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1999–2000 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Washington, DC, January 21–23, 2000

A Winter Meeting of the Association for Symbolic Logic was held January 21–23, 2000, in Washington, DC, at the Marriott Wardman Park Hotel and George Washington University, in conjunction with the joint mathematical meetings of the American Mathematical Society, the Mathematical Society of America, and the Society for Industrial and Applied Mathematics. The Program Committee consisted of Valentina Harizanov, Jean Larson (chair), Chris Laskowski, Carl Posy, and Marion Scheepers. Harizanov and Laskowski functioned as an informal local organizing committee. The program included five invited 45-minute talks, seven invited 25-minute talks, ASL Special Sessions in Computable Model Theory and Ramsey Theory each consisting of eight 20-minute talks, a Symposium on the Brouwer-Hilbert Foundational Controversy consisting of four 40-minute talks and a panel discussion, twelve contributed talks, and three talks contributed by title. Three invited speakers were unable to come at the last minute: Jeffrey Remmel, Anil Nerode, and Ilijas Farah. Douglas Cenzer filled in for Remmel: Cenzer used Remmel's slides to give a 45-minute talk about their joint work. Suman Ganguli, a student of Nerode, spoke in his place in the special session on Computable Model Theory. The talk of Ilijas Farah was cancelled.

There was an AMS Special Session on the History of Topology in honor of Ralph Krause, on the occasion of his retirement, who has done so much for the field of logic as National Science Foundation program director for Topology and Foundations. As a token of appreciation, the ASL Program Committee presented Krause with a framed mock review of his work in the style of reviews of individual NSF grant proposals.

The following invited 45-minute addresses were presented at the meeting:

Timothy J. Carlson (Ohio State University), *Combinatorial aspects of patterns of resemblance*.

Douglas Cenzer (University of Florida), *Complexity theoretic algebra and model theory*.

Saul Kripke (Princeton University), *The collapse of the Hilbert program: Variations on the Gödelian theme*.

Bradd Hart (McMaster University), *The logic of hyperimaginaries*.

Dirk van Dalen (Utrecht University), *L. E. J. Brouwer: How mathematics is rooted in life*.

The following invited 25-minute addresses were presented at the meeting:

Meir Buzaglo (Hebrew University), *Forced expansions*.

James Cummings (Carnegie Mellon University), *Large cardinals and small sets*.

Willem L. Fouché (University of Pretoria), *Symmetry and the Ramsey degree of finite relational structures*.

J. Karel Lambert (University of California, Irvine), *Set theory and definite descriptions: Four solutions in search of a problem*.

Manuel Lerman (University of Connecticut), *Homomorphisms and quotients of degree structures*.

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Zachary Robinson (Eastern Carolina University), *Rigid sybanalytic sets*.

Kevin Wald (University of Connecticut), *Automorphisms and noninvariant properties of the computably enumerable sets*.

The following talks were presented in the Special Session on Computable Model Theory organized by Valentina Harizanov:

Douglas Cenzer (University of Florida), *Complexity of decidable theories and models*.

Suman Ganguli (Cornell University), *Recursive model theory of modal logics*.

Jeffry Hirst (Appalachian State University), *Models and reverse mathematics*.

Steffen Lempp (University of Wisconsin), *The Lindenbaum algebra of the class of finite models*.

Charles F. McCoy (University of Notre Dame), *Categoricity in linear orderings and Boolean algebras*.

Timothy H. McNicholl (University of Dallas), *Intrinsic reducibilities*.

Alexandra Shlapentokh (East Carolina University), *New results concerning Hilbert's tenth problem over number fields*.

Walker White (Cornell University), *Complexity of model theoretic properties*.

The following talks were presented in the Special Session on Ramsey Theory organized by Jean A. Larson and Marion Scheepers:

Lorenz J. Halbeisen (University of California), *A Ramsey type theorem and its corollaries*.

Neil Hindman (Howard University), *A set partition regular for 3-term arithmetic progressions which has large subsets of any finite set with no 3-term arithmetic progressions* (joint work with V. Bergelson).

Albin L. Jones (Kenyon College), *Partitioning triples*.

Maria Losada (Universidad Antonio Nariño, Bogota), *Chains and antichains in Boolean algebras*.

Justin Tatch Moore (University of Toronto), *Continuous colorings associated with certain characteristics of the continuum*.

L. M. Pretorius (University of Pretoria), *Partition theorems for ascending parameter words*.

Rene Schipperus (Universität Bielefeld), *Partition ordinals*.

William Weiss (University of Toronto), *Ramsey theory of metric and non-metric spaces*.

The following talks were presented in the Symposium on the Brouwer-Hilbert Foundational Controversy organized by Carl Posy:

Dennis Hesseling, *The reception of Brouwer's intuitionism in the Foundational Controversy*.

Paolo Mancosu (University of California, Berkeley), *In the middle of the war of the factions: Weyl at a crossroads*.

David McCarthy (University of Indiana, Bloomington), *Brouwer's intuitionism in the foundation debate: From out of the past*.

Wilfried Sieg (Carnegie Mellon University), *The Foundational Controversy: Hilbert's reductive structuralism*.

Abstracts of contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
JEAN A. LARSON

Abstracts of invited talks

► MEIR BUZAGLO, *Forced expansions*.
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Almost all mathematical functions can be expanded in a non-arbitrary way. In my paper I wish to subject certain aspects of this process to logical analysis. I define within model theory a family of non-arbitrary expansions, classify them and offer two logics (one incomplete and one complete) which emerge from these definitions. I will also prove a theorem which shows that there is a deep relation between the notion of non-arbitrary expansion and deduction. I shall show how this accounts for historical cases of conceptual expansion in mathematics. Thus I will argue against Frege's opposition to the idea of conceptual expansion, an opposition which survived in some modern views.

► DOUGLAS CENZER, *Complexity of decidable theories and models*.
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Complexity-theoretic model theory is the study of resource-bounded structures and isomorphisms and their relation to computable structures and isomorphisms. We will give some recent results on theories with low complexity (for example, exponential space or time). In particular, we show how to construct a model with complexity not much more than the complexity of the theory.

► JAMES CUMMINGS, *Large cardinals and small sets*.
Mathematical Sciences Department, Carnegie Mellon University, Weam Hall, Pittsburgh, Pennsylvania 15213, USA.

Large cardinal axioms are statements which increase the strength of ZFC set theory by postulating the existence of very large sets. I discuss the role of large cardinal axioms in settling questions about comparatively small sets such as $\aleph_1$ and $\mathbb{R}$. The talk is aimed at a general audience of logicians.

► BURKHARD ENGLERT, MANUEL LERMAN, AND KEVIN WALD, *Homomorphisms and quotients of degree structures*.
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We survey results on quotients and homomorphisms of degree structures in various signatures, with an emphasis on the signature of *pseudolattices* (posets with least (and greatest) elements, a join operation and $(n-1)$-ary relations $M_n(a_1, \dots, a_n, b)$ to be interpreted as every element $\leq$ every a_i is also $\leq b$). Sample results are:

THEOREM 1. *There are no finite pseudolattice quotients of the degrees below* $\mathbf{0}'$.

THEOREM 2. *Every finite distributive lattice is a pseudolattice quotient of the c. e. degrees.*

► ILIJAS FARAH, *Prime maps on finite-dimensional cubes*.
Department of Mathematics, Rutgers University, 110 Frelinghuysen Road, Piscataway, New Jersey 08854, USA.
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We investigate the question: When does a function $f : N^d \to N$ (where d is a natural numbers) essential depend on at most one variable? For example, the function $f(m, n) = m$, if m is even, and $f(m, n) = n$, if n is odd, depends on both of its variables, but the domain can be partitioned into two rectangles such that f depends on one variable on each of them. We give a theorem that gives the sufficient and necessary conditions that the domain of f can be partitioned into finitely many rectangles so that f depends on at most one variable on each one of them. We also discuss some applications and possible extensions of this result.

► WILLEM L. FOUCHÉ, *Symmetry and the Ramsey degree of finite relational structures*.
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We introduce a measure of the extent to which a finite combinatorial structure is a Ramsey object in the class of objects with a similar structure. We show for classes of finite relational structures, including posets, trees, distributive lattices and graphs, how this measure depends on the symmetries of the structure.

[1] W. L. FOUCHÉ, *Symmetry and the Ramsey degrees of finite relational structures*, ***Journal of Combinatorial Theory***, Series A, vol. 85 (1999), pp. 134–147.

[2] ———, *Symmetry and the Ramsey degree of posets*, ***Discrete Mathematics***, vol. 167–168, pp. 309–315.

► LORENZ J. HALBEISEN, *A Ramsey type theorem and its corollaries*.
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E-mail: halbeis@math.berkeley.edu.

A generalized version of Carlson's Lemma is proved from which the Finite and the Infinite Ramsey Theorem, the Hales-Jewett Theorem, as well as some other combinatorial results are derivable. So, the main result—which can be seen as the dual form of the Infinite Ramsey Theorem—covers the classical theorems in Ramsey Theory.

► BRADD HART, *The logic of hyperimaginaries*.
Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario, Canada L8S 4K1.
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For some time now, model theorists have realized the importance of definable quotients of definable sets; these are called imaginaries. Over the past few years, a more general object has gained in importance—type-definable quotient objects. In this talk, I will say something about the logic of these objects and where they appear in current model theory.

► DENNIS E. HESSELING, *The reception of Brouwer's intuitionism in the Foundational Controversy*.
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The foundational controversy started with the publication of Weyl's polemical 1921 paper, *Über die neue Grundlagenkrise der Mathematik*. In my lecture, I sketch the historical development of the foundational debate. By analysing the role played by—often political—metaphors used in the debate, we will see that its zenith lay around 1928. I also point out the main themes of the debate, namely the question of mathematical existence and the status of logic and the principle of the excluded middle. Additionally, I treat some contextual factors, like the differences in reaction in Europe and the US. In doing so, I will focus on the reception of Brouwer's intuitionism and thus present the controversy through an intuitionist's eye. Seen from this perspective, the debate turns out to be one-sided, focussing on negative aspects of intuitionism while almost completely ignoring its positive contributions. Philosophers, however, on the whole reacted more positively to intuitionism than mathematicians. I present some of the arguments used for and against the intuitionistic criticism of the unrestricted use of the principle of the excluded middle.

Various authors interpreted intuitionism as introducing a three-valued logic, indicating the unclarity and misunderstandings which formed an important part of the controversy. Finally, I argue that the controversy was not, as it is often presented, mainly between Brouwer's intuitionism and Hilbert's formalism. At the beginning of the debate, Hilbert had not yet drawn the ultimate consequences of the formalistic point of view. What was at stake were the contentual claims of classical mathematics. Formalism developed to a large degree in reaction to the criticism thereof.

► NEIL HINDMAN (joint work with Vitally Bergelson), *A set partition regular for 3-term arithmetic progressions which has large subsets of any finite set with no 3-term arithmetic progressions*.
Department of Mathematics, Howard University, Washington, DC 20059, USA.
E-mail: `nhindman@howard.edu`.

At the July meeting in Budapest in memory of Paul Erdős, Nešetríl asked the following question:

> *Do there exist* $X \subseteq \mathbb{N}$ *and* $\varepsilon > 0$ *such that*
> 1. *whenever* X *is finitely colored, there must exist a monochrome 3-term arithmetic progression, and*
> 2. *for all finite* $Y \subseteq X$ *there exists* $Z \subseteq Y$ *such that* $|Z| \geq \varepsilon \cdot |Y|$ *and* Z *does not contain any 3-term arithmetic progression*?

We produce a simply described set X satisfying property (1) and having no 4-term arithmetic progressions at all. At this writing, we suspect that it satisfies (2) with $\varepsilon = 1/2$, but can only prove something weaker than (2).

► JEFFRY L. HIRST, *Models and reverse mathematics*.
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Model theory and structures from computability theory play an important role in the study of reverse mathematics. For example, the set universes of ω-models of RCA_0, WKL_0 and ACA_0 are ideals, Scott sets, and jump ideals, respectively. Also, coded models can be used while working in the subsystems, resulting in streamlined proofs.

► ALBIN L. JONES, *Partitioning triples*.
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We will summarize the state of the art (as we know it) in the partition calculus of triples. As an example, consider an arbitrary partition of all triples of countable ordinals into two classes. We have recently proven that either for each integer m the first class contains all triples from a set of order type $\omega + m$, or for each integer n the second class contains all triples of an n element set. In other words, we have proven that $\omega_1 \to (\omega + m, n)^3$ for all natural numbers m and n. In fact, we have shown that this is true when ω_1 is replaced with an arbitrary non-ω-special partial order. (For any infinite cardinal κ, a partial order P is non-κ-special if it satisfies the partition relation $P \to (\kappa)^1_\kappa$.) We have also been able to generalize our proof of this result to show that if κ is any measurable cardinal, and P is a non-κ-special partial order, then $P \to (\kappa + \mu, n)^3$ for all ordinals $\mu < \kappa$ and all natural numbers n. We will briefly examine these results, the history of the problem which inspired them, and the prospects for strengthening them in the future.

► J. KAREL LAMBERT, *Four solutions in search of a problem*.
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The naïve theory of definite descriptions is the theory obtained by adding the principle

$$\text{(NDD)}: (\forall x\ (x = IyF(\forall y)\ ([F(x)\ \&\ (\forall y\ (F(y)(y = x)])$$

to classical predicate logic with identity. It entails the naïve principle of set abstraction with the aid of an appropriate definition of set abstracts. So the naïve theory of definite descriptions is defective. But that theory can be shown to be inconsistent (without the intervention of any definition of set abstracts) via the paradox of the round square. There are four well known ways out of the Russell paradox emanating from naïve abstraction. Surprisingly, these four ways also apply to the paradox of the round square, a paradox that infects NDD, and, indeed, these resolutions of the paradox of the round square are reflected in the four major approaches to the logical treatment of definite descriptions developed over the last century. The result is that a common understanding of the four traditional approaches to the logical treatment of definite descriptions is now available; they can be seen as four distinct answers to the question "How is the paradox of the round square to be avoided?" Finally, an open problem is sketched for the way out of the paradox of the round square provided by free theories of definite descriptions, theories that are useful in the treatment of partial functions vis-à-vis the development of programming languages.

► STEFFEN LEMPP, *The Lindenbaum algebra of the class of finite models*.
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Presenting joint work with Mikhail Peretyat'kin and Reed Solomon, I will give a characterization of the Lindenbaum algebra of the class of finite models over a rich finite signature (i.e., a finite signature containing at least two unary function symbols, or at least one n-ary function or relation symbol for some $n > 1$).

It turns out that this Lindenbaum algebra is an atomic Boolean algebra, and its quotient algebra modulo the Fréchet ideal is a Σ^0_2-universal Boolean algebra (i.e., a Σ^0_2-Boolean algebra containing a copy of any Σ^0_2-Boolean algebra as an initial segment). This uniquely determines the (classical) isomorphism type of this Lindenbaum algebra.

The proof involves coding Turing machines into finite models.

► MARIA LOSADA, *Chains and antichains in Boolean algebras*.
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In this talk we give a positive answer to problem DJ from Fremlin's list which asks whether $\mathrm{MA}_{\aleph_1}$ implies that every uncountable Boolean algebra has an uncountable set of pairwise incomparable elements.

► DAVID MCCARTY, *Brouwer's intuitionism and the foundation debate: From out of the past*.
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One often assesses the events of the foundations debate and the views then put forward, among them Brouwer's intuitionism and Hilbert's proof theory, with respect to events which occurred only later, e.g., the proof of Gödel's First and Second Incompleteness Theorems or Turing's analysis of computability. One also attempts to understand the debate by using concepts not generally available in the 1920s, e.g., formal semantics for logical systems. In this presentation, we try to assess the foundations debate not by means of its future but from out of its own recent past, placing the views of the debate's participants within the intellectual landscape of the latter half of the preceding century. New perspectives open up once we compare Brouwer's and Hilbert's positions on some of the larger intellectual issues of the 19th Century, among them the possibility of mechanism, and once we compare their logical innovations to the views of 19th Century logicians such as Lotze. From these perspectives, Brouwer begins to appear less as Weyl's "revolutionary" and more as a conservative, a defender and elaborator of traditional themes and ideas.

► CHARLES F. MCCOY, *Categoricity in linear orderings and Boolean algebras*.
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Goncharov completely characterized computable categoricity in many classes of structures, including linear orderings and Boolean algebras. We discuss Δ^0_2-categoricity in these two classes, emphasizing how an examination of relativized categoricity provides not only a statement of the unrelativized result, but also a useful guide for the priority constructions required. We progress to Δ^0_3-categoricity and note the sharp difference that appears between these two types of structures with respect to this notion. Finally, we explore the possibility of a meaningful examination of Δ^0_n-categoricity in Boolean algebras.

► TIMOTHY H. MCNICHOLL, *Intrinsic reducibilities*.
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Let $\alpha < \omega_1^{\mathrm{CK}}$. We show that for many reducibilities, the requirement that a relation be intrinsically reducible to the αth jump of a countable model has a syntactic equivalent. Furthermore, we show that many reducibilities coincide in such a situation.

► PAOLO MANCOSU, *"In the middle of the war of the factions": Weyl at a crossroads*.
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Weyl's role in the debate between Hilbert and Brouwer in the 1920s cannot be overestimated. Around 1920 Weyl fell under the spell of Brouwer and abandoned the foundational position he had defended in Das Kontinuum (1918). However, already by 1924 Weyl was trying to find a middle ground between Brouwer and Hilbert. In 1925 Bernays wrote to Hilbert that Weyl was no longer opposed to his foundational position. This development in Weyl's foundational position is also connected to his changes in attitude towards phenomenology. Starting in 1924 Weyl grew increasingly skeptical, and by 1928 was downright negative, on the ability of phenomenology to account for pure mathematics and mathematical physics. The stages of this development remain obscure in Weyl's published writings. However, a much more detailed and interesting picture of Weyl's changing attitute to phenomenology can be gathered by exploiting the long correspondence between Weyl and Oskar Becker, which is still preserved in the Wissenschaftshistorische Sammlungen of the ETH in Zurich.

The talk will be divided into two parts. The first part will describe Weyl's polemic against positivism (in particular against Moritz Schlick) and his espousal of Husserlian phenomenology as the background for his conversion to Brouwer's intuitionism in 1920. The second part will analyze, exploiting the correspondence with Becker described above, Weyl's abandonment of intuitionism and phenomenology, and his reevaluation of Hilbert's program, after 1924.

► JUSTIN TATCH MOORE, *Continuous colorings associated with certain characteristics of the continuum*.
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A coloring $c: [X]^n \to Z$ is said to be irreducible if for every $Y \subseteq X$ of equal cardinality $c''[Y]^n = Z$. The focus of this talk will be to discuss continuous irreducible colorings on sets of reals associated with various cardinal characteristics of the continuum.

► ANIL NERODE, *Recursive model theory of modal logics*.
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A report on the work of my student Suman Ganguli. Previously Isihara, Nerode, and Khoussainov established that decidable constructive first order theories have decidable Kripke models (deciding forcing relation). There was an obstacle for more general Kripke models. There was an obstacle for modal logics in general because of the lack of a deduction theorem. This has been overcome using an essential modal idea.

► L. M. PRETORIUS, *Partition theorems for ascending parameter words*.
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We discuss the homogeneities of factorisations of finite ascending parameter words. The results are established in primitive recursive arithmetic and are extensions of results by Milliken and, independently, by Nešetřil and Rödl on the Ramsey properties of fixed rank partitions of finite sets. This is joint work with W. L. Fouché and C. J. Swanepoel.

[1] W. L. Fouché, L. M. Pretorius, and C. J. Swanepoel, *Partition theorems for factorisations of ascending parameter words*, ***Discrete Mathematics***, vol. 197–198 (1999), pp. 331–350.

[2] K. R. Milliken, *Ramsey's theorem with sums or unions*, ***Journal of Combinatorial Theory***, Series A, vol. 18 (1975), pp. 276–290.

[3] J. Nešetřil and V. Rödl, *Dual Ramsey type theorems*, ***Proceedings of the 8th winter school on abstract analysis***, Prague, 1980, pp. 121–123.

► JEFFREY REMMEL, *Complexity theoretic algebra and model theory*.
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We will survey a number of recent results in complexity theoretic algebra and model theory. Computable algebra and model theory has traditionally focused on the study of the effective content of constructions and theorems in model theory and algebra. The study of complexity theoretic algebra and model theory also studies the effective content of constructions and theorems in model theory and algebra but, in contrast to computable algebra and model theory, one limits the underlying model of computation to be one of standard complexity classes such as polynomial time, polynomial space, exponential time, nondeterministic polynomial time, etc. Thus in the study of complexity theoretic algebra and model theory, one must carefully consider resource bounds. We shall show that this area is not just a mere imitation of computable model theory and algebra but that a number of new phenomena and questions arise when the issue of resource bounds are incorporated into the study of the effective content of mathematics.

► RENE SCHIPPERUS, *Partition ordinals*.
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We survey past and recent work on the subject of countable and uncountable Partition Ordinals. We describe some of the current open questions and indicate further research directions.

► ALEXANDRA SHLAPENTOKH, *New results concerning Hilbert's tenth problem over number fields*.
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We discuss new Diophantine definability and undecidability results over "large" subrings of algebraic number fields with exactly one pair of complex conjugate embeddings.

► WILFRIED SIEG, *The foundational controversy: Hilbert's reductive structuralism*.
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The logico-philosophical analysis of Hilbert's finitist means—as viewed in the twenties—for securing mathematics was and remains an important issue. However, the substance of what Hilbert strove to secure—over a lifetime—has not received equal attention: it is not classical mathematics as it evolved until the early 19th century, but rather modern mathematics as it resulted from a radical transformation during the second half of that century. Howard Stein called it a transformation "so profound that it is not too much to call it a second birth of the subject." Stein argued, and I agree, that it was effected mainly by the work of Gauss, Dirichlet, Riemann, and Dedekind. I intend to describe this broad context of the foundational controversy and thus to analyze, *what* the programmatic constructivist Hilbert defended against his flamboyant fellow constructivist Brouwer, and *how* he tried to do so.

► DIRK VAN DALEN, *L. E. J. Brouwer: How mathematics is rooted in life*.
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We sketch a picture of Brouwer's development, his mathematics and philosophy. Contrary to popular belief, he was preoccupied with the foundations of mathematics right from the beginning.

When he was still a student, his mystic leanings became apparent. They were indeed an integral part of his philosophy, and even his mathematics was influenced by it. His dissertation contains his first foundational program, in which the negative aspects were rather stressed. From the dissertation (1907) to World War I he developed his topology. On the basis of this work he got a chair in Amsterdam and a seat on the editorial board of the *Mathematische Annalen*.

During the war he returned to the foundations, which resulted in his second foundational program, called 'intuitionism'. Between 1918 and 1924 the basic foundational insights were obtained: continuity principle, bar induction, fan theorem, continuity of all real functions.

His foundational program was strongly disapproved by Hilbert, whom Brouwer had revered from the beginning of his career. When Weyl joined Brouwer's "revolution", the so-called Grundlagenstreit followed. This conflict between intuitionism and formalism ended, in practice, in 1928, when Hilbert removed Brouwer from the editorial board of the *Mathematische Annalen*. From then on Brouwer virtually withdrew from mathematics, to return briefly after World War II.

Brouwer's life was rich in conflicts: he fought at one time or another with Lebesgue, Koebe, Denjoy, Menger, and Hilbert. Between the two wars he was active for the cause of the re-admission of German scientists into the international community.

Brouwer's personal life style was bohemian; as an academic he was impeccably dressed and very precise. He had a great interest in language, both philosophically and privately; the first led him to join the Signific Circle. He was a staunch friend and an implacable enemy. His many conflicts had almost always been caused by his extreme sense of justice and his passion for the underdog.

[1] D. van Dalen, ***Mystic, geometer, and intuitionist. The life of L. E. J. Brouwer, Volume I: The dawning revolution***, Oxford Press, 1999.

► KEVIN WALD, *Automorphisms and noninvariant properties of the computably enumerable sets*.
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The computably enumerable (c. e.) sets form a lattice $\mathcal{E}$ under inclusion. We will discuss how the automorphisms of $\mathcal{E}$ relate to the computational properties of the c. e. sets, with reference to some of the techniques used in the construction of $\mathcal{E}$-automorphisms. Particular examples will include several new results showing the noninvariance under $\mathcal{E}$-automorphism of various properties of c. e. sets (quasicreativity, n-creativity, subcreativity, and effective simplicity), and some related results.

► WILLIAM WEISS, *Ramsey theory of metric and non-metric spaces*.
Department of Mathematics SB-4037, University of Toronto, 3359 Mississauga Road N, Mississauga, Ontario, Canada L5L 1C6.
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We examine results involving partitioning pairs of points in a metric space and recent work which tries to extend this to the non-metric case. We look at open problems and recently closed ones.

► WALKER WHITE, *Complexity of model theoretic properties*.
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Both classical and computable model theory are interested in what properties are syntactically definable. A standard example in classical model theory is the Scott sentence, defining models up to isomorphism. Less work, however, has been done in determining when properties are not syntactically definable. This gives us some motivation for studying index sets of model theoretic properties. For any property P, we can define the set

$$S_P = \{\, e \mid \phi_e \text{ codes a computable structure with property } P \,\}.$$

If P is definable by an effective infinitary sentence, such as a Scott sentence, S_P must be hyperarithmetical. So, if S_P is complete at some level in the analytical hierarchy, it can have no such definition.

Studying these index sets is also interesting from the viewpoint of computability theory. Elementary equivalence and similar properties create natural index sets that are complete at nonarithmetical levels of the hyperarithmetical hierarchy. For example, the index sets for homogeneity and atomic structures are both $\Pi_{\omega+2}$ complete.

This talk will go over the motivation for studying index sets of computable models, and the subtleties that are involved in their study. It will also present the machinery used in the more interesting hyperarithmetical index set reductions.

Abstracts of contributed talks

► YUUKI ANDOU, *Structural aspects and Church-Rosser property of classical natural deduction*.
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We investigate reductions for classical natural deduction systems which have full logical connectives including disjunction and the existential-quantifier. If the system has these connectives, the treatment of removing maximal formulas is rather complicated even in the intuitionistic case, since some structural parts arise in the deductions ([5]). First we introduce a reduction defined in [1] for full classical natural deduction, in the notation of extended lambda-calculus ([2]). Then we prove the Church-Rosser property of our reduction. The proof uses the extended method of parallel reduction ([6]) with the extended notion of Prawitz's segment ([5]) to treat the structural reductions.

[1] Y. Andou, *A normalization-procedure for the first order classical natural deduction with full logical symbols*, ***Tsukuba Journal of Mathematics***, vol. 19 (1995), pp. 153–162.

[2] ———, *A canonical extension of Curry-Howard isomorphism to classical logic*, ***Anales of WAIT '99*** (1999), pp. 5–10.

[3] ———, *Church-Rosser property of a simple reduction for full first order classical natural deduction*, submitted (preliminary version) and in preparation (revised version).

[4] G. Gentzen, *Untersuchungen über das logische Schliessen*, ***Mathematische Zeitschrift***, vol. 39 (1935), pp. 176–210 and 405–431.

[5] D. Prawitz, ***Natural deduction—A proof theoretical study***, Almqvist & Wiksell, Stockholm, 1965.

[6] M. Takahashi, *Parallel reduction in lambda-calculus*, ***Information and Computation***, vol. 118 (1995), pp. 120–127.

► STEVEN AWODEY, *Topological completeness results for type theories and higher-order logic*.
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The recent spatial covering theorem for toposes [1] permits a significant sharpening of past completeness results for classical and intuitionistic higher-order logic and type theories like the λ-calculus. In place of Henkin and Kripke models, one achieves completeness with respect to models that are *sheaves* over topological spaces. This talk presents some of these new topological completeness results, the simplest of which can be stated as follows:

Theorem. *For any theory in the simply-typed λ-calculus, there is a model in sheaves on a topological space that is universal, in the sense that it satisfies exactly those equations that are derivable from the theory in the usual calculus of λ-conversion*.

[1] C. Butz and I. Moerdijk, *Topological representation of sheaf cohomology of sites*, Preprint 973, Department of Mathematics, Utrecht University, September 1996, 17 pp., to appear in ***Compositio Mathematica***.

► B. ENGLERT, *A necessary and sufficient condition for embedding principally decomposable finite lattices preserving greatest element into the computably enumerable degrees*.
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In [1] a necessary and sufficient condition for embedding principally decomposable finite lattices into the computably enumerable degrees was presented. We show that the same condition is also necessary and sufficient for embedding principally decomposable finite lattices into the computably enumerable degrees preserving greatest element. The proof, however, requires significant modifications.

[1] M. LERMAN, *A necessary and sufficient condition for embedding principally decomposable finite lattices into the computably enumerable degrees*, to appear in ***Annals of Pure and Applied Logic***.

► KEJIA HO, *Information-theoretic characterizations of generalized computably enumerable sets*.

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The main purpose of this work is to characterize computably enumerable (c. e.) sets and generalized c. e. sets from the perspective of Kolmogorov complexity. Given a set A, we consider $\chi_A \upharpoonright m$, the initial segment of length m of the characteristic sequence of A. It is known ([1], [2], and [3]) that for any c. e. set A, the unconditional (condition, uniform, resp.) Kolmogorov complexity of $\chi_A \upharpoonright m$ has optimal upper bound $2 \log m$ ($\log m$, $\log m$, resp.). We show that these optimal upper bounds for c. e. sets apply to n-c. e. sets as well. We study also the corresponding optimal upper bounds for x^n-c. e. and 2^x-c. e. sets. We show that there is a strongly effectively simple set A such that the unconditional (conditional, uniform, resp.) Kolmogorov complexity of $\chi_A \upharpoonright m$ is as high as what is possible for a computably enumerable set; but not all strongly effectively simple sets have this property. We show that for any hypersimple set A, the uniform Kolmogorov complexity of $\chi_A \upharpoonright m$ cannot reach the upper bound of that for computably enumerable sets. In addition, we show that for any Gödel numbering φ, the set of all nonrandom strings with respect to φ is Q-complete.

[1] J. BARZDIN, *Complexity of programs to determine whether natural numbers not greater than n belong to a recursively enumerable set*, ***Soviet Math. Dokl.***, vol. 9 (1968), pp. 1251–1254.

[2] M. KUMMER, *Kolmogorov complexity and instance complexity of recursively enumerable sets*, ***SIAM Journal of Computation***, vol. 25 (1996), pp. 1123–1143.

[3] M. LI and P. VITÁNYI, ***An introduction to Kolmogorov complexity and its applications***, Springer-Verlag, Berlin, 1997.

► JESSE HUGHES, *The coalgebraic analogue of Birkhoff's variety theorem*.

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One of the earliest theorems in universal algebra is Garrett Birkhoff's Variety Theorem [1]. It states that a class of algebras **V** is closed under homomorphic images, subalgebras and products if and only if **V** is the collection of all algebras satisfying some set of equations.

The classical definition of Σ-algebras generalizes to the category theoretic notion of Γ-algebras for an endofunctor Γ. This, in turn, leads to the dual notion of Γ-coalgebras. Coalgebras have proven useful in modeling processes and objects in computer science [3]. It is natural to ask whether such a basic theorem as Birkhoff's variety theorem can be dualized to a "Co-Birkhoff Covariety Theorem" for coalgebras.

In order to dualize the Birkhoff Variety Theorem, we first prove a general variety theorem in an abstract setting, showing that it holds for a wide class of categories. This theorem also yields the Birkhoff theorem in the classical setting.

The principle of categorical duality then yields two covariety theorems. On the one hand, the abstract version of the classical theorem leads to an abstract covariety theorem that applies to all categories that satisfy some basic factorization properties. On the other hand, this abstract version specializes to the following theorem for certain categories of coalgebras (see [2] for a version of this over **Set**):

THEOREM. *A full subcategory* **V** *of a category of coalgebras is closed under images, subcoalgebras and coproducts if and only if* **V** *is the class of all coalgebras that satisfy a collection of coequations.*

The new notion of a "coequation" can be interpreted as a constraint on the internal structure of the coalgebra.

[1] G. BIRKHOFF, *On the structure of abstract algebras*, ***Proceedings of the Cambridge Philosophical Society***, vol. 31 (1935), pp. 433–454.
[2] H. PETER GUMM and TOBIAS SCHRÖDER, *Covarieties and complete covarieties*, ***Electronic Notes in Theoretical Computer Science***, vol. 11 (1998), pp. 43–56.
[3] BART JACOBS and JAN RUTTEN, *A tutorial on (co)algebras and (co)induction*, ***Bulletin of the European Association for Theoretical Computer Science***, vol. 62 (1997).

► JUKKA KERANEN, *Parallelism: A case study in methodology of mathematics*.
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In spite of repeated calls for "naturalism," philosophy of mathematics in the 20th century was a predominantly philosophy-driven enterprise. I propose that we should be receptive to the possibility of mathematics-driven philosophy of mathematics: that philosophically interesting questions can be raised (and perhaps even answered) about expressly "mathematical," rather than ontological or epistemological, aspects of actual mathematical theories.

As an illustration of this possibility, I will present a case study in the methodology of mathematics. It is well-known that progress in mathematics often occurs when one area of mathematics is brought to bear on another, but so far there have been no serious attempts to explain why this should be so. Since this question is too broad to be treated in detail here, I will focus on a (apparently) narrow case of the applicative relationship I call "parallelism."

Parallelism occurs when there is a one-to-one correspondence between the elements of two systems of mathematical objects whereby the structure of one of the systems can be used to acquire information about the structure of the other. Thus, isomorphic systems are parallel, but parallelism goes well beyond isomorphisms. I will discuss three examples: Galois theory, the theory of Lie groups and Lie algebras, and algebraic topology. Although I take parallelism to be an important, indeed pervasive form of mathematical theorizing, my main point is not that parallelism in fact is a widely used intellectual strategy. Rather, I wish to examine the specific intra-theoretical reasons that make parallelism so fruitful in mathematical research.

The underlying ideology of this paper is that philosophy of mathematics should be informed by actual mathematics. First, recognizing the richness of actual mathematics can open new directions for philosophical reflection. Second, failing to recognize such richness increases the risk of generating poor philosophy. For example, as I have argued elsewhere ([1] and [2]), realist structuralism fails because it cannot account for structures with non-trivial automorphisms.

[1] J. KERANEN, *The individuation problem: Realist structuralism is dead*, a paper presented at the Annual Meeting of the Association For Symbolic Logic, San Diego, California, 1999.
[2] ———, *The individuation problem*, a paper presented at the 11th International Congress of Logic, Methodology and Philosophy of Science, Krakow, Poland, 1999.

► MOSES KLEIN, *Set theory of the continuum and subgroups of the Baer-Specker group*.
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We study relationships between subgroups of the Baer-Specker group $\mathbf{Z}^{\omega}$ and the set theory of the continuum. It has long been known that all countable subgroups are free abelian groups, and that no stronger results in terms of cardinality are possible, since there exist both non-free subgroups as small as $\aleph_1$ and free subgroups as large as $2^{\aleph_0}$. In this paper we eliminate another candidate for a necessary condition for freeness by constructing, in ZFC, a free non-meager pure subgroup. We also extend an independence result of Dugas and Irwin about groups which can be embedded into the Baer-Specker group.

► SATORU KURODA, *Weak arithmetic for slow growing depth circuits*.

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We will define weak bounded arithmetic theories whose provably total functions are computed by polynomial size circuits of depth $\log(\log(\dots\log n))$.

Let LD^i be the class consisting of functions which are computable by polynomial size, $(\log^{(i)} n)^{O(1)}$ depth unbounded fan-in circuits, where $(\log^{(i)} n)^{O(1)}$ denotes the i iterations of log. Then a weak form of length induction of the form

$$\varphi(0) \wedge \forall x\ (\varphi(x) \to \varphi(x+1)) \to \forall x\ \varphi(|x|_i)$$

is used to characterize the theory L_2^i, where $|x|_i$ is the binary length function iterated i times.

We also show the separation of L_2^i from the weaker theory AC^0CA, whose provably total functions are AC^0 computable. This separation follows from the famous KPT witnessing theorem ([3]) and a theorem from complexity theory stating that the parity function is not in AC^0.

[1] P. Clote and G. Takeuti, *First order bounded arithmetic and small circuit complexity classes*, in ***Feasible mathematics II***, Birkhäuser, 1995, pp. 154–218.

[2] J. Håstad, *Computational limitations on small depth circuits*, ***Ph.D. thesis***, Massachusetts Institute of Technology, 1986.

[3] J. Krajíček, P. Pudlák, and G. Takeuti, *Bounded arithmetic and the polynomial hierarchy*, ***Annals of Pure and Applied Logic***, vol. 52 (1998), pp. 143–153.

► CLAUDE LAFLAMME AND CHRISTOPHER C. LEARY, *Filter games on the natural numbers*.

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We characterize winning strategies in various infinite games involving filters on the natural numbers in terms of combinatorics or structural properties of the given filter. Our results continue earlier work of the first author and generalize several ultrafilter games of Galvin.

[1] Claude Laflamme, *Filter games and combinatorial properties of winning strategies*, ***Contemporary Mathematics***, vol. 192, American Mathematical Society, Providence, Rhode Island, pp. 51–67.

► SAUNDERS MAC LANE, *Logical foundations in controversy*.

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For more than 50 years Zermelo Fraenkel set theory has been generally regarded as the most effective "Foundation" of mathematics, doubted only by a few constructionists and occasional advocates of type theories. Recently, however, F. W. Lawvere has proposed that mathematics might better be founded on axioms on functions, as formulated in the axioms for the category of all sets and functions—technically as the axioms for a well pointed topos with a natural number object. These axioms, not generally so well known, have recently become the occasion of a noisy controversy presented in a web site "FOM" (Foundations of Mathematics), organized by Steven Simpson (Penn State) with the support of Harvey Friedman (Ohio State University). The issue is under discussion, but the recent discussion in this web site has been one-sided and raucous. At one point several junior experts on category theory were cajoled into putting up their axioms—with the promise that the web site would count the number of letters used in the categorical foundation in order to compare this with the count for Zermelo-Fraenkel! Apparently this vacuous test was never complete. More recently the sponsors of the web site have been sharply critical of some other logicians—who were urged to give up their current research interests in order to work on Friedman's project.

