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The annual conference of the Australasian Association for Logic was held at Newman College, University of Melbourne, from Friday, July 2, through Sunday, July 4, 1999. Twenty-one papers were presented at the meeting, and one paper was presented by title. We welcomed two international visitors: Professor David Lewis of Princeton University and Dr. Manuel Correia of Pontificia Universidad Catolica de Chile. Bob Meyer, Rod Girle and Errol Martin gave tributes to Paul Thistlewaite, a former student of University of Queensland, La Trobe University, and Australian National University, and former staff member of ANU, who died last February. The Conference was organized by Ross Brady of La Trobe University and Graham Oppy of Monash University.

Abstracts of contributed talks given (in person or by title) follow.

Conference Organizer
ROSS BRADY

Abstracts of contributed talks

► KATALIN BIMBÓ, *Kripke semantics for structurally free logics without distributivity*.
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Structurally free logics (LC) were introduced by Dunn and Meyer [3]. Syntactically these systems were formulated as generalised Gentzen calculi, where the generalisation amounts to dropping structural rules and introducing combinatory rules instead. This generalisation makes precise the analogy between structural rules and combinators observed long ago by Curry. The introduction of combinators forces the structural connective (',') to be a non-associative binary operation, since application is not associative in combinatory logic. In this framework one can model a wide range of the so-called 'substructural logics' by varying the set of combinatory rules. Another way to arrive at different systems is to consider extensions by further sentential connectives (originally, there were only three intensional operators in the system, $\circ$ (fusion), $\rightarrow$ and $\leftarrow$ (two implications)). Bimbó and Dunn [2] extended LC by conjunction and disjunction, both stipulating and omitting distributivity of $\wedge$ and $\vee$. It is relatively straightforward to give a Kripke-style semantics for the distributive system. In this paper I shall present two Kripke-style semantics for the system without distributivity. The crucial part of the semantics is the lattice representation and its seamless combination with the relation R, which gives rise to the intensional operations. The first representation presented is closely related to two other ones: Hartonas and Dunn [4] and Urquhart [5]. The

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ternary relation for the binary intensional operators is defined in the Routley-Meyer fashion. The second one generalizes some observations from the first representation and the one by Birkhoff and Frink [1].

[1] G. BIRKHOFF and O. FRINK, *Representation of lattices by sets*, ***Transactions of the American Mathematical Society***, vol. 64 (1948), pp. 299–316.

[2] K. BIMBÓ and J. M. DUNN, *Two extensions of the structurally free logic LC*, ***The Logic Journal of IGPL***, vol. 6 (1998), pp. 403–424.

[3] J. M. DUNN and R. K. MEYER, *Combinators and structurally free logic*, ***The Logic Journal of IGPL***, vol. 5 (1997), pp. 505–537.

[4] C. HARTONAS and J. M. DUNN, *Stone duality for lattices*, ***Algebra Universalis***, vol. 37 (1997), pp. 391–401.

[5] A. URQUHART, *A topological representation theory for lattices*, ***Algebra Universalis***, vol. 8 (1978), pp. 45–58.

► ARCADY BLINOV, *Chance moves in semantical games: Uses and varieties*.
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In Game Theoretical Semantics (GTS), change moves are one source of truth-value gaps (another is branching quantifiers and informational independence at large). One central use of chance moves is in chance-prenexed games that are GTS-counterparts of supervaluations. I am trying to (i) answer the question "Why is the supervaluational use of chance moves syntactically rigid?", and (ii) explain some well-known facts about supervaluations in terms of the syntactical rigidity of chance moves in chance-prenexed games. Then I discuss another, non-supervaluational and syntactically flexible use of chance moves. Some further topics of discussion include: GTS-modelling of gluts; chance moves and duality; chance moves combined with informational independence.

