.

In [2] an interesting "gap behaviour" of tree resolution for sequences of clause-sets derived from first-order sentences has been shown: Either the sequence has *polynomial* size tree resolution refutations, or every resolution tree refutation is of (sub-) *exponential* size.

I will further investigate this phenomenon, and relate it to recent *general* upper and lower bounds for relativised (tree) resolution (see [1]).

Supported by the Natural Sciences and Engineering Research Council of Canada and by the Communications and Information Technology Ontario.

[1] Oliver Kullmann, *Investigating a general hierarchy of polynomially decidable classes*

of CNF's based on short tree-like resolution proofs, ***Technical Report*** TR99-041, Electronic Colloquium on Computational Complexity (ECCC), October 1999.

[2] SØREN RIIS, *A complexity gap for tree-resolution*, ***Technical Report*** RS-99-29, BRICS Basic Research in Computer Science, September 1999.

► ALBERTO MARCONE, *Wqo and bqo theory in subsystems of second order arithmetic*.
Dipartimento di Matematica e Informatica, Università di Udine, viale delle Scienze 206, 33100 Udine, Italy.
E-mail: marcone@dimi.uniud.it.

The concept of well quasi-ordering (wqo) is the most natural generalization of the notion of well ordering to partial orderings. However for many purposes the notion of wqo is too weak and this led to the introduction by Nash-Williams in the '60s of the notion of better quasi-ordering (bqo). Both concepts have received quite a bit of attention from logicians. In this talk we look at wqo theory and bqo theory from the viewpoint of *reverse mathematics* ([6]).

We start with a survey of results by Friedman [4], Simpson [5], Shore [3] and the author [2]. Then we deal with some more recent work which focuses on the interplay between Ramsey theory and the elementary parts of these subjects. In particular the most elementary results of wqo theory appear to be related to the Ramsey theorem for exponent 2 (which has been the object of much recent work [1]), while the basics of bqo theory are connected with the clopen Ramsey theory.

[1] P. A. CHOLAK, C. G. JOCKUSCH and T. A. SLAMAN, *On the strength of Ramsey's theorem for pairs*, ***Journal of Symbolic Logic***, to appear.

[2] A. MARCONE, *On the logical strength of Nash-Williams' theorem on transfinite sequences*, in ***Logic: from foundations to applications***, Clarendon Press, 1996, pp. 327–351.

[3] R. A. SHORE, *On the strength of Fraïssé's conjecture*, in ***Logical methods***, Birkhäuser, 1993, pp. 782–813.

[4] S. G. SIMPSON, *Nonprovability of certain combinatorial properties of finite trees*, in ***Harvey Friedman's research on the foundations of mathematics***, North-Holland, 1985, pp. 87–117.

[5] ———, *Ordinal numbers and the Hilbert basis theorem*, ***Journal of Symbolic Logic***, vol. 53 (1988), pp. 961–974.

[6] ———, ***Subsystems of second order arithmetic***, Springer-Verlag, 1999.

Abstracts of invited talks in the Special Session in Set Theory

► JÖRG BRENDLE, *Random reals and club principles*.
Graduate School of Science and Technology, Kobe University, Rokko-dai 1-1, Nada, Kobe, 657-8501, Japan.
E-mail: brendle@pascal.seg.kobe-u.ac.jp.

This talk is about the effect of cardinal invariants of the continuum on combinatorial properties of uncountable cardinals.

Given regular cardinals $\kappa < \lambda$, let $\clubsuit^\kappa_\lambda$ be the club principle for guessing subsets of λ at limit points of cofinality κ, i.e., $\clubsuit^\kappa_\lambda$ asserts there is a sequence $\langle A_\alpha ; A_\alpha$ is cofinal in $\alpha,\ \alpha < \lambda$, $\mathrm{cf}(\alpha) = \kappa \rangle$ such that for each $A \in [\lambda]^\lambda$ there is α with $A_\alpha \subseteq \alpha$. As usual, $\clubsuit$ denotes $\clubsuit^\omega_{\omega_1}$. Let $\mathfrak{c}$ be the cardinality of the continuum. A result implicit in work of Fuchino, Shelah, and Soukup [2] says that, given a model of *GCH* with regular cardinals $\kappa < \lambda$, there is a forcing extension satisfying $\lambda < \mathfrak{c}$, $\clubsuit^\kappa_\lambda$ and Martin's axiom *MA* for countable partial orderings (i.e., for Cohen forcing).

This suggests one investigates whether a similar consistency result can be obtained for other (possibly stronger) forms of Martin's axiom as well. For random forcing we have

THEOREM 1 ([1]). *It is consistent $\mathfrak{c}$ is large, and ♣ and MA for random forcing both hold.*

The argument for this result is similar to, yet much more involved and less general than, the one of [2]. The reason for the lack of generality is

THEOREM 2 ([1]). *Let $\kappa \leq \lambda$ be uncountable cardinals. Assume $\mathcal{F} \subseteq [\lambda]^\kappa$ is such that for all $A \subseteq \lambda$ there is $B \in \mathcal{F}$ with $B \subseteq A$ or $B \subseteq \lambda \setminus A$. Then the size of $\mathcal{F}$ is at least the covering number of the null ideal.*

This immediately entails

COROLLARY. *If $\kappa < \lambda < \mathfrak{c}$ are regular uncountable and MA holds for random forcing, then $♣_\lambda^\kappa$ fails.*

Note that, by an old result of Truss, $\mathfrak{c} \geq \aleph_2$ and MA for both random and Cohen forcing imply ♣ fails. A number of results similar to either Theorem 1 or Theorem 2 can be proved with MA(random) replaced by a corresponding statement about some other cardinal invariant of the continuum.

In the talk, we shall give an outline of the proofs of Theorems 1 and 2.

[1] J. BRENDLE, *Cardinal invariants of the continuum and combinatorics on uncountable cardinals*, in preparation.

[2] S. FUCHINO, S. SHELAH, and L. SOUKUP, *Sticks and clubs*, ***Annals of Pure and Applied Logic***, vol. 90 (1997), pp. 57–77.

► SU GAO, *Classifications of metric structures and descriptive set theory*.
Department of Mathematics, California Institute of Technology, Pasadena, California 91125, USA.
E-mail: sugao@its.caltech.edu.

In this talk we survey some recent results on the classification problems for various classes of Polish metric spaces. The research is joint with A. S. Kechris.

Classical results of Gromov showed that compact metric spaces and general Polish metric-measure spaces are concretely classifiable. It was speculated that the classification problem for the class of *all* Polish metric spaces is enormously complicated. Here we quantify this enormity by proving that the isometry for Polish metric spaces is universal for all orbit equivalence relations induced by Borel actions of Polish groups. This also seems to provide the first natural example of equivalence relations of this complexity.

The classification problems for various special classes of Polish metric spaces are also studied. In particular, we consider special classes obtained by imposing one or more properties in the following list: local compactness, connectedness, pseudo-connectedness, homogeneity, ultrahomogeneity, zero-dimensionality and ultrametrizability.

The framework in which the complexity of these classification problems can be precisely measured is the theory of Borel reducibility for definable equivalence relations.

► ALBIN JONES, *Polarized partition relations via elementary substructures*.
Department of Mathematics, Kenyon College, Gambier, Ohio 43022, USA.
E-mail: jones@kenyon.edu.

If one is given a family $\{ f_i : \lambda \to \mu \}_{i<\kappa}$ of partitions of λ, a natural question is: Can one find α many of the f_i's which are constant and equal at β many points? An organized assault on questions of this sort was begun in 1956 by P. Erdős and R. Rado when they introduced the polarized partition calculus in their paper, *A partition calculus in set theory*. In this and later work, Erdős and Rado (together with A. Hajnal) almost completely codified the polarized partition theory of cardinals (that is, where each of κ, λ, α, and β above are cardinals) in the presence of the GCH. Only a few outstanding problems in this area remain. In contrast, the polarized partition theory of cardinals in the absence of the GCH and that of more exotic types (e.g., ordinals, graphs, partial orders) seem to remain largely unresolved.

In this talk we will describe several recent results and open problems in the polarized partition calculus of ordinals. A new technique which uses elementary substructures to prove polarized partition relations will be demonstrated.

► ANDRZEJ ROSŁANOWSKI, *World of creatures*.
Department of Mathematics, University of Nebraska at Omaha, Omaha, Nebraska 68182-0243, USA.
E-mail: roslanowski@unomaha.edu.

The method of building forcing notions using *norms on possibilities* was introduced in Shelah [4]. A systematic study of this technique was presented in Rosłanowski and Shelah [3] (the non-ccc case) and [2] (the ccc case).

We will present highlights of this method and its most recent applications. We will discuss aspects of Martin's Minimum (see Bartoszyński and Rosłanowski [1]), *Sweet and Sour* properties (see Rosłanowski and Shelah [2]), and some questions related to the sup-measurability problem.

[1] TOMEK BARTOSZYŃSKI and ANDRZEJ ROSŁANOWSKI, *Towards Martin's minimum*, ***Archive for Mathematical Logic***, accepted, http://front.math.ucdavis.edu/math.LO/9904163.

[2] ANDRZEJ ROSŁANOWSKI and SAHARAON SHELAH, *Sweet & sour and other flavours*, submitted, http://front.math.ucdavis.edu/math.LO/9909115.

[3] ———, *Norms on possibilities I: forcing with trees and creatures*, ***Memoirs of the American Mathematical Society***, vol. 141 (1999), no. 671.

[4] SAHARON SHELAH, *On cardinal invariants of the continuum*, in ***Axiomatic set theory*** (Boulder, Colorado, 1983) (J. Baumgartner, D. Martin, and S. Shelah, editors), Contemporary Mathematics, vol. 31, American Mathematical Society, Providence, Rhode Island, 1984, Proceedings of the Conference in Set Theory, Boulder, June 1983, pp. 183–207.

► ERNEST SCHIMMERLING, *Core models*.
Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213-3890, USA.
E-mail: eschimme@andrew.cmu.edu.

Extenders are certain directed systems of filters and core models are universes constructed from sequences of extenders. Jensen style fine structure and its generalizations have been used to prove that certain core models satisfy combinatorial principles and covering properties. Answers to questions about lower bounds on large cardinal consistency strength typically involve core models, and core models provide important test cases when it comes to upper bounds. We will discuss aspects of the general theory and give some applications.