It is unfortunate that the world wide web is used for such restricted and needlessly partisan purposes. My talk will, however, be concerned with the foundation by functions and its uses. This may help to make the issues clear.

[1] F. W. Lawvere, *An elementary theory of the category of sets*, Proceedings of the National Academy of Sciences of the United States of America, vol. 50 (1963), pp. 869–873.

[2] Stephen G. Simpson, ***Subsystems of second order logic***, Perspectives in Mathematical Logic, Springer-Verlag, Berlin, Heidelberg, and New York, 1999, 445 pp.

► AMY K. C. S. VANDERBILT, *Alternate forms of the common derivation*.
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Various questions concerning extensions of Nonmonotonic Rule Systems have been explored in detail by Marek, Nerode, Remmel, Cenzer and others. The problem of finding a common derivation d for a formula ϕ occurring in each extension of a nonmonotonic rule system $\langle U, N\rangle$ will be explored. We will further explore alternate forms of the common derivation that ensure certain properties for the resulting nonmonotonic rule system $\langle U, N \cup \{d\}\rangle$.

This work is part of the author's dissertation, which is in preparation.

Abstracts presented by title

► RAYMOND D. GUMB, *The lazy logic of partial terms*.
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The Logic of Partial Terms LPT is a strict negative free logic that provides an economical framework for developing many traditional mathematical theories having partial functions. In these traditional theories, all functions and predicates are strict. For example, if a unary function (predicate) is applied to an undefined argument, the result is undefined (respectively, false). On the other hand, every practical programming language incorporates at least one nonstrict or lazy construct, such as the if-then-else, but nonstrict functions cannot be either primitive or introduced in definitional extensions in LPT. Consequently, lazy programming language constructs do not fit the traditional mathematical mold inherent in LPT. A nonstrict (positive free) logic is required to handle nonstrict functions and predicates.

Previously developed nonstrict logics are not fully satisfactory because they are verbose in describing strict functions (which predominate in many programming languages), and some logicians find their semantics philosophically unpalatable. The newly developed Lazy Logic of Partial Terms LL is as concise as LPT in describing strict functions and predicates, and nonstrict functions and predicates can be introduced in definitional extensions of traditional mathematical theories. LL is "built on top of" of LPT, and, like LPT, admits only one domain in the semantics. In the semantics, if the argument of a unary function f in an LL theory T is permitted to be nonstrict, we have $\models_T\ f(\bot) = y \leftrightarrow \forall x\ (f(x) = y)$, where $\bot$ is a canonical undefined term. Correspondingly, before introducing function f with an explicitly definition, an "indifference" (to the value of the argument) condition must be established: If f is to be introduced with the explicit definition $f(x) = y \leftrightarrow D(x, y)$, we must show $\vdash_T\ D(\bot, y) \leftrightarrow \forall x\ D(x, y)$. The price paid for LL's naturalness is that it is tailored for describing a specific area of computer science, program specification and verification, possibly limiting its role in explicating traditional mathematical and philosophical subjects.

► NATHANIEL S. HELLERSTEIN, *Diffracting Brown's modulator: Paradox logic and how to count to two*.
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This paper uses diamond, a four-valued paradox logic, to prove that G. S. Brown's modulator circuit does indeed count to two. Topics include: diamond definitions, diamond circuitry, laws, and fixed-point lattices; the J gate, diamond interferometry, Brown's modulator, Kauffman's modulator, modulators diffracted, rotor circuits, pumps, tapes, and phase computation.

► CYRUS F. NOURANI, *Descriptive definability and the Beth tableaux*.
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Defining descriptive computing on generic diagrams allows us to correspond admissible sets to descriptive computing and prove certain admissible sets are descriptive computable. Since Beth definability is basic to proving the descriptive computing theorem its computational significance is further explored by connecting it to the Beth tableaux for automated deduction by a model-theoretic theorem. The techniques and the basics on computation with generalized model diagrams are presented for applications to planning as goal satisfiability examples. Beth definability is applied to prove the following theorem applying Hanf numbers and admissible sets on cardinals ([1]). We want to explore its Tableaux computational significance further. From [3] we have:

THEOREM 3. *For A an admissible computable set, A is descriptive computable.*

THEOREM 4. *A formula p in L is provable by the tableaux method if and only if it is explicitly definable by $G\{P_1, \ldots, P_n\}$, where $G\{P_1, \ldots, P_n\}$ is the set of sentences of the language $L \cup \{P_1, \ldots, P_n\}$; and $P_1, \ldots, P_n$ are n-placed relation symbols for relations defining the Skolem functions applied by the tableaux.*

[1] J. BARWISE, ***The syntax and semantics of infinitary languages***, Lecture Notes in Mathematics, no. 72, Springer-Verlag, 1968.

[2] EWERT W. BETH, *Semantical entailment and formal derivability*, ***Mededlingen der Koninklijke Nederlandse Akademize van Westenschappen***, 1955.

[3] C. F. NOURANI, *Descripting computing—The preliminary definition*, Summer Logic Colloquium, July 1996, San Sebastian, Spain, see American Mathematical Society, April 1997, Memphis.

[4] ———, *Descriptive definability—A model computing perspective on the tableaux*, November 1998.