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1999 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

LOGIC COLLOQUIUM '99

Utrecht, The Netherlands, August 1–6, 1999

Logic Colloquium '99, the 1999 European Meeting of the Association for Symbolic Logic, took place at the University of Utrecht in The Netherlands on August 1–6, 1999. The Program Committee consisted of Erik Barendsen, Jan van Eijck, Sergey Goncharov, Wilfrid Hodges (Chair), Dick de Jongh, Alexander Kechris, Peter Koepke, Manuel Lerman, David Marker, Jaap van Oosten, Andrew Pitts, and Albert Visser. The Organizing Committee consisted of Peter Blok, Jan van Eijck (Chair), Vincent van Oostrom, Simone Panka, Marc Pauly, Maarten de Rijke, Frans Snijders, Yde Venema, and Marco de Vries. The main program consisted of twelve invited lectures, four series of three-lecture tutorials, and three series of parallel sessions for contributed talks.

The following invited talks were presented:

Samson Abramsky (Edinburgh), *Concurrent games and full completeness*.
Alessandro Andretta (Torino), *Countable Borel equivalence relations*.
Sergei Artëmov (Cornell and Moscow), *Explicit provability or How to circumvent Gödel's incompleteness theorem*.
Joan Bagaria (Barcelona), *Reflection and absoluteness in set theory*.
Lev Beklemishev (Münster), *Reflection principles in formal arithmetic*.
Peter Cholak (Notre Dame, Indiana), *Some recent results on the computably enumerable sets*.
Deirdre Haskell (Holy Cross, Worcester, Massachusetts), *Valued fields and elimination of imaginaries*.
Dale Miller (Pennsylvania State), *A meta-logic for sequent calculus*.
Andrei Morozov (Novosibirsk), *Symmetry and computability*.
Jan Rutten (CWI, Netherlands), *Coinduction and behavioural differential equations*.
Patrick Speissegger (Toronto), *The Pfaffian closure of an o-minimal structure*.
Andreas Weiermann (Münster), *Proof theory and subrecursive hierarchies*.

The series of tutorials were:

Greg Hjorth (University of California, Los Angeles), *Group actions and countable models*.
Jan Willem Klop (Free University, Netherlands), *Ten topics in term rewriting*.
Ieke Moerdijk (Utrecht), *Categories and types*.
Anand Pillay (University of Illinois, Urbana-Champaign), *Geometric model theory*.

There were also two evening lectures arranged by members of the Dutch logic community:

Johan van Benthem (Amsterdam), *Guarded quantifiers: a new look at decidable fragments*.
N. G. de Bruijn, *Teaching logic in a style inspired by Automath*.

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Abstracts of invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committe
WILFRID HODGES

Abstracts of invited tutorials

► GREG HJORTH, *Group actions and countable models*.
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Some aspects of the study of countable models can be framed in the context of continuous actions of the infinite symmetric group on an appropriate Polish space of countable structures. Following work of Vaught in the 1970s this has lead to a general investigation of the continuous actions of Polish groups on Polish spaces. I will survey the recent work in this area, with emphasis on the still open Vaught conjecture and how ideas from model theory have continued to provide inspiration in the study of general Polish group actions.

► JAN WILLEM KLOP, *Ten topics in term rewriting*.
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In this tutorial we aim to give a survey of term rewriting systems, ranging from abstract rewriting to infinitary rewriting and term graph rewriting. We start in the first hour with abstract rewriting, illustrating the basic notions with the examples of knots (Reidemeister moves) and braids (Artin). Next we treat a powerful theorem due to De Bruijn and van Oostrom to prove confluence by decreasing diagrams. This theorem generalizes several well-known lemmas such as Newman's lemma, Huet's strong confluence lemma and the like. The proof will be explained using infinite reduction diagrams, arising by unsuccessful attempts to prove confluence.