Abstracts of invited talks in the Special Session in Constructivism and Categorical Logic

► ANDREJ BAUER, *A logic for computable analysis and topology*.
School of Computer Science, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA.
E-mail: andrej.bauer@cs.cmu.edu.

One approach to developing computable analysis and topology is to work in a logic that does not allow any non-constructive reasoning. On the other hand, the development of analysis is smoother in a logic that validates many reasoning principles, such as choice principles and Markov's principle, and makes it easy to construct many types, such as function spaces, subspaces, quotient spaces, and inductively defined structures. This poses a design challenge: to have as rich as possible a logic that can still be interpreted constructively.

I will present a predicative first-order intuitionistic logic with dependent types, subset types, quotient types, and inductive and coinductive types. In addition, there is a 'computability' operator which operates on types and singles out the computable elements of a type.

The logic has a constructive interpretation in categories of modest sets and computable morphisms. Types are interpreted as arbitrary modest sets and may contain non-computable points, whereas the interpretation of functions always gives computable morphisms.

There are several schools of computable and constructive analysis, among others recursive analysis, type 2 effectivity, effective topological spaces, domain theory, equilogical spaces, and the SSB real model. These are all examples of the general framework presented in this talk.

► CARSTEN BUTZ, *A saturated model of Heyting arithmetic*.
Department of Mathematics and Statistics, Burnside Hall, McGill University, 805 Sherbrooke Street West, Montreal, Quebec, Canada, H3A 2K6.
E-mail: butz@math.mcgill.ca.

In this talk we will give a constructive proof that every consistent intuitionistic first-order theory has a saturated model. More precisely, we will prove the following theorem [1]:

THEOREM. *Let $\mathbb{T}$ be a consistent first-order theory formulated in intuitionistic logic. Then there is a Heyting-valued model $\mathfrak{M}$ of $\mathbb{T}$ with the following two properties*:

1. *The model $\mathfrak{M}$ is conservative, that is, for every first-order formula φ the model $\mathfrak{M}$ satisfies φ if and only if φ is derivable from $\mathbb{T}$.*
2. *The model $\mathfrak{M}$ is saturated, that is, for a set $\{\varphi_i(x)\}_{i\in I}$ of formulae, if*

$$\mathfrak{M} \models \exists x \bigwedge_{i\in I_0} \varphi_i(x)$$

for every finite subset I_0 of I, then

$$\mathfrak{M} \models \exists x \bigwedge_{i\in I} \varphi_i(x).$$

In addition we give a complete axiomatization of the infinitary first-order theory of the model $\mathfrak{M}$.

Next we apply this theorem to Heyting arithmetic (HA). The saturated model of (HA) that we obtain from the theorem interprets the following theory (HAI), which is therefore a conservative extension of (HA) [2]: We extend the language of Heyting arithmetic by two unary predicates St and I standing for standard numbers and infinitely large numbers. The axioms of (HAI) are as follows:

(1) (HA);
(2) $\mathrm{St}(x_1) \wedge \cdots \wedge \mathrm{St}(x_n) \to \mathrm{St}(f(x_1, \ldots, x_n))$, for every function symbol f in the language of (HA);
(3) $\mathrm{St}(x) \leftrightarrow \neg\, \mathrm{I}(x)$, and $\mathrm{I}(x) \leftrightarrow \neg\, \mathrm{St}(x)$;
(4) $\exists x\ \mathrm{I}(x)$;
(5) $\neg\forall x\ (\mathrm{St}(x) \vee \neg\, \mathrm{St}(x))$;
(6) external induction;
(7) overspill;
(8) underspill.

In addition the model validates the ω-rule.

The type of models we construct have been used before in ad-hoc constructions to get models of intuitionistic theories. For example, [2] construct models of Heyting arithmetic, [3] constructs models for intuitionistic analysis, [4] implicitly uses saturated models to derive interpolation and completeness results, and I. de Freitas Druck described in her thesis a saturated model of synthetic differential geometry, an intuitionistic analogue of non-standard analysis.

[1] C. Butz, *The filter construction revisited*, preprint, July 1999.

[2] I. Moerdijk and E. Palmgren, *Minimal models of Heyting arithmetic*, ***Journal of Symbolic Logic***, vol. 62 (1997), pp. 1448–1460.

[3] E. Palmgren, *Developments in constructive nonstandard analysis*, this Bulletin, vol. 4 (1998), pp. 233–272.

[4] A. M. Pitts, *Conceptual completeness for first-order intuitionistic logic: an application of categorical logic*, ***Annals of Pure and Applied Logic***, vol. 41 (1989), pp. 33–81.

► TODD H. TRIMBLE, *String diagram calculus for first-order logic*.
Department of Mathematics, Northwestern University, Evanston, Illinois 60208, USA.
E-mail: `trimble@math.nwu.edu`.

By the end of the 19th century, the American logician C. S. Peirce had developed a remarkable graphical calculus, called Existential Graphs, for conducting calculations in first-order relational algebra. Guided by analogies with chemistry, Peirce pictured a relation of type $A_1 \times \cdots \times A_n$ as a configuration of "valency" n, and composites of relations as a "bonding" between two or more configurations, much like the bonding of several atoms in a molecular configuration.

This talk describes joint work with Geraldine Brady on a modern-day formulation of Peirce's system. First, the first-order calculus of relations is described in terms of a free "locally Boolean-valued discrete cartesian bicategory," following work of Carboni-Walters. Second, this type of free structure is presented geometrically, using the method of string diagrams as given in work of Joyal-Street (cf. Feynman and Penrose diagrams). In fact, the string diagrams relevant here are properly regarded as 1-dimensional notation for 2-dimensional cobordisms, thus exemplifying the sometimes deep interaction between logical syntax and low-dimensional geometry.

Abstracts of contributed talks

► MATTHIAS ASCHENBRENNER AND LOU VAN DEN DRIES, *Continuity properties of differential polynomials over H-fields*.
University of Illinois at Urbana-Champaign, 1409 West Green Street, Urbana, Illinois 61801, USA.
E-mail: `maschenb@math.uiuc.edu`.
E-mail: `vddries@math.uiuc.edu`.

In [1], H-fields were defined as certain ordered differential fields, with the aim of capturing the first-order content of the notion of a Hardy field. (In the same way as the axioms for ordered fields capture the algebraic content of "subfield of the reals".) The derivation on an H-field K is continuous with respect to the order topology, so every differential polynomial gives rise to a continuous function on K. We will present a few results about these differential-polynomial functions, especially when K is Liouville closed, where they show behavior much like ordinary polynomials over real closed fields. In particular, we will show that over a Liouville closed H-field, the zeros of a non-zero differential polynomial (in one variable) are isolated. The proof uses a generalization of an old (unpublished) result of van den Dries, with its main idea going back to E. Borel.

[1] *H-fields and their Liouville closures*, submitted.

► JEREMY AVIGAD AND JEFFREY HELZNER, *Transfer principles in nonstandard intuitionistic arithmetic*.
Department of Philosophy, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA.
E-mail: avigad@cmu.edu.
E-mail: helzner+@andrew.cmu.edu.

Using a slight generalization of sheaf semantics due to Palmgren, we present a term-model construction which assigns a model to any first-order intuitionistic theory. A modification of this construction then assigns a nonstandard model to any theory of arithmetic, enabling us to reproduce conservation results of Moerdijk and Palmgren for nonstandard Heyting arithmetic. Internalizing the construction allows us to strengthen these results with additional transfer rules; we then show that even trivial transfer axioms or minor strengthenings of these rules destroy conservativity over HA. The analysis also shows that nonstandard HA has neither the disjunction property nor the explicit definability property. A weak theory of intuitionistic nonstandard arithmetic, conservative over primitive recursive arithmetic, is also considered.

► L. BELAIR, *Model theory of pseudovaluation domains*.
Dep. de mathematiques, UQAM, C. P. 8888 Succ. Centre-ville, Montreal, Quebec, Canada H3C 3P8.
E-mail: belair.luc@uqam.ca.

Fix a field k and a proper subfield k_0. Take the subring of the power series ring $k[\![T]\!]$ consisting of the series whose coefficient of order 0 belongs to k_0. Pseudovaluation rings are characterized as obtained similarly from valuation rings. They are local rings and their model theory is close to that of valuation rings, with similar transfer principles à la Ax-Kochen-Ershov for henselian ones. In particular, the independence property is determined by that of the pair k_0 included in k.

[1] L. BELAIR, *Pseudovaluation domains with Vapnik-Chervonenkis classes of definable sets*, ***Communications in Algebra***, to appear.

► JEAN-YVES BEZIAU, *Universal logic: Towards a general theory of logics*.
Centre for the Study of Language and Information, Ventura Hall, Stanford University, Stanford, California 94305-4115, USA.
E-mail: jybeziau@hotmail.com.

By universal logic, we mean here a general theory of logics considered as fundamental mathematical structures in the sense of Bourbaki (cf. [4]). The aim of this talk is to explain the philosophical and historical background of such a theory and to give some examples of technical problems that are dealt with.

We use the expression "universal logic" by analogy with the expression "universal algebra": universal algebra was developed as a very general theory of algebras considered as abstract structures. As Birkhoff explains (cf. [3]), he was lead to the now standard notion of abstract algebra (a set with a family of operators) by the need of systematization of heterogeneous structures.

During the XXth century, numerous logics were developed (intuitionistic logic, many-valued logic, relevant logic, paraconsistent logic, quantum logic, modal logic, linear logic, etc.). The basic idea of universal logic is to consider logics as structures and study general properties of these structures and their relationships (cf. [1], [2]).

[1] J.-Y. BEZIAU, *Universal logic*, in ***Logica '94: Proceedings of the 9th International Symposium*** (T. Childers and O. Majer, editors), Czech Academy of Science, Prague, 1994, pp. 73–93.

[2] ———, *Recherches sur la logique universelle*, ***Ph. D. thesis***, Department of Mathematics, University of Paris 7, 1995.

[3] G. Birkhoff, *Universal algebra*, in ***Selected papers on algebra and topology by Garret Birkhoff*** (G.-C. Rota and J. S. Oliveira, editors), Birkhäuser, Basel, pp. 111–115.

[4] N. Bourbaki, *The architecture of mathematics*, ***American Mathematical Monthly***, vol. 57 (1950), pp. 221–232.