► ROSS BRADY, *Interpretational meta-logic and the semantics of quantified relevant logics*.
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There is a part of the meta-theory that corresponds most closely to the object-logic, that is, the logic of the meta-logical connectives and quantifiers used in expressing the interpretation statements. We have the following interpretation statements for classical logic, with primitives, $\wedge$, $\vee$, $\sim$, $\forall$, $\exists$:

(i) $I(A \wedge B) = T$ if and only if $I(A) = T$ and $I(B) = T$,
(ii) $I(A \vee B) = T$ if and only if $I(A) = T$ or $I(B) = T$,
(iii) $I({\sim}A) = T$ if and only if not $I(A) = T$, i.e., $I(A) = F$.
(iv) $I(\forall x\ A) = T$ if and only if $\mathrm{Id}/x(A) = T$, for all elements d in the domain D.
(v) $I(\exists x\ A) = T$ if and only if $\mathrm{Id}/x(A) = T$, for some element d in the domain D.

A classical meta-logic could be built up from these meta-logical connectives, as it is part of the Hilbertian usage of meta-logic. This meta-logic could reasonably be seen as the interpretation of the corresponding object-logic and thus we will call it the interpretational meta-logic. This is just the logic of the interpretation statements, without reference to the underlying meta-logic. Moreover, we could separate off this interpretational meta-logic from the underlying meta-logic, even though in this case both logics are classical.

This method provides a semantics for quantified relevant logics, incorporating an interpretational meta-logic. We then prove that classical negation '$-$', fusions '$\emptyset$', '$\emptyset c$', and the sentential constant 't' can be conservatively added to these quantificational logics. This then enables us to Gentzenize quantified relevant logics, using the methods I have previously used for the sentential logics.

► MARTIN BUNDER, *Expedited Broda-Damas bracket abstraction*.
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A bracket abstraction algorithm is a means of translating lambda terms into combinators, and so, via the Curry-Howard isomorphism, of translating natural deduction style proofs in implicational intuitionistic logic into Hilbert style proofs. Broda and Damas (in [1]) introduce a new, rather natural set of combinators and a new form of abstraction which introduces at most one combinator for each lambda abstraction. This leads to particularly compact combinatory terms (and so proofs). A disadvantage of their abstraction process is that it includes the whole original Schonfinkel algorithm plus two mappings which convert the Schonfinkel abstract into the new abstract. This paper shows how the new abstraction can be done more directly, in fact, using only $2n - 1$ algorithm steps if there are n occurrences of the variable to be abstracted in the term. Some properties of Broda-Damas combinators are considered and the logic corresponding to the new combinators is shown to be represented most simply by two axioms and three rules (none of them *modus ponens*).

[1] Broda and Damas, ***Journal of Symbolic Logic***, vol. 62 (1997), pp. 729–740.

► SAM BUTCHART AND SUSAN ROGERSON, *Naive comprehension and contracting implications*.
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In his paper [1], Greg Restall conjectured that a logic supports a naive comprehension scheme if and only if it is robustly contraction free, that is, if and only if no contracting connective is definable in terms of the primitive connectives of the logic. In this paper, we will describe our attempts to find a counter-example to the conjecture. We will present some purely implicational logics which are robustly contraction free, but which trivialize naive comprehension.

[1] G. Restall, *How to be really contraction free*, ***Studia Logica***, vol. 52 (1993), pp. 38–391.