In the second hour we deal with orthogonal first-order rewriting, with Combinatory Logic as its paradigm example. Basic theorems such as Church's theorem for non-erasing orthogonal term rewriting systems (TRSs) are treated, and attention is paid to strategies (e.g., leftmost reduction, parallel-outermost reduction). The issue of (strong) sequentiality is discussed, after Huet and Levy. We treat the new technique of 'origin tracking' to prove some of the fundamental results. We also hope to mention in this hour a technique for proving termination by the RPO (recursive path order) method.

In the third hour we turn to some extended rewrite formats. First, infinitary rewriting, both for the first-order case and for lambda calculus. Boehm Trees, Levy-Longo Trees, and Berarducci Trees are encountered as infinite normal forms. We apply the technique of origin tracking for infinite lambda terms to understand the sequentiality property of lambda calculus (after Berry). Next, we introduce a framework for higher-order rewriting originally due to Aczel, that encompasses (typed) lambda calculi and first-order TRSs, mentioning a technique to reduce proofs of SN (strong normalization) to proofs of WN (weak normalization). Finally some results in term graph rewriting are covered. Here we have terms with subterm sharing, and containing cycles.

► IEKE MOERDIJK, *Categories and types*.
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In this series of lectures, which are meant for a general logical audience, I will explain the relation between various kinds of logics and the corresponding categorical structures, which lies at the basis of all uses of category theory in logic. I will begin with some classical cases such as the correspondence between typed lambda calculus and Cartesian closed categories, and gradually work towards more recent advances related to set theory and predicative type theory.

► ANAND PILLAY, *Geometric model theory*.
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I will discuss the contemporary model-theoretic approach to the study of (mainly first order) theories and structures. I will discuss issues of 'pure theory' such as stability and simplicity, as well as examples and applications such as differential and difference equations.

Abstracts of invited plenary talks

► SAMSON ABRAMSKY, *Concurrent games and full completeness*.
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Game semantics has emerged as a powerful paradigm for giving semantics to a variety of logical systems and programming languages. It has been used to prove "full completeness" results—completeness at the level of proofs, not merely provability—and to give the first syntax-independent constructions of fully abstract models for a spectrum of programming languages ranging from purely functional languages to languages with non-functional features such as control operators and references. There are also applications to the semantics of branching quantifiers and "independence-friendly" logics. In this talk we will present a concurrent games model of Linear Logic, and a full completeness result will be described.

► ALESSANDRO ANDRETTA, *Countable Borel equivalence relations*.
via Palazzo di Città 8, 10123 Torino, Italy.
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A Borel equivalence relation E on $\mathbb{R}$ is said to be countable if each equivalence class is countable. Many equivalence relations arising in mathematics are of this form, and in recent years there has been a lot of activity in descriptive set theory trying to understand the structure of such Es. We will survey some of these developments, looking in particular at countable universal Borel equivalence relations.

► JOAN BAGARIA, *Reflection and absoluteness in set theory*.
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Reflection Principles state, roughly, that what is true in the universe of all sets is true in some initial segment of it, whereas Principles of Generic Absoluteness state that what is true in a generic extension of the universe is already true in it. Both types of principles are closely related and they usually appear disguised under different forms, e.g., as large cardinal axioms, as forcing axioms, or simply as purely combinatorial principles. The study of such principles yields not only interesting connections between the different kinds of axioms, but it also leads to new and natural axioms for Set Theory.

► LEV BEKLEMISHEV, *Reflection principles in formal arithmetic*.
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Reflection principles provide a uniform approach to the proof-theoretic analysis of formal systems. We discuss applications of some general results on reflection principles to the study of fragments of Peano Arithmetic. Advantages and limitations of the method are discussed.

► PETER A. CHOLAK, *Results on the computably enumerable sets*.
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We will survey some of the recent work on the computably enumerable sets. In particular, we will focus on the following result:

THEOREM (Cholak, Harrington). *Let J be REA in $\mathbf{0}''$. Then there is a $\mathcal{L}_{\omega_1,\omega}$ formula φ_J in the language of the computably enumerable sets (just inclusion $\subseteq$) such that*

1. *For every computably enumerable degree* $\mathbf{a}$, *if* $\mathbf{a}'' \geq_T J$ *then there is a computably enumerable set A in* $\mathbf{a}$ *with* $\varphi_J(A)$.
2. *Furthermore if $\varphi_J(B)$ then $B'' \geq J$.*

This has some very nice corollaries including:

COROLLARY. *For all $n \geq 2$, the high$_n$ ($\overline{\text{low}_n}$) computably enumerable degrees are invariant.*

► N. G. DE BRUIJN, *Teaching logic in a style inspired by automath*.
Eindhoven.