► DOUGLAS CENZER AND FARZAN RIAZATI, *Recent advances in the lattice of Π^0_1 classes*.
Department of Mathematics, University of Florida, P. O. Box 118105, Gainesville, Florida 32611, USA.
E-mail: `cenzer@math.ufl.edu`.

We will discuss some results involving minimal and thin extensions of Π^0_1 classes. After introducing the notion of "Preservation property" for a pair of these classes, we will see how minimal and thin extensions are related to the automorphisms of the above lattice.

In a related effort to reveal more structural properties of the Lattice, we will examine the effective content of Polish group actions. Recalling the notion of a "tame" Polish group from the works of H. Becker, A. Kechris and G. Hjorth, we look at the actions where the "orbit equivalence relation" turns out to be a hyperarithmetical set.

► JOHN CHISHOLM, *Using the structure of $\aleph_0$-categorical linear orders to count the number of countable models of a countable linear order*.
Department of Mathematics, Western Illinois University, Macomb, Illinois 61455, USA.
E-mail: `john_chisholm@ccmail.wiu.edu`.

It is known that any countable linear order L (with finitely many unary predicates) is either $\aleph_0$-categorical, or has a model containing a "self-additive" interval, from which it follows that L has continuum many nonisomorphic countable models. We show how, in fact, self-additive intervals arise naturally as a direct limit of $\aleph_0$-categorical subintervals of L.

► STEPHEN COOK AND ANTONINA KOLOKOLOVA, *A second-order system for polytime reasoning based on Grädel's theorem*.
Department of Computer Science, University of Toronto, Toronto, Ontario M5S 3G4, Canada.
E-mail: `sacook@cs.toronto.edu`.
E-mail: `kol@cs.toronto.edu`.

We introduce a second-order system V_1-**Horn** of bounded arithmetic based on Grädel's [1] second-order Horn characterization of the polynomial-time predicates. Our system closely resembles Zambella's [2] system Σ^p_1-*comp* (and somewhat resembles the standard system V^1_1), except that the comprehension scheme is restricted to *SO*$\exists$-*Horn* formulas of the form

$$\exists P_1 \ldots \exists P_k \; \forall x_1 \le t_1 \ldots \forall x_m \le t_m \; \phi \tag{1}$$

where $P_1, \ldots, P_k$ are unary relation symbols, $x_1, \ldots, x_k$ range over the natural numbers, $t_1, \ldots, t_m$ are terms, and ϕ is a quantifier-free CNF formula in which every clause contains no more than one unnegated atom of the form $P_i(t)$ for some term t. It follows from Grädel's theorem that formulas of the form (1) represent the polynomial-time predicates in their free second-order variables, when these variables are interpreted as finite sets of numbers in the standard model. Our system turns out to be equivalent to QPV and Zambella's $\mathcal{P}$-*def* but less powerful than V^1_1 and S^1_2 (unless the polynomial hierarchy collapses).

[1] E. Grädel, *Capturing complexity classes by fragments of second-order logic*, ***Theoretical Computer Science***, vol. 101 (1992), pp. 35–57.

[2] D. Zambella, *Notes on polynomially bounded arithmetic*, ***Journal of Symbolic Logic***, vol. 61 (1996), pp. 942–966.

► PAUL CORAZZA, *Reports that the existence of a $j: V \to V$ is inconsistent have been greatly exaggerated.*
6000 Shepherd Mountain Cove, Apt. 1110, Austin, Texas 78730, USA.
E-mail: pcorazza@kdsi.net.

In the 1960s, Reinhardt asked whether there could be a nontrivial elementary embedding $j: V \to V$. In 1970, K. Kunen showed that the existence of such an embedding leads to an inconsistency. Since the appearance of Kunen's theorem, many other innovative proofs of the same result have also been published. However, there has been no general agreement about how to state this result formally, though there is general agreement that it cannot be formulated as a theorem of ZFC alone. In this paper, we propose a formal context for stating Kunen's Theorem. This context makes it clear that the assertion "there exists a nontrivial elementary embedding $j: V \to V$" can have a number of interpretations. Some of these interpretations are quite weak; one interpretation leads to Kunen's inconsistency; and yet another interpretation leads to a statement that has consistency strength strictly between the axiom I_3 and the existence of a cardinal that is super-n-huge for every n.

► LOU VAN DEN DRIES AND PHILIP EHRLICH, *Exponentiation for surreal numbers.*
Department of Mathematics, University of Illinois, Urbana, Illinois 61801, USA.
E-mail: vddries@math.uiuc.edu.
Department of Philosophy, Ohio University, Athens, Ohio 45701, USA.
E-mail: ehrlich@ohiou.edu.

Conway's field *No* of surreal numbers [1] extends the field $\mathbb{R}$ of real numbers. In his book [3, Ch. 10], Gonshor, following suggestions by Kruskal, defined an exponential function $\exp: No \to No$ such that $\exp(x) = e^x$ for $x \in \mathbb{R}$. We show that *No* with exp is an elementary extension of the real exponential field. The proof consists in equipping *No* with even further structure, by extending the restricted analytic functions from $\mathbb{R}$ to *No*, and verifying that the axioms in [2] for the model-complete theory $T_{\text{an,exp}}$ are satisfied by *No* thus expanded. Others have told us that they were aware of this argument, so that the result can be viewed as (recently acquired) "folklore". Still, it seems to us of sufficient interest to provide the details and record in print, in conjunction with some further results including the following two. Let $No(\lambda)$ be the set of surreals of length [3, p. 3] less than the ordinal λ: (i) the ordinals λ for which $No(\lambda)$ is closed under the field operations and exponentiation of *No* are exactly the ε-numbers, that is, the ordinals λ such that $\omega^\lambda = \lambda$; (ii) for each ε-number λ the set $No(\lambda)$ with the induced exponential field operations is an elementary extension of the real exponential field (and an elementary substructure of the exponential field *No*).

[1] J. H. Conway, ***On numbers and games***, Academic Press, 1976.

[2] L. van den Dries, A. Macintyre, and D. Marker, *Restricted analytic fields with exponentiation*, ***Annals of Mathematics***, vol. 140 (1994), pp. 183–205.

[3] H. Gonshor, ***An introduction to the theory of surreal numbers***, London Mathematical Society Lecture Note Series, vol. 110, Cambridge University Press, 1986.

► FERNANDO FERREIRA, *Basic analysis in a feasible theory, with an application to interpretability in Q.*
Departamento de Matematica, Universidade de Lisboa, Rua Ernesto de Vasconcelos, bloco c1, 3, P-1749-016 Lisboa, Portugal.
E-mail: ferferr@alf1.cii.fc.ul.pt.

This is partly joint work with A. Marques Fernandes.

We study some basic analysis over a weak base theory of second-order arithmetic BTFA (an acronym for "Base Theory for Feasible Analysis") whose provably total functions are the polynomial time computable functions. Some basic theorems of analysis are studied, the intermediate value theorem among them. The import of Weak Konig's lemma in our weak setting is also discussed, as well as some reversals. We briefly address some computational and philosophical issues, and relate our work with the work of other people. Finally, as an application, we show how to interpret the base theory BTFA in Robinson's arithmetic Q. This result yields the interpretability of the theory of real closed fields in Q.

[1] FERNANDO FERREIRA, *A feasible theory for analysis*, ***Journal of Symbolic Logic***, vol. 59 (1994), pp. 1001–1011.

► LEV GLEBSKY AND EVGENII GORDON, *On approximation of locally compact fields by finite algebraic systems*.

Department of Mathematics, University of Illinois, 1409 West Green Street, Urbana, Illinois 61801, USA.
E-mail: gordon3@math.uiuc.edu.

In [1] it was proved that any LCA group $\mathcal{G}$ may be represented as a quotient group of a σ-subgroup G_b of a hyperfinite (in the sense of nonstandard analysis) group G over its normal π-subgroup G_0. In terms of standard mathematics this is equivalent to some natural property of approximation by finite groups. We show here that locally compact fields are not approximable in the similar sense by finite associative and commutative rings and discuss some hyperfinite algebraic systems, which approximate $\mathbf{R}$ (hyperfinite computer arithmetics). We also introduce one more complicated definition of approximability according to which the field $\mathbf{R}$ is approximable by finite rings.

[1] E. GORDON, ***Nonstandard methods in commutative harmonic analysis***, American Mathematical Society, Providence, Rhode Island, 1997, 168 pp.

► THOMAS HOFWEBER, *Equivalence principles and neo-Fregean approaches to the philosophy of mathematics*.

Department of Philosophy, University of Michigan, 435 S. State St., Angell Hall 2215, Ann Arbor, Michigan 48109-1003, USA.
E-mail: hofweber@umich.edu.

In neo-Fregean approaches to mathematics, especially arithmetic, equivalence, or abstraction, principles play an important role. They bridge talk about how many things of a kind there are to talk about the number of things of that kind, and to talk about numbers in general. Taking recourse to equivalence principles neo-Fregeans try to defend a version of Platonism which allegedly has advantages over "Gödelian" forms of Platonism, and which refutes other approaches to the philosophy of mathematics.

Whatever one says about such equivalence principles leads to some difficulties. Several proposals about this have been made, including reductionism, denying the equivalence principles, and endorsing Fregean Platonism. I will present evidence for a further option which asserts the literal truth of the equivalence principles (and of the left and right hand side of certain instances), denies reductionism, and at the same time denies that this supports Fregean Platonism, or other forms of Platonism. Instead a positive account of their function will be proposed.

► KAREL HRBACEK, *Platonism, nonstandard set theory, and large cardinals*.

Department of Mathematics, City College of New York, New York, New York 10031, USA.
E-mail: hrbacek@math0.sci.ccny.cuny.edu.

Platonist point of view has been a guiding light of set theory since the time of Cantor. Here we apply it to foundations of nonstandard analysis, and argue that intuition about nonstandard entities, buttressed by extrinsic reasons similar to those used by standard set theorists, justifies acceptance of the following basic axioms for nonstandard set theory (BNST):

ZFC in the standard universe, Existence of internal nonstandard sets, Transfer, Standardization, and ZFC except Regularity in the external universe.

The key remaining issue for a "nonstandard" platonist is the extent of existence of "ideal" entities. BNST is inconsistent with "full" Idealization, but, on similar evidence, we propose axioms $A(0)$, $A(1)$, $A(2)$ (and even some stronger ones), where $A(0)$ [$A(1)$, $A(2)$, resp.] asserts:

> For arbitrarily large [all regular, all, resp.] infinite cardinals k there exist internal sets that are k-constrained, but not m-constrained for any $m < k$.

DEFINITION. x is *k-constrained* if x belongs to $^{*}\!A$ for some standard set A of cardinality less than or equal to k.

For example, in BNST, $A(1)$ is equivalent to: "All standard ordered fields have infinitesimals," a prerequisite for any nonstandard theory of the continuum and its generalizations.

THEOREM 1. BNST $+ A(0) \vdash$ *There exists an inner model with a measurable cardinal.*

[This uses deep results of Jensen and Donder on nonregular ultrafilters.]

THEOREM 2. Con(ZFC $+$ *There is a proper class of measurable cardinals*) $\Longrightarrow$ Con(BNST $+ A(0)$).

Consistency strength of the stronger axioms is unknown.

The extent to which BNST can serve as a convenient vehicle for practice of nonstandard analysis is also considered.

► TAMARA J. HUMMEL AND CARL G. JOCKUSCH, JR., *Ramsey's theorem for computably enumerable colorings*.

Mathematics Department, Allegheny College, 520 N. Main Street, Meadville, Pennsylvania 16335, USA.
E-mail: thummel@alleg.edu.
Mathematics Department, University of Illinois, 1409 W. Green St., Urbana, Illinois 61801, USA.
E-mail: jockusch@math.uiuc.edu.

Ramsey's theorem [4] states that every k-coloring of $[\omega]^n$ (the set of all n-element subsets of ω) has an infinite homogeneous set. Jockusch [3] proved that every computable k-coloring of $[\omega]^n$ has an infinite Π^0_n homogeneous set, as well as an infinite homogeneous set A such that $A' \leq_T 0^{(n)}$. Say that a 2-coloring of $[\omega]^n$ is *computably enumerable* if there is a color c such that the set of codes for the n-element sets which are given color c is computably enumerable. We show in [2], using methods from [1], that Jockusch's results remain true when computable 2-colorings are replaced by computably enumerable 2-colorings and that these results are best possible in various senses.

[1] T. HUMMEL and C. G. JOCKUSCH, JR., *Generalized cohesiveness*, ***Journal of Symbolic Logic***, vol. 64 (1999), pp. 489–516.

[2] ———, *Ramsey's theorem for computably enumerable colorings*, ***Journal of Symbolic Logic***, to appear.

[3] C. G. JOCKUSCH, JR., *Ramsey's theorem and recursion theory*, ***Journal of Symbolic Logic***, vol. 37 (1972), pp. 268–280.

[4] F. P. RAMSEY, *On a problem in formal logic*, ***Proceedings of the London Mathematical Society***, vol. 30 (1930), pp. 264–286.

► JOSÉ IOVINO, *Classification theory for the asymptotic geometry of sequences in a Banach space*.

Division of Mathematics and Statistics, The University of Texas at San Antonio, San Antonio, Texas 78249-0664, USA.
E-mail: iovino@math.utsa.edu.

A central concept in Banach space geometry is that of spreading model associated with a sequence. Every basic sequence in a Banach space X has a subsequence that generates a Banach space extension of X called a spreading model, and the spreading models generated by a sequence are the standard tool to analyze the asymptotic geometry of the sequence.

The central question in the geometry of spreading models is to identify under what conditions a spreading contains either of the classical sequence spaces ℓ_p and c_0. Up to date, most of the results in this direction have been negative. (See, for example, [3] and [4]). We will state a result that shows that the geometric structure of a spreading model is intimately connected with its model theoretical properties:

THEOREM. *Every basic sequence in a Banach space X has a subsequence* (x_n) *for which the following three conditions are equivalent.*

1. (x_n) *has a normalized block basis that is cofinally isometric over* X *to either* c_0 *or* ℓ_p, *for some* p *with* $1 \leq p < \infty$.
2. (x_n) *has a normalized block basis that generates a spreading model which is stable in the sense of model theory.*
3. (x_n) *generates a spreading model which is stable in the sense of model theory.*

We will discuss implications of this theorem in connection with the question raised in Banach space theory (see [1] and [2]) whether every Banach space whose norm is defined by an explicit formula is guaranteed to contain ℓ_p or c_0 almost isometrically.

[1] W. T. GOWERS, *Recent results in the theory of infinite-dimensional Banach spaces*, in *Proceedings of the International Congress of Mathematicians, Vol. 1 and 2 (Zürich, 1994)*, Birkhäuser, Basel, 1995, pp. 933–942.

[2] E. ODELL, *On subspaces, asymptotic structure, and distortion of Banach spaces: Connections with logic*, to appear.

[3] E. ODELL and TH. SCHLUMPRECHT, *On the richness of the set of p's in Krivine's theorem*, in ***Geometric aspects of functional analysis (Israel, 1992–1994)***, Operator Theory Advances and Applications, vol. 77, Birkhäuser, Basel, 1995, pp. 177–198.

[4] ———, *A problem on spreading models*, ***Journal of Functional Analysis***, vol. 153 (1998), no. 2, pp. 249–261.

► RENLING JIN, *Sumset phenomenon*.
Department of Mathematics, College of Charleston, Charleston, South Carolina 29424, USA.
E-mail: jinr@cofc.edu.

Using nonstandard model of arithmetic, I prove a theorem which implies several old and new results in the standard world. Those results exemplify a common phenomenon that if A and B are large in terms of "measure", then $A + B$ is not small in terms of "order-topology". For example, the theorem mentioned above implies that if both A and B are subsets of real numbers with positive Lebesgue measure, then $A + B$ covers a non-empty open interval. The main theorem also implies that if both A and B are subsets of natural numbers with positive upper Banach density, then $A + B$ is piecewise syndetic. The first result is well-known but the second result is new. The second result complements an old result in combinatorial number theory which says that if A has positive upper Banach density, then $A - A$ is syndetic. The definitions of upper Banach density, syndeticity and piecewise syndeticity can be found in H. Furstenberg's book ***Recurrence in Ergodic Theory and Combinatorial Number Theory***, Princeton University Press, 1981.

► JOHN T. KEARNS, *The priority of denial to negation*.
Department of Philosophy and Center for Cognitive Science, University of Buffalo, The State University of New York, Baldy Hall, Buffalo, New York 14260, USA.
E-mail: kearns@acsu.buffalo.edu.

A system of illocutionary logic is developed based on [1]. The language contains atomic and compound sentences formed with the connectives for conjunction and disjunction. In addition there are illocutionary operators for asserting a statement/sentence, for denying a sentence, for supposing a sentence true, and for supposing a sentence false. Illocutionary operators cannot be iterated, and a sentence with an illocutionary operator cannot be a component of a larger sentence. The semantic account for the language provides both truth conditions and commitment conditions for sentences. It is commitment conditions which provide the basis for evaluating arguments, because asserting, denying, and supposing some sentences will commit a person to asserting, denying, and supposing others. Illocutionary logic is the area of logic that is most relevant to human cognition, because the cognitive practice of making deductive inferences and arguments is based most directly on commitment conditions, and only incidentally on truth conditions. The deductive system provides inference principles (rules) for the connectives and for the illocutionary operators. Principles for denial and negative supposition are adequate. It is conjectured that natural languages originally lacked resources for negating sentences (statements). It is shown that the negative illocutionary operators provide a sufficient basis for introducing a negation connective and characterizing its inferential properties. And this in spite of the fact that introducing a negation connective increases the expressive power of the language. This priority of denial and negative supposition to sentential negation helps explain why introduction and elimination rules seem the most natural or appropriate rules for natural deduction.

[1] JOHN T. KEARNS, *Propositional logic of supposition and assertion*, ***Notre Dame Journal of Formal Logic***, vol. 38 (1997), pp. 325–349.

► RICHARD O. KETCHERSID, *On the strength of CH + an ω_1-dense ideal on ω_1*.
Department of Mathematics, University of California, Berkeley, 731 Evans Hall #3840, Berkeley, California 94720-3840, USA.
E-mail: ketchers@math.berkeley.edu.

In this paper I show that from the continuum hypothesis together with a technical strengthening of "there exists an ω_1-dense ideal on ω_1", that $AD^+ + \Theta_0 < \Theta$ holds in a proper-class inner-model containing $\mathbb{R}$. By a result of H. Woodin this yields non-tame Mitchell-Steel mice, and hence inner-models of "ZFC + there is a proper-class of strong cardinals the least of which is a limit of Woodin cardinals." The argument I give generalizes to yield a proper-class inner-model containing $\mathbb{R}$ of $AD_{\mathbb{R}}$, and probably $AD_{\mathbb{R}} + DC$. This represents a first step in proving a converse to a theorem of Woodin that states that assuming $AD_{\mathbb{R}} + \Theta$ is regular there is a generic extension of V in which "ZFC + CH + there is an ω_1-dense ideal on ω_1" holds.

► LINDA LAWTON, *Minor superclasses for Π^0_1 classes*.
800 Cedar Ridge Lane, Apartment 302, Richton Park, Illinois 60471, USA.
E-mail: lawton@math.uiuc.edu.

We explore an analogue of major subsets for c. e. sets.

Let P and Q be Π^0_1 classes with $P \subset Q \subset 2^\omega$ and $Q - P$ infinite. Then Q is a *minor superclass* of P if for any Π^0_1 class R, whenever $R \cap P = \emptyset$, then $R \cap Q$ is finite. Q is a *strong minor superclass* of P if for any Π^0_1 class R, whenever $R \cap P$ is finite, then $R \cap Q$ is finite.

Every noncomplemented Π^0_1 class has a decidable minor superclass. There are homeomorphic Π^0_1 classes P, $Q \neq 2^\omega$ such that P has a strong minor superclass and Q does not. We show that if a Π^0_1 class P contains infinitely many boundary points (under the product topology) then P has a strong minor superclass.

► BENEDIKT LÖWE, *Partition cardinals below δ^1_5 under* AD.
Mathematisches Institut, Rheinische Friedrich-Wilhelms-Universität Bonn, Beringstrasse 6, 53115 Bonn, Germany.
E-mail: loewe@math.uni-bonn.de.

Eugene Kleinberg linked the theory of partition cardinals to the Axiom of Determinacy AD by showing that the first $\omega + 1$ infinite cardinals satisfy certain large cardinal properties defined via partition relations.

More generally, Kleinberg showed:

THEOREM 1. *Let κ be a cardinal with the strong partition property and μ be a normal ultrafilter on κ. Let $\kappa_1 := \kappa$ and $\kappa_{n+1} := \kappa_n^\kappa/\mu$ and call $\langle \kappa_n; n \in \omega \rangle$ the **Kleinberg sequence relative to** μ. Then all κ_n are Jónsson cardinals, and* $\sup\{ \kappa_n; n \in \omega \}$ *is a Rowbottom cardinal.*

The deep structural results of Jackson's computation of $\boldsymbol{\delta}^1_5$ immediately provided many applications of Kleinberg's theorem under AD: All odd projective ordinals $\boldsymbol{\delta}^1_{2n+1}$ are starting points for Kleinberg sequences, and hence there are many Jónsson and Rowbottom cardinals.

Where exactly are these Jónsson and Rowbottom cardinals, i.e., can we compute the cardinality of the members of these additional Kleinberg sequences?

We shall answer this question and compute the Kleinberg sequences derived from the ω_1-cofinal and the ω_2-cofinal measures on $\boldsymbol{\delta}^1_3$. An important ingredient here is the exact knowledge of cofinalities of successor cardinals between $\boldsymbol{\delta}^1_3$ and $\boldsymbol{\delta}^1_5$ provided by [1].

THEOREM 2. *Assume* AD. *Then the cardinals $\aleph_{\omega \cdot n+1}$ and $\aleph_{\omega^\omega \cdot n+1}$ are Jónsson for every $n \in \omega$. Furthermore, the cardinals $\aleph_{\omega^2}$ and $\aleph_{\omega^\omega \cdot \omega}$ are Rowbottom.*

[1] STEVE JACKSON and FARID T. KHAFIZOV, *Descriptions and cardinals below $\boldsymbol{\delta}^1_5$*, ***Journal of Symbolic Logic***, to appear.

► BILLY JOE LUCAS, *The logic of categorical rights*.
Philosophy Department, Manhattanville College, 2900 Purchase, Purchase, New York 10577, USA.
E-mail: billyjoelucas@email.com.

Let L include $\sim$, $\supset$, a deontic obligation operator O, and define P as $\sim O \sim$. A common reading of $O{\sim}\alpha$ $({\sim}O{\sim}\alpha)$: α is (not) prohibited. One thereby also sometimes reads, for example $P(x \text{ does } y)$: x has a *right* to do y. We read $OP\alpha$: α is (expresses) a *categorical right*. (One might write "α is a *mere* right": $P\alpha \cdot {\sim}OP\alpha$.) We thus distinguish two kinds of rights claim: $P\alpha$, $OP\alpha$.

QUESTION 1. Are all obligations also categorical rights?

QUESTION 2. Are all categorical rights also rights?

BCM (Basic Categorical Morality, defined below) is the minimal deontic logic that guarantees a yes answer to both questions. OCR (CRR) is the minimal deontic logic closed under M. P., Uniform Substitution, and Deontic Necessitation that extends deontic K by addition of the axiom scheme $O\alpha \supset OP\alpha$ $(OP\alpha \supset P\alpha)$. OCR is canonically characterized by frames with R: $\forall x\, \forall y\, [x\, R\, y \cdot \forall z\, (y\, R\, z \supset x\, R\, z)]$. CRR is canonically characterized by frames with R: $\forall x\, \exists y\, [x\, R\, y \supset \exists z\, (x\, R\, z \cdot y\, R\, z)]$. BCM is OCR plus CRR. CRR extends KD, and BCM is properly included in KD4. Adding wff R $(PO\alpha \supset OO\alpha)$ to CRR gives KDR4, the full logic of categorical morality.

► SAUNDERS MAC LANE, *Foundations by using categories with elements*.
Mathematics Department, University of Chicago, 5734 University Avenue, Chicago, Illinois 60637, USA.
E-mail: saunders@math.uchicago.edu.

Lawvere and others have proposed to use suitable categories as a foundation of mathematics alternative to the usual Zermelo-Fraenkel set theory. Such categories, however, seem a bit unnatural because the objects of a category do not normally have elements like the familiar elements of a set. To avoid this, one can propose a further categorical structure which provides such elements for the objects of a category. This can be done by axioms for a well-pointed category in which certain monomorphisms are called "inclusions," in such a way that every map in the category has a unique factorization as an epimorphism followed by an inclusion. With suitable axioms, an "element" of a category object A can then be taken to be an inclusion into A. This can be combined with other axioms to provide a flexible and intuitive categorical axiomatics.

► JOHN N. MARTIN, *The logic of Proclus*.
Department of Philosophy, University of Cincinnati, Cincinnati, Ohio 45221-0374, USA.
E-mail: john.martin@uc.edu.

This paper investigates the views of Proclus (412–458 A. D.) on the logic of order and negation. From the second to the fifth centuries Neoplatonic philosophers wrote commentaries on Aristotle in which they seem to condone his logic with its non-Platonic semantics that on its face is inconsistent with Neoplatonic metaphysics. The paper details how Proclus extended the notions of order and negation to harmonize his logical views with his ontology. He advocated a complex linear ordering $<$ (*causation*) with a supremum (*the One*) and which is divided into segments by fixed points (*monads*), each segment being order-isomorphic to any other. In addition the order is "dense" in a special sense that for any points a and b such that $b \ll a$ (i.e., *b is the successor/predecessor of a*) there are points c and d such that $b \ll c \ll d \ll a$ with a a fixed point (c, d, a is called a *triad*). Moreover Proclus posited relative complement operators ("negations") on the order. One such operation (*hyper-negation*) maps a point to the corresponding point in the next higher segment (relative to a fixed point), and another (*privative negation*) maps the point to the corresponding position in the next lower segment (relative to a fixed point). Truth-conditions for the sentence forms of the syllogistic can be defined in terms of the order so that they validate the correct moods, and which directly yields the Proclean theorem that predication is causation. At the molecular level however the logic is non-traditional because it violates laws like double negation (a version of which Proclus attributes to Aristotle), contraposition and *modus pones*. These negations are used in the Neoplatonic tradition, by Proclus and others, to explicate the *via negativa* in theology (hyper-negation) and the evil of the material world (through privation). They are of some current interest because Proclus' versions are special cases of scalar predicate negations found in natural language, and investigated by Laurence R. Horn [2], Lennart Åqvist [1], and John N. Martin [3].

[1] Lennart Åqvist, *Predicate calculi with adjectives and nouns*, ***Journal of Philosophical Logic***, vol. 10 (1981), pp. 1–26.

[2] Laurence R. Horn, ***A natural history of negation***, University of Chicago Press, Chicago, 1989.

[3] John N. Martin, *Existence, negation, and abstraction in the neoplatonic hierarchy*, ***History and Philosophy of Logic***, vol. 16 (1995), pp. 169–196.

► G. MINTS, *Skolemization and quick cutelimination*.
Department of Philosophy, Stanford University, Stanford, California 94305, USA.
E-mail: mints@csli.stanford.edu.

It is proved in [1] that cuts over *monotone* formulas (constructed from atomic formulas [including $\perp$] by $\&, \vee, \forall, \exists$) can be quickly (almost exponentially) eliminated from derivations of quasi-monotone formulas.

In this note we simplify proofs in [1] for derivations of atomic formulas from Horn axioms using pruning transformations (underlying Harrop's theorem, cf. also [2]) which allow drastically "skolemize" formulas proved from Horn axioms.

We consider derivations in classical predicate logic LK. $l(d)$ is the number of sequents in the derivation d.

A *majorant* μA of a monotone formula A is any formula obtained from A by retaining exactly one of the disjuncts in any disjunction and instantiating existential quantifiers by abstract terms and universal quantifiers by fresh variables (which do not occur in terms instantiating preceding existential quantifiers).

μ cut is the rule: from $\Gamma \Longrightarrow \mu A,\ A, \Delta \Longrightarrow B$ derive $\Gamma, \Delta \Longrightarrow B$.

LEMMA. *Let H be a finite sequence of universal Horn formulas, $\mathbf{M}$ be a finite sequence of monotonic formulas. Then every cut-free derivation of $H \Longrightarrow \mathbf{M}$ can be pruned (by deleting formulas, sequents and rules) into a cut-free intuitionistic derivation of $H \Longrightarrow \mu M$ for some majorant μM of $M \in \mathbf{M}$.*

PROOF. Induction on the derivation using the fact that $\vee$, $\exists$-antecedent are not present.

THEOREM. *Let $f : H \Longrightarrow \mathbf{P}$ for a sequence $\mathbf{P}$ of atomic formulas with exactly $k > 0$ cuts, all with monotone cut-formulas. Then there is a cut-free $g : H \Longrightarrow \mathbf{P}$ with*

$$l(g) < (l(d))^{2^k} < (l(d))^{2^{l(d)}}.$$

PROOF. Eliminate the lowermost cut using the previous Lemma.

THEOREM. *Monotone cuts do not define a cut-reduction class.*

PROOF. As in [1]: cut-elimination for derivations in Statman's sequence is not of elementary complexity.

[1] M. BAAZ and A. LEITSCH, *Cut normal forms and proof complexity*, ***Annals of Pure and Applied Logic***, vol. 97 (1999), pp. 127–177.

[2] G. MINTS, *The Skolem method in intuitionistic calculi*, ***Proc. Inst. Steklov***, vol. 121 (1974), pp. 73–109.

► RAHIM MOOSA, *On difference fields and quantifiers elimination*.
Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 West Green St., Illinois 61801, USA.
E-mail: moosa@math.uiuc.edu.

Let $\mathcal{L}$ be the language of fields augmented by a new unary function symbol σ. A *difference field* is a field equipped with a distinguished automorphism (which we will also denote by σ), viewed as an $\mathcal{L}$-structure. In this talk we are concerned with difference fields that satisfy various properties of a model theoretic nature; superstability and quantifier elimination in particular.

A difference field is called *trivial* if the automorphism is an integer power of the Frobenius automorphism (meaning the identity in characteristic 0). Note that an algebraically closed difference field is trivial exactly when the automorphism is definable in the pure field language. E. Hrushovski has shown (this is a consequence of Proposition 3 in [1]) that all superstable difference fields in characteristic 0 are trivial. Conjecturally, *all* superstable difference fields are trivial.

While we are not able to resolve this conjecture in general, our main result considers the following special case.

THEOREM. *All difference fields admitting quantifier elimination (in the language $\mathcal{L}$) are trivial.*

Recall that difference fields with quantifier elimination are in particular superstable (see S. Nonvidé [2]).

[1] E. HRUSHOVSKI, *On superstable fields with automorphisms*, in ***The model theory of groups*** (A. Nesin and A. Pillay, editors), University of Notre Dame Press, 1989.

[2] S. NONVIDÉ, *Corps aux différences finies*, ***Comptes Rendus de l'Académie des Sciences Paris*** (1992), pp. 423–425.

► ALAN PARIS, *Lauchli realizability and gauge theories*.
CDI Corporation, 271 Front St. #4, Owego, New York 13827, USA.
E-mail: alan@ccsis.com, alan.paris@lmco.com.

On the one side is the physicist's belief in the "laws of Nature", guilded with symmetry, as the most fundamental bedrock of logic in the Universe .... Set against this is some picture of abstract computation ... as the most fundamental of all notions .... The great unsolved puzzle for the future is to decide which is more fundamental: symmetry or computation.

John D. Barrow, Theories of Everything

Soundness of a realizability is just careful bookkepping applied to (coded) proof trees. But proving a formula A unrealizable (the conclusion of a proof of completeness) requires a property posessed by all realizers but not posessed by any hypothetical realizer of A. Kleene uses the property "partial recursive". Lauchli uses "invariant under the group $(\mathbb{Z}, +, 0)$".

I generalize Lauchli's result to the topos $\mathrm{Sh}(G, X)$ of G-equivalent sheaves, G a topological group and X a G-space. The key lemma is: Every $\mathrm{Sh}(G, X)$ is projective in $\mathrm{Presh}(G, X)$.

Lauchli's Theorem is the restriction to first-order formulae of $G = \mathbb{Z}$, $X =$ the 1-point space.

I generalize the classical-to-intuitionist double-negation translation to a "sheafification translation" of $\mathrm{Sh}(G, X)$ into $\mathrm{Presh}(G, X)$.

This "realizability $\Longleftrightarrow$ sheafification translation" duality allow the two universes to comprehend each other as consistently misinterpreting Truth.

I generalize the Stone Representation Theorem to non-distributive lattices L to produce *two* spaces: a space $X = \mathrm{Spec}(L)$ of "points" and a totally disconnected space U of "atoms". The G-action on U induces both a topology on G and a G-space structure on X.

Distributive examples: Cohen forcing, Mostowski models, Scott analysis, algebraic geometry.

Non-distributive examples: classical Galois theory, "quantum logic", Klein's Erlanger Programme.

Let $X = \mathrm{Spec}(\text{Lagrangian})$ and $G =$ gauge group of a Yang-Mills bundle over X, Lauchli realizability provides a link between "laws of Nature" (formulae forced in $\mathrm{Presh}(G, X)$) and "computations" (gauge-invariant realizers).

► DAVID A. ROSS, *Nonstandard proofs of a lemma in constructive measure theory*.
Department of Mathematics, University of Hawaii, Honolulu, Hawaii 96822, USA.
E-mail: ross@math.hawaii.edu.

Nonstandard proofs are given for a central lemma in Bishop's constructive measure theory. The nature of these proofs has implications both for recent work on the constructivisation of nonstandard analysis and for the general question of what constitutes a constructive proof.

► WIM RUITENBURG, *End-extension and cofinal extension Kripke models*.
Department of Mathematics, Marquette University, P. O. Box 1881, Milwaukee, Wisconsin 53201, USA.
E-mail: wimr@mscs.mu.edu.

This is joint work with Mohammad Ardeshir and Saeed Salehi. We present a theory which is sound and complete for the class of Kripke models whose node structures are end-extensions of lower node structures. We present a similar result for cofinal extension Kripke models. Both results are applications of a general theorem which generalizes a well-known result about constant domain Kripke models.

Heyting Arithmetic extends the theory above of end-extension Kripke models, hence is sound and complete for its end-extension Kripke models.

[1] SABINE GÖRNEMANN, *A logic stonger than intuitionism*, ***Journal of Symbolic Logic***, vol. 36 (1971), pp. 249–261.

[2] DIETER KLEMKE, *Ein Henkin-Beweis für die Vollständigkeit eines Kalküls relativ zur Grzegorczyk-Semantik*, ***Archiv für Mathematische Logik und Grundlagenforschung***, vol. 14 (1971), pp. 148–161.

► RICHARD SOMMER, *Weak well-foundedness*.
CSLI, Stanford University, Stanford, California 94305-4115, USA.
E-mail: sommer@csli.stanford.edu.

It is straightforward to show that, over $I\Delta_0$, transfinite induction up to ω^2 for bounded arithmetic formulas proves the totality of each primitive recursive function (using a standard representation of ordinals up to ω^2). In other words, transfinite induction for small ordinals and low-complexity sets yields functions of relatively high growth rates. Here we analyze weaker well-foundedness principles that assert there are no descending sequences in an ordinal α (represented by a standard notation system) given by functions of a particular complexity. We give the precise relative strengths of such principles in terms of α and for various classes of functions of low complexity.

► SERGEI TUPAILO, *Realization of constructive set theory in explicit mathematics*.
Institut für Informatik und angewandte Mathematik, Bern, Switzerland.
E-mail: sergei@iam.unibe.ch.

Systems of Explicit Mathematics (the theory T_0 and others) were introduced by S. Feferman in the '70s as a foundation for Bishop-style constructive mathematics (see [2]). Although Explicit Mathematics has both set-theoretic and recursion-theoretic models, the situation on the other side, that is interpreting other theories *in* Explicit Mathematics, was not so well-developed, the strongest systems having been interpreted reaching in strength only $\Pi^1_1 - \mathrm{CA}$. This became apparent again, when Jäger and Studer [3] introduced a theory T_m extending T_0 by a Mahlo axiom and built its model in Kripke-Platek Set Theory KPM, but the question of lower bound was left open.

We show that *this can be solved by a realizability interpretation of Aczel's Constructive Set Theory* CZF *enriched by a corresponding Mahlo principle*. The final result is the following:

THEOREM 5. CZFM *is realizable in intuitionistic* T_M,

where CZFM is a Mahlo-extension of CZF introduced in [4] and T_M is an extension of T_0 by a Mahlo-universe.

This yields the following corollary:

$$|\mathrm{CZFM}| \leq |T_M \text{ intuitionistic}| \leq |T_M \text{ classical}| \leq |\mathrm{KPM}^+|,$$

where $|\cdot|$ stands for proof-theoretic strength and KPM^+ is a certain minor extension of KPM. Since it's known that $|\mathrm{CZFM}| = |\mathrm{KPM}^+|$, all these inequalities are in fact exact, proving in particular that T_M inherits a special for this strength feature of Explicit Mathematics: proof-theoretically the Law of Excluded Middle doesn't matter!

[1] P. ACZEL, *The type theoretic interpretation of constructive set theory*, in ***Logic colloquium '77*** (A. MacIntyre, L. Pacholski, and J. Paris, editors), 1978, pp. 55–66.

[2] S. FEFERMAN, *Constructive theories of functions and classes*, in ***Logic colloquium '78***, 1979, pp. 159–224.

[3] G. JÄGER and T. STUDER, *Extending the system T_0 of explicit mathematics: the limit and Mahlo axioms*, submitted.

[4] M. RATHJEN, *Interpreting Mahlo set theory in Mahlo type theory*, preprint.

[5] S. TUPAILO, *Realization of analysis into explicit mathematics*, submitted.

[6] ———, *Realization of constructive set theory into explicit mathematics: a lower bound for impredicative Mahlo universe*, preprint.

► AMY K. C. S. VANDERBILT, *Locally determined nonmonotonic rule systems*.
Department of Mathematics, University of Florida, 358 Little Hall, P. O. Box 118105, Gainesville, Florida 32611-8105, USA.
E-mail: akcsv@math.ufl.edu.

Nonmonotonic Rule Systems have been extensively studied in papers by Cenzer and Remmel [1], and by Marek, Nerode, and Remmel [2, 3]. We introduce the notion of levels for a nonmonotonic rule system $\mathcal{U} = \langle \mathcal{U}, \mathcal{N} \rangle$. Based on this we define Locally Determined Nonmonotonic Rule Systems and examine various results brought about or enhanced by this type of system. This is work in conjunction with Douglas A. Cenzer and J. B. Remmel.

[1] D. CENZER and J. B. REMMEL, Π^0_1*-classes in mathematics*, ***Handbook of recursive mathematics***, vol. 2 (Yu. L. Ershov, S. S. Goncharov, A. Nerode, and J. B. Remmel, editors), Studies in Logic and the Foundations of Mathematics, vol. 139, Elsevier, 1998, pp. 623–822.

[2] W. MAREK, A. NERODE, and J. B. REMMEL, *Nonmonotonic rule systems I*, ***Annals of Mathematics and Artificial Intelligence***, vol. 1 (1990), pp. 241–273.

[3] ———, *Nonmonotonic rule systems II*, ***Annals of Mathematics and Artificial Intelligence***, vol. 5 (1992), pp. 229–263.

► EVGUENI VASSILIEV, *Pairs of models of simple theories*.
Department of Mathematics, University of Notre Dame, Notre Dame, Indiana 46556, USA.
E-mail: evassili@hilbert.helios.nd.edu.

Given a complete first order theory T, a pair of models of T is a structure (M, P) in the language $L(T) \cup \{P\}$, where P is a new unary predicate symbol, $M \models T$, and P is interpreted by an elementary submodel of M. In this talk, we will focus on the question of preserving simplicity under such pair expansions, and discuss some properties of the pairs of models of simple theories.

► DEJIA WANG, *On ω-models of* RCA_0 *plus the DNR axiom*.
Department of Mathematics, University of Wisconsin, Madison, Wisconsin 53706, USA.
E-mail: dwang@math.wisc.edu.

An ω-model of RCA_0 is essentially the same thing as a Turing ideal, i.e., $\mathcal{M}$ is an ω-model of RCA_0 if and only if $\mathcal{M}$ is nonempty and closed under $\oplus$ and $\leq_T$. We will describe several ω-models of RCA_0 plus the DNR axiom. For instance there exists a *low* ω-model $\mathcal{M}$ such that $\mathcal{M} \models \mathrm{RCA}_0 + \mathrm{DNR} + \neg\mathrm{WKL}$. Furthermore we will describe how such models are connected to some open questions in reverse mathematics.

[1] MARIAGNESE GIUSTO and STEPHEN G. SIMPSON, *Located sets and reverse mathematics*, ***Journal of Symbolic Logic***, vol. 37 (1999), to appear.

[2] C. G. JOCKUSCH, JR. and R. I. SOARE, Π^0_1*-classes and degrees of theories*, ***Transactions of the American Mathematical Society***, vol. 173 (1972), pp. 33–55.

[3] C. G. JOCKUSCH, JR. and FRANK STEPHAN, *A cohesive set which is not high*, ***Mathematical Logic Quarterly***, vol. 39 (1993), pp. 515–530.

[4] A. KUČERA, ***Measure,*** Π^0_1***-classes, and complete extensions of PA***, Lecture Notes in Mathematics, vol. 1141, Springer-Verlag, 1986, pp. 245–259.

[5] ———, *On the use of diagonally nonrecursive functions*, ***Logic Colloquium '87*** (H. D. Ebbinghaus et al., editors), Elsevier Science Publishers B. V., North-Holland, 1989.

[6] STEPHEN G. SIMPSON, *Open problems in reverse mathematics*, a handout at the AMS-INS-SIAM conference *Computability Theory and Applications*, June 13–17, 1999, Boulder Colorado.

► OSAMU WATARI, KOJI NAKATOGAWA, AND TAKESHI UENO, *Normalization theorems for substructural logics in Gentzen-style natural deduction*.
Division of Systems and Information Engineering, Hokkaido University, 060-8628 Sapporo,

Japan.
E-mail: watari@main.eng.hokudai.ac.jp.
Department of Philosophy, Hokkaido University.
Department of Mathematics, Hokkaido University.

In [3], we introduced a formal system, called **NFL**, of Gentzen-style natural deduction, which is equivalent to the full Lambek calculus, where no structural inference rules are allowed. For **NFL**, we have obtained the following results:

1. In **NFL** itself, which has $\supset$, $\supset'$, $\neg$, $\neg'$, $*$, $\wedge$, $\vee$ as logical connectives, one cannot normalize every proof, due to the fact that a pair of the applications of $\supset$-introduction and $\supset$-elimination cannot be eliminated if at least one application of either $\vee$-elimination or $*$-elimination occurs between that pair.

2. Let **NFL** $-\{\supset,\neg\}$ be a system which results from **NFL** by deleting $\supset$ and $\neg$. Then, normalization holds for **NFL** $-\{\supset,\neg\}$.

3. Let $\widetilde{\textbf{NFL}}$ be a system which results from **NFL** by modifying the elimination rules of $*$ and $\vee$ in such a way that the tree-like structure of the paths from the assumptions of the minor-premise(s) of $*$-elimination ($\vee$-elimination, resp.) to that minor-premise(s) is nullified so that one can use these assumptions to carry out further inferences while neglecting the tree-like structure among them. Then, normalization holds for $\widetilde{\textbf{NFL}}$.

[1] K. Nakatogawa and T. Ueno, *On structural inference rules for Gentzen-style natural deduction, Part I*, in ***Proceedings of the sixth Asian logic conference*** (Beijing, 1996) (C. Chong, M. Yasugi, et al., editors), World Scientific, 1998, pp. 199–221

[2] A. S. Troelstra and H. Schwichtenberg, ***Basic proof theory***, Cambridge University Press, 1996.

[3] T. Ueno, O. Watari, and K. Nakatogawa, *On structural inference rules for Gentzen-style natural deduction, Part II*, submitted to ***Proceedings of the seventh Asian logic conference***, 1999. http://logic.let.hokudai.ac.jp/~watari/Research/7thALC/proceeding-nfl.pdf.

► FRANK WROBLEWSKI, *Quine's definition of class resolves Russell's paradox*.
1662 Coughlin St., Laramie, Wyoming 82072, USA.
E-mail: estellew@uwyo.edu.

In ***Set Theory and its Logic***, Quine defines the subject-predicate class-in-use notation $\beta \in \{\,\alpha \mid \varphi(\alpha)\,\}$ to be the abbreviation of a subject-predicate proposition equivalent to the class condition $\varphi(\beta)$, where α and β are variables. To identify the definition of class, it remains to match the terms on either side of the '$\in$' copulas, the notation for the 'is' of predication.

To resolve Russell's paradox, we use Quine's result to prove that Russell's set cannot be an element of any class including itself. Quine proves that substituting classes for variables requires the support of comprehension $\exists\beta\;(\beta = \{\,\alpha \mid \varphi\,\})$. This enabled Quine to prove the characterization of class-in-use:

$$\zeta \in \{\,\alpha \mid \varphi(\alpha)\,\} \equiv \exists\beta\;(\beta = \zeta).\varphi(\zeta), \quad \text{where } \zeta \text{ is a class or variable.}$$

When ζ is a variable, the characterization reduces to the naive axiom. Here again, since Russell's set falsifies comprehension, Russell's paradox fails and undercuts the reason for axiomatizing classes.

Time permitting, we will discuss the empirical account of mathematics which explains the indispensability of mathematics to science. That mathematics is found to be true of the world is compelling evidence that mathematics is true under some empirical interpretation. Accordingly, we identify nature's bearers of mathematical terms, the variables and classes. The variables denote subatomic particles and the classes are locutions articulating their identifying properties. The truth-maker driving mathematics proper turns out to be the particle-property relation expressed by comprehension. Comprehension $\exists\beta\ (\beta = \{\alpha \mid \varphi\})$ explains the particle-property duality of nature, where for any natural property there exists a subatomic particle identifying with that property. Thus, truth in mathematics is empirical whose certainty is merely the temporal stability of the duality of nature. In this way, mathematics translates to natural language mediating particle-property relations.

► ALADDIN M. YAQUB, *Tarski's semantic conception of truth*.
Department of Philosophy, University of New Mexico, Albuquerque, New Mexico 87131-1151, USA.
E-mail: ayaqub@unm.edu.

There seem to be two basic intuitions underlying our ordinary understanding of the predicate 'True' as applied to sentences, statements, and beliefs. I shall refer to these intuitions as "the description intuition" and "the assertion intuition". The description intuition may roughly be expressed as follows: to say that a sentence (or a statement) is true is to say that it correctly describes the way things are, or are not, and to say that it is false is to say that it incorrectly describes the way things are, or are not. The assertion intuition may be stated as follows: the condition under which a sentence (or a statement) is true is given by what the sentence asserts (or is given by the statement itself). I use the phrase 'conceptual deflationism' to refer to the thesis that the assertion intuition captures the concept of truth completely and correctly. It is conceptual deflationism because it does not go beyond what is asserted by a sentence in order to define the truth status of that sentence. In this paper I first argue that at the center of Tarski's work in truth theory is a concept of truth that is very close, if not identical, to conceptual deflationism. In making this argument, I propose a different interpretation of Tarski's Convention-T, and I clarify the relation of his formal account of truth to his semantic conception. I then go on to show how the revision theory of truth (and *only* the revision theory of truth) permits Tarski's (reconstructed) conception to meet the demands of logical consistency and material adequacy.

Abstracts presented by title

► M. BURGIN, *Topological computations*.
Department of Mathematics, University of California, Los Angeles, California 90095-1555, USA.
E-mail: mburgin@math.ucla.edu.

A topological Turing machine is a mathematical model of algorithms with the structure of the conventional Turing machine but the result of which is defined by means of the topology on the set W of all output words [1]. Limit Turing machines are such topological Turing machines in which the result is defined by the convergence. When topology on W is discrete, we have inductive Turing machines [2]. Here we consider decision power of topological and inductive Turing machines with recursive memory and $W = \mathbb{N}$.

THEOREM 1. *For any set A from $\mathbb{N}$, there is such a limit Turing machine T such that $T(\mathbb{N}) = A$, i.e., there is a limit Turing machine with the range A.*

It means that without additional conditions on topology, limit Turing machines are much more powerful than inductive Turing machines.

DEFINITION. A topology on W is called locally recursive (recursively decidable) if for any point a from $\mathbb{N}$ there is a recursively defined filter of (recursively decidable) neighborhoods of a converging to a.

THEOREM 2. *The range R_T of the limit Turing machine T with a complete recursively decidable topology is recursively enumerable, i.e., there is an inductive Turing machine with recursive memory that T enumerates A, that is, x is in A if and only if T is defined for x.*

[1] M. S. BURGIN, *Universal limit Turing machines*, ***Notices of the Russian Academy of Sciences***, vol. 325 (1983), pp. 654–658.
[2] ———, *Inductive Turing machines*, ***Notices of the Russian Academy of Sciences***, vol. 270 (1983), pp. 1289–1293.

► KONSTANTINOS GEORGATOS AND RUILI YE, *Specificity orderings on proper denoting terms*.
Department of Mathematics, John Jay College of Criminal Justice, City University of New York, 445 West 59th Street, New York, New York 10019, USA.
E-mail: `lesalpes@yahoo.com`.

By a *proper denoting term* (cf. [1]) we mean either a definite description or a name. We give a logical account for what happens when proper denoting terms occur in ordinary conversations, and deal with some puzzles relating to belief, by modeling proper denoting terms using a *specificity ordering*. For each believing agent, the specificity ordering is a partial ordering $\leq$ on the set of all proper denoting terms. The more specific a proper denoting term is for the agent, the higher its place is in the ordering, which means the less candidates for its reference there are for the agent. The place of names in the ordering is determined by the epistemic state. For example, for every agent,

> *the man who won the Nobel Prize in Physics* $\leq$ *the man who won the Nobel Prize in Physics in 1921*,

whereas for the agent who believes that Einstein was the unique winner of the Nobel Prize in Physics in 1921, the following also holds:

> *the man who won the Nobel Prize in Physics in 1921* $=$ *Einstein*.

We explore the nature of such orderings and their relationship to epistemic states. This provides us with a suitable framework for studying belief change, since the change of belief, caused by new information, leads to the change of the specificity ordering.

[1] B. RUSSELL, *On denoting*, ***Mind***, vol. 14 (1905), pp. 479–493.

► DANIEL LEIVANT, *Program verification in substructural logic and computational complexity*.
Computer Science Department, Indiana University, Bloomington, Indiana 47405, USA.
E-mail: `leivant@cs.indiana.edu`.

A link between first order verification of functional programs and computational complexity was established in the author's LCC '94 paper (Springer Lecture Notes in Computer Science 960). In that framework, the characterization of feasible complexity (i.e., poly-time) required a restriction of Induction to existential formulas. Recently, research by Hofmann, as well as Aehlig, Bellantoni, Niggl, and Schwichtenberg, showed how to accommodate induction for arbitrary formulas (or, equivalently, recurrence in all types, in an appropriate lambda calculus), without transgressing the boundaries of poly-time, provided the formalism is extended into affine linear logic.

We shall exhibit a simpler approach, in which the linear control resides in the use of substructural inference rules rather than in the explicit use of linear modalities. The simplification bodes well for implementation of these ideas in static analysis of functional programs, and in environments for program synthesis.

► PAOLO LIPPARINI, *Still more on regular ultrafilters*.
Università di Tor Vergata, Dipartimento di Mathematica, Viale della Ricerca Scientifica I-00133 Rome, Italy.
E-mail: lipparin@axp.mat.uniroma2.it.

Throughout, D is an ultrafilter, and κ, μ, λ are infinite cardinals.

THEOREM. *Suppose that either*:

(a) *κ is strongly inaccessible and D is not κ-decomposable, or*
(b) *κ is singular strong limit and D is $(\mathrm{cf}\,\kappa)^+$-complete.*

Then the following are equivalent:

(i) $\left|\prod_D \kappa\right| > \kappa$;
(ii) *there are arbitrarily large $\mu < \kappa$ such that D is μ-decomposable*;
(iii) *D is 2^κ-decomposable*;
(iv) $\left|\prod_D 2^\kappa\right| > 2^\kappa$;
(v) *there is $\mu < \kappa$ such that $\left|\prod_D \mu\right| > 2^\kappa$.*

If the above conditions hold then 2^κ is regular.

PROPOSITION.

(a) $\left|\prod_D 2^\kappa\right| \le \left|\prod_D 2^{<\kappa}\right|^{\mathrm{cf}\left(\prod_D \langle\kappa,<\rangle\right)}$.
(b) *If $\left|\prod_D 2^\kappa\right| > 2^\lambda$, $\left|\prod_D 2^{<\kappa}\right| \le 2^\lambda$ and D is (κ^+,λ)-regular then D is (κ,λ)-regular.*
(c) *If $2^\kappa = \kappa^+$, $2^\lambda > \lambda^+$ and D is (κ^+,λ)-regular then $\left|\prod_D \kappa\right| > \lambda$.*
(d) *If D is (κ^{++},λ)-regular, $\left|\prod_D \kappa^+\right| > \lambda$ and $\left|\prod_D \kappa\right| < \left|\prod_D \kappa^+\right|$ then D is (κ^+,λ)-regular.*
(e) *If $\left|\prod_D 2^{\kappa^{+n}}\right| > 2^{\kappa^{+n}}$ then D is μ-decomposable for some μ with $\kappa \le \mu \le 2^\kappa$.*

PROBLEMS.

(a) Suppose that $2^\kappa < \mu < 2^{\kappa^+}$ and D is μ-decomposable. Is D necessarily λ-decomposable for some λ with $\kappa \le \lambda \le 2^\kappa$?
(b) Can we get D (κ,λ)-regular from the hypotheses of (c)?

[1] PAOLO LIPPARINI, *More on regular ultrafilters in ZFC*, re-submitted.

► GIAN ARTURO MARCO, *PC-classes and analytic G-spaces*.
C. so Manacorda 44, 15033 Casale Monferrato (AL), Italy.
E-mail: gianarturomarco@tin.it.

Generalizing a theorem of J. Barwise [1] on resplendent models and limits of Borel approximation to analytic sets, this paper characterizes which PC-classes of structures admit an $L_{\omega_1,\omega}$ description. The approximation problem is an instance of the more general problem of finding an answer to the question: when can we completely recover Borel elementary properties of a nonelementary structure from the topological properties of its automorphism group?

Let XL be the space of (codes of) structures of a countable language L and S the symmetric group of permutations on ω with the pointwise topology. We show that if $S \subset XL$ is analytic G-invariant for G a closed subgroup of S such that G is isomorphic to $\mathrm{Aut}(M)$ for some automorphism group of a structure M whose code is in S then:

1. If G is a c. l. i. group (complete left invariant metric group) then S has the approximation property.
2. If S has the approximation property then the intersection between $\mathrm{Aut}(M)_X$ (the automorphism group pointwise fixing a subset X of elements of M) and the automorphism group fixing setwise a relation R defined on the domain of the structure M whose code is in S has the finite index property.

The proof is based on a topological version of the classical omitting types theorem in the realm of strong Choquet G-spaces.

[1] J. Barwise, *Some applications of Henkin quantifiers*, ***Israel Journal of Mathematics***, vol. 25 (1976).

► CYRUS F. NOURANI, *Functorial fragment definable models*.
METAAI and UCSB, P. O. Box 278, Cardiff by the Sea, California 92007, USA.
E-mail: project_metaai@compuserve.com.

Specific models with an infinitary variant to Robinson consistency with the new functorial model theory provide new functorial consistency proof techniques. Starting with $L_{\omega_1,B}$ language fragments and the consistency theorem [3] define Functorial ω-Chain models [4]. A functor $F\colon L_{\omega_1,B}\langle \mathrm{Op}\rangle \to \mathrm{Set}$ can be defined by sets F_i, where the F_i's are defining a free structure on some subfragment of $L_{\omega_1,B}$ called FLC-models [3].

Definition. For functions f and g in a structure for a language L, define the morphic preorder $f \ll g$ if and only if there are formulas p and q from L such that the formulas define f and g respectively, and p is a subformula of q in the sense of Keisler [1].

Let a FLC model be the limit model defined by the elementary chain on the L-reducts of the models defined by functors on fragments [2]. A specific FLC model is defined by Theorem 1's proof.

Theorem 1. *There is an elementary chain FLC model for L, where L is $L_{\omega_1,B}$.*

Theorem 2. *FLC models with the morphic preorder property are always definable provided the functions ordered are definable with the exact fragment with which the FLC model is obtained from and defined for.*

Theorem 3. *The morphic preorder functor defines FLC models for $L_{\omega_1,B}$ provided the functions ordered are definable with the exact fragment with which the FLC model is obtained from and defined for.*

[1] H. J. Keisler, ***Model theory for infinitary logic***, North-Holland, Amsterdam, 1971.
[2] C. F. Nourani, *Functorial models, admissible sets, and generic rudimentary fragments*, Summer Logic Colloquium, 1997, Leeds.
[3] ———, *Functorial models and infinitary Gödel consistency*, Gödel Conference, CRIM, Barcelona, Spain, March 1999.
[4] ———, *Functorial ω-chain models*, 1998, Holiday Mathematics Symposium, January 1999, Las Cruces.

► SAEED P. SALEHI, *Decidable formulas of intuitionistic primitive recursive arithmetic*.
Institute of Mathematics, Polish Academy of Sciences, ul. Śniadeckich 8, P. O. Box 137, 00-950 Warsaw, Poland.
E-mail: salehi@impan.gov.pl.

By formalizing some classical facts about provably total recursive functions in intuitionistic primitive recursive arithmetic ($iPRA$), we prove that the set of decidable formulas of $iPRA$ and of $i\Sigma_1^+$ (intuitionistic Σ_1-induction in the language of PRA) coincides with the set of its provably Δ_1-formulas and coincides with the set of its provably atomic formulas. For the case of $i\Sigma_1$ (in the language of arithmetic) the decidable formulas are just its provably Δ_1-formulas.

By the same methods, we shall give another proof of a theorem of Marković and De Jongh [1]: the decidable formulas of HA are its (provably) Δ_1-formulas.

We also show that provably *total* functions of HA are precisely provably *recursive* functions of PA (that is $< \varepsilon_0$-recursive functions), which corresponds to a theorem of Wehmeier [2]: provably total functions of $i\Sigma_1$ are primitive recursive functions, which are the provably recursive functions of $I\Sigma_1$.

[1] Zoran Marković, *On the structure of Kripke models of Heyting arithmetic*, ***Mathematical Logic Quarterly***, vol. 39 (1993), pp. 531–538.

[2] Kai F. Wehmeier, *Fragments of HA based on Σ_1-induction*, ***Archive for Mathematical Logic***, vol. 37 (1997), pp. 37–49.

► RALF SCHINDLER, *Inner models and* $0^{\P}$.
Institut für formale Logik, Universität Wien, Währinger Str. 25, 1090 Wien, Austria.
E-mail: rds@logic.univie.ac.at.

I indicate the significance of $0^{\P}$ ("zero hand-grenade") for inner model theory. $0^{\P}$ corresponds to the existence of indiscernibles for an inner model with a proper class of strong cardinals. $0^{\P}$ marks the point at which linear comparability breaks down in a general sense.

We survey the theory of the core model below $0^{\P}$, as has been developed by the speaker.

► CHAZ SCHLINDWEIN, *Shelah's theorems on iterated Namba forcing*.
Division of Mathematics and Computer Science, Lander University, Greenwood, South Carolina 29649, USA.
E-mail: chaz@lander.edu.

In this talk we survey the proofs of three forcing preservation theorems from Chapters XI and XV of [1], with slight improvements, particularly to the definitions of S-condition (Chapter XI) and UP-condition (Chapter XV). These preservation theorems apply to certain non-semi-proper forcings, including Namba forcing.

[1] Saharon Shelah, ***Proper and improper forcing***, Perspectives in Mathematical Logic, Springer-Verlag, 1998.

► D. SINGH, *A note on some combinatorial aspects of multisets*.
Mathematics Department (Faculty of Science), Ahmadu Bello University (ABU), Zaria (Kaduna), Nigeria.
E-mail: mdsingh@abu.edu.ng.

The purpose of this note is to delineate some insight into the concept of multisets along with a couple of combinatorial results related to multisets. The paper indicates that a general formula needs to be worked out for determining the cardinality of the 'Set' (whose elements may be multisets but do not repeat) of all multisubsets of a finite multiset $[x, y, z, \dots]_{m,p,t,\dots}$ in which x occurs at most m times, y occurs at most p times, and so on. It outlines some directions provided in the literature and points out that they all turn out to be inefficient. Finally, a general formula to this effect is provided along with a couple of other related results.

[1] W. Blizard, *Multiset theory*, ***Notre Dame Journal of Formal Logic***, vol. 1 (1989), pp. 36–66.