► MANUEL A. CORREIA, *Existential import of categorical propositions in ancient semantics*.
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Aristotle's logic has scarce considerations concerning how its categorical propositions should be interpreted, especially whether they imply or not the existence of objects denoted by the subject term of a sentence. There are some 100 years of discussion about paradoxes derived from his meagre explanations. M. Wedin in [1] pointed out the importance of *Cat*. 13b. 27–35 in developing an Aristotelian doctrine on this issue, but he thought that the ancient commentators had not added anything substantial to its comprehension, for they did not notice the difficulties that existential import creates in Logic. But Wedin seems wrong in this point, for—as I argue—those commentators gave specific comments on other crucial passages of Aristotle's writings (i.e., *PeriH*. 16b. 15; 21a. 25–28; and 21a. 32–33), which help us to understand the problem and even to resolve all the paradoxes and misunderstandings. The point is to distinguish a *kata auto* way of predicating 'is' in a categorical proposition from a *kata sumbebekos* way of doing it. According to the first sense, non-existence produces contradictions in affirmations, but not in negations; according to the second sense, neither existence nor non-existence produce contradictions in propositions (either affirmative or negative ones), for this sense is entirely predicative and without existential commitment, so that truth or falsity depend only on the opinion (*doxa*) producing the proposition.

[1] M. WEDIN, *Aristotle on existential import of singular sentences*, ***Phronesis***, vol. 23 (1978), pp. 179–194.

► ALLEN P. HAZEN, *The "strictly nominalistic constructivity" of PM 1925, App. B*.
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In the Introduction and Appendix B to the 1925 second edition of *Principia Mathematica*, Russell suggested a logical system strictly stronger than Ramified Type Theory (as formalized in, e.g., [1]). The first commentator to notice this, Gödel, remarked in [3] that the new logic was acceptable from the point of view of a "strictly nominalistic" sort of constructivism. It seems reasonable to interpret this (and Landini's [4] remarks about "nominalistic semantics") as claiming that the new logic, like Ramified Type Theory, can be given an interpretation in which higher-order quantification is interpreted substitutionally, as in [5]. [2] claimed this, on the basis of an argument subsequently seen to contain a lacuna. (The oversight was Hazen's; Davoren expressed reservations at the time but her doubts were dismissed by her co-author.) The current paper fills the gap (thereby confirming that Russell's 1925 logic has a mathematically precise and philosophically interesting interpretation), but shows that the extensionality axioms proposed for it in [4] are not valid on this interpretation.

[1] A. CHURCH, *A comparison of Russell's resolution of the semantic antinomies with that of Tarski*, ***Journal of Symbolic Logic***, vol. 41 (1976), pp. 747–760; reprinted in ***Recent essays on truth and the liar paradox*** (R. L. Martin, editor), Oxford University Press, New York, 1983.

[2] J. M. DAVOREN AND A. P. HAZEN, *Russell, Gödel, and Skolem* (abstract), ***Journal of Symbolic Logic***, vol. 56 (1991), pp. 1108–1109.

[3] K. GÖDEL, *Russell's mathematical logic*, in ***The philosophy of Bertrand Russell*** (P. A. Schilpp, editor), Northwestern University, Evanston, 1944.

[4] G. LANDINI, *The definability of the set of natural numbers in the 1925* ***Principia Mathematica***, ***Journal of Philosophical Logic***, vol. 25 (1996), pp. 597–615.

[5] C. PARSONS, *A plea for substitutional quantification*, ***Journal of Philosophy***, vol. 68 (1971), pp. 231–237; reprinted in C. Parsons, ***Philosophy of Mathematics***, Cornell University Press, Ithaca, 1983.

► LLOYD HUMBERSTONE, *An intriguing logic with two implicational connectives*.
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We provide a commentary, from the perspective of Kripke models for modal logic, on the logic described in the paper presented to this conference by Matthew Spinks (see his abstract).

► KEITH HUTCHISON, *What are conditional probabilities conditional upon?*.
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This paper rejects a traditional epistemic interpretation of conditional probability. Suppose some chance process produces outcomes X, Y, ..., with probabilities $P(X)$, $P(Y)$, ... If later observation reveals that outcome Y has in fact been achieved, then the probability of outcome X cannot normally be revised to $P(X|Y) \equiv [P(X \wedge Y)/P(Y)]$. This can only be done in exceptional circumstances when more than just knowledge of Y-ness has been attained. The primary reason for this failure is that the weight of a piece of evidence varies with the means by which it is provided, so knowledge of Y-ness does not have uniform impact on the probability of X. A better updating of the probability of X is provided by $P(X|Y^*)$, where Y^* is not an outcome of the chance process being observed, but the sentence 'the outcome Y has been observed', an "outcome" of the subsequent observation process.

This alternative formula is widely endorsed in practice, but not well recognised in theory, where it has generated some unsatisfactory consequences.

[1] K. HUTCHISON, *What are conditional probabilities conditional upon?*, ***British Journal for the Philosophy of Science***, vol. 50 (1999), no. 4, pp. 665–695.

► WILLIAM JAMES, *Paraconsistency, co-exponentiation and categories*.
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Algebras for paraconsistent logic arrive in the structure of mathematical categories via the categorical property of co-exponentiation. One so-derived structure is discussed in terms of its place in a programme for the reconstruction of mathematics and set theory.

► DAVID LEWIS, *Tensed quantifiers*.
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A presentist is likely to analyse a tensed quantifier phrase, such as 'there has been' or 'there will be' as an untensed (or present tensed) quantifier within the scope of a tense operator: 'it has been that (there is ...)' or 'it will be that (there is ...)'. This works smoothly when the quantifiers are singular, but breaks down when the quantifiers are plural. There have been several kings of England named George; but never has it been the case that there are several kings of England named George. What alternative treatments of tensed plural quantifiers are there that conform to presentist doctrines?

► JOHN LOVE, *Redescribing language use records*.
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A process of interpretation generally benefits from having access to object language use records. However, in order for an interpreter to optimise their resource of accessible records, it is necessary that records which are not directly available, can nonetheless be obtained or redescribed from direct knowledge of what other object language uses have occurred.

By viewing object language use records as encoded by natural numbers, we are able to analyse this situation formally, using recursion theory. We wish to establish various limits as to when redescription might be effectively described as possible in principle. Our interest is to determine whether a process of redescription can be effectively described as occurring, regardless of various practical restraints which might prevent this. A central aspect of this analysis is to establish whether the relative informational contents of respective record classifications supervene the possibility of redescription.

► ROBERT K. MEYER, MARIANGIOLA DEZANI, J. MICHAEL DUNN, YOKO MOTOHAMA, AND RICHARD SYLVAN, *The key to the universe—intersection types, lambda, CL and relevant entailment*.
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Virtual* co-author: (St.) Alasdair Urquhart, Toronto.

*In view of their work on the semantics of entailment not only Urquhart but also Kit Fine ought to be co-authors of this paper. They have, however, so far rejected my invitation. Urquhart, reduced to canonical form by the Logicians Liberation League for his many miracles (the first of which was a semantics for relevant logics), is listed anyway as a "virtual" co-author. It is hoped that eventually he will accept full co-authorship.

Sylvan (nee Routley) and Meyer developed a (ternary relational) semantics for relevant logics from the early 1970s. On this semantics there was a natural minimal positive logic

$B+$, which yielded also a Dunn-style algebraic analysis. Since almost all relevant (and all irrelevant) logics are naturally viewed as extensions of $B+$, we looked for principles motivating one logic or another. These reduced mainly to preferences (or dispreferences) for certain *combinators*, in the sense of Curry's CL. (Or of Church's *lambda*.) This intrusion of CL into relevant semantics so impressed Meyer that he dubbed it "the key to the Universe" as long ago as 1972. We sought to interest the *lambda*-CL community in $B+$. But Dezani and her associates had *independently* invented a $B+$-like system ITD (for Intersection Type Discipline) to analyze *types* in lambda models. ITD corresponds to B^T (say "bat"), the $\rightarrow$ & fragment of $B+$ with Church constant T. We note an Urquhart-Routley-Fine style binary operational semantics for B^T, which makes the *lambda* connections clear. And two projects thought *unsemantical* years ago have (in the end) the *same* semantics!

► CHRIS MORTENSEN, *Inconsistent pictures*.
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Inconsistent pictures have been around since 1934, and popularised by M. C. Escher since 1957. Escher and the Penroses collaborated to discover a range of new forms. This paper explores the varieties of inconsistent pictures and offers some diagnoses about their logical structure.

► GRAHAM PRIEST, *Objects of thought*.
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The stiffest problem for those who believe that there are non-existent objects is to give a principled account of what properties they do and do not have. This paper attempts just such an account. According to this, non-existent objects have only intensional properties at this world. They have their characterising properties at other worlds, determined by people's intensional attitudes.

► GREG RESTALL, *Models for substructural arithmetics*.
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This paper is dedicated to Professor Robert K. Meyer.

In substructural arithmetics, like $R\#\#$ and its cousins, conditionals like $0 = 2 \rightarrow 0 = 1$ are unprovable. However, their negations $\sim(0 = 2 \rightarrow 0 = 1)$ are also unprovable, since these arithmetics are no stronger than classical arithmetic. On a natural interpretation, the conditional should be false (there is no way, using arithmetic operations in successor, addition and multiplication, to infer $0 = 1$ *from* $0 = 2$).

This leads us to look for natural models in which the conditionals are indeed false. Meyer and I have shown how to extend $E\#\#$ to a complete theory $TE\#$ ("Strenge" Arithmetic) in which conditionals like these are genuinely false. The proof relies on metavaluations, and only works for the logic E of entailment, and not stronger logics like R. In this paper I show how to construct complete models of R arithmetic in which these conditionals are false.

► GREG RESTALL AND J. C. BEALS, *Contents: Or, what do you do when one set of truth conditions isn't enough*?
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This paper is about content, the content of a (declarative) utterance. But the paper arises from considerations about consequence, i.e., logical consequence. We understand consequence thus:

(V) B is a consequence of A if and only if every circumstance in which A is true is a circumstance in which B is true.

We have argued elsewhere [1, 2, 3] for logical pluralism, or pluralism about consequence. Logical pluralism stems from the observation that different classes of circumstance are appropriate for substitution into (V). For example, using complete and consistent circumstances yields classical consequence; incomplete but consistent circumstances afford constructive logic; incomplete and inconsistent circumstances give rise to relevant and other paraconsistent logics; and so on. But what are these circumstances of which there are many kinds? In the end, they are truth conditions—conditions under which claims (utterances) are true. This is where issues of content arise. We assume that the content of a claim is its truth conditions, which in turn are the classes of circumstances in which the claim is true. But, then, given our pluralism about such "circumstances" we find ourselves committed to pluralism about content.

This paper explores the costs and benefits of pluralism about content. Among some of the main questions addressed are these: Does pluralism about content face any obvious (or not so obvious) problems? Are there any reasons, independent of logical pluralism, to accept pluralism about content? Does such pluralism help to avoid standard difficulties facing the doctrine that "knowing the meaning of B is knowing B's truth conditions"?

[1] J. C. Beall and G. Restall, *Defending logical pluralism*, presented at the Society of Exact Philosophy Conference, 1999.

[2] ———, *Logical pluralism* submitted to the Year 2000 special issue on Logic of the ***Australasian Journal of Philosophy***, 1999.

[3] G. Restall, *Negation in relevant logics*: *How I stopped worrying and learned to love the Routley star*, in ***What is negation?*** (Dov Gabbay and Heinrich Wansing, editors), Applied Logic Series, no. 13, Kluwer Academic Publishers, 1999, pp. 53–76.

► JERRY SELIGMAN, *Proof systems for hybrid languages*.
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Hybrid languages are propositional modal languages extended with variables ranging over modal contexts (worlds, times, states, etc.) and various devices for binding them. In expressive power they are intermediate between propositional modal languages and first-order predicate languages. Some hybrid logics are decidable, with complexity close to the base modal logic. The favourable trade-off between expressivity and computational tractability makes them well-suited to applications.

This paper presents a uniform method for providing proof systems for hybrid logics, based on Gentzen's sequent calculus for predicate logic. Cut-elimination and interpolation theorems are given. An analysis of the proof systems sheds light on some well-known problems in the axiomatisations of propositional modal logics.

► NICHOLAS J. J. SMITH, *C^C-valued logic and semantic closure*.
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My aim is to develop an extension of classical logic which accommodates two features of ordinary language that classical logic has difficulty accommodating: vagueness (including higher-order vagueness) and semantic closure. This paper (i) presents a C^C-valued semantics for a standard first-order language C = continuum many), (ii) shows how this language may be extended so as to be able to express the concepts used in giving this semantics, and (iii) discusses the relationship between C^C-valued logic and classical logic. Vagueness is discussed in my paper at the AAP conference.

► MATTHEW SPINKS, *A non-classical extension of classical implicative propositional logic*.
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We extend the fragment of classical logic over the language $\Longrightarrow$ with a second implication operation $\longrightarrow$. The resulting logic, called BCSK logic, has the property that every tautology of classical implicative logic is derivable in the fragments of BCSK logic over the languages $\longrightarrow$ and $\Longrightarrow$, yet the two implication operations are distinct. We show that the logic is algebraisable in the sense of Blok and Pigozzi, with equivalent quasivariety semantics the class of implicative BCSK-algebras; an implicative BCSK-algebra is an algebra $\langle A; \Longrightarrow, \longrightarrow, 1\rangle$ of type $\langle 2, 2, 0\rangle$ such that the following conditions are satisfied: (i) the reduct $\langle A; \Longrightarrow, 1\rangle$ is an implicative BCK-algebra; (ii) the reduct $\langle A; \longrightarrow, 1\rangle$ is an implicative BCS algebra; and (iii) the implicative BCS partial order and the implicative BCK partial order coincide. We describe several results for BCSK logic and for implicative BCSK-algebras, including: completeness and deduction theorems for BCSK logic; a proof that the lattices of ideals and congruences are isomorphic in any implicative BCSK-algebra; and a proof that the class of implicative BCSK-algebras has equationally definable principal congruences (and hence is congruence distributive). We characterise the subdirectly irreducible implicative BCSK-algebras, showing that every subdirectly irreducible implicative BCSK-algebra is the $\langle \Longrightarrow, \longrightarrow, 1\rangle$ reduct of a subdirectly irreducible skew Boolean union algebra, thereby demonstrating that the class of reduced matrices for BCSK logic have a particularly simple form.

► KOJI TANAKA, *Tableaux for modal relevant logics* (*and some of their rivals*).
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The talk concerns a tableaux system for modal relevant logics (and some of their rivals). The tableaux system for Dunn's semantics of First Degree Entailment is developed first. Entailment is then added to the system to give a tableaux system for the basic system, which is analogous to classical modal logic K. Extensions of the basic system are developed subsequently. Since the basic system is built on a four-valued semantics, the extensions that we obtain are not the systems of the standard family of relevant logics. Finally, I add non-normal worlds to the semantics and provide a tableaux system for a non-normal relevant logic.

Abstract presented by title

► HONGXUE WANG, *A typed first order logic with five quantifiers*.
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In this paper, we present an extended first order logic in which some quantity aspects of knowledge can be handled more easily than in the ordinary first order logic. The extension is made by bringing type and five quantifiers into ordinary first order logic. We first explain why it is necessary to use types and additional quantifiers in our logic, followed by presenting the extended logic, including the syntax and semantics. We then add some axioms and inference rules to the logic, and prove the completeness and the soundness of these inference rules. We also show by examples how the extended first order logic can be used to formalise and reason about some subtle aspects of knowledge relevant to quantity. In our discussion, we summarise our contributions and argue that such a logic can serve as a logical foundation for constraint logic programming.