The talk will be about two groups of ideas; both were inspired by the structure of the mathematics checking system Automath (developed around 1970). They turned out to be quite successful in the education of mathematicians and computer scientists in Eindhoven since about 1980.

The first group is about the systematic use of Fitch-style natural deduction as a first introduction into propositional and predicate logic. It is intended for freshmen, but parts of it might be transported to middle school education. The second group of ideas is about bridging the gap between standard mathematical vernacular (the mixture of words and formulas used in mathematics) and formalized languages. This uses a simple grammatical system and some new notation, particularly adapted to a type-theoretical approach.

► DEIRDRE HASKELL, *Valued fields and elimination of imaginaries*.
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It is useful to know that a structure eliminates imaginaries when one wants to study objects which are interpretable, but not obviously definable, over the structure. Valued fields in general do not eliminate imaginaries in the natural language. Any structure eliminates imaginaries over $\mathbf{M}^{\mathrm{eq}}$. In recent work, E. Hrushovski, D. Macpherson and I have shown that algebraically closed valued fields eliminate imaginaries in a language with a small part of $\mathbf{M}^{\mathrm{eq}}$ added—sorts for the closed and open balls. In this talk, I will try to explain why the property of eliminating imaginaries is important, and give some idea of the arguments that go into the proof.

► DALE MILLER, *A meta-logic for sequent calculus*.
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Park, Pennsylvania 16802, USA.
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Linear logic can be used as a meta-logic to specify a variety of proof systems for classical, intuitionistic, and linear logic. The specification of inference rules translates naturally to certain linear logic programming clauses. The operational semantics of linear logic arising from goal-directed search translates directly into object-level proof search semantics. Girard's Logic of Unity (LU) can be motivated and specified naturally using this meta-logic approach. Finally, we shall use the meta-theory of linear logic to prove various properties of object-level proof systems.

► ANDREI MOROZOV, *Symmetry and computability*.
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The results on symmetries arising in the study of computability are presented.

► JAN RUTTEN, *Coinduction and behavioural differential equations*.
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In the semantics of programming, finite data types such as finite lists have traditionally been modelled by initial algebras. Later final coalgebras were used in order to deal with infinite data types. Coalgebras, which are the dual of algebras, turn out to be suited, moreover, as models for many types of automata and, more generally, for transition-, discrete event-, and dynamical systems. An important property of initial algebras is that they satisfy the familiar principle of induction. Dually, final coalgebras satisfy a proof principle called coinduction. It is formulated in terms of bisimulations, a notion originally stemming from the world of concurrent programming languages, which is in a precise sense the coalgebraic counterpart of the notion of congruence relation on algebras. The main goal of this talk will be to illustrate the use of coinduction, both as a proof and as a definition principle, by giving a coalgebraic presentation of formal power series, which are functions from A^* to a semiring such as the Booleans or the reals. Coinductive definitions of operators on power series (such as product, sum, star, shuffle) take the shape of what we have called behavioural differential equations, after Brzozowski's notion of input derivative. This is more than just an analogy: many classical differential equations for (analytic) functions over the reals, appear in the present setting as a special case. The use of behavioural differential equations leads, amongst others, to easy definitions of and proofs about (existing and new) operators on power series, as well as to the construction of syntactic finite automata, implementing them.

► PATRICK SPEISSEGGER, *The Pfaffian closure of an o-minimal structure*.
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Let R be an o-minimal expansion of the real field. The author shows that there is an o-minimal expansion $P(R)$ of R which is closed under solutions to Pfaffian equations in the following strong sense: whenever U is an open and connected subset of Euclidean n-space definable in R, $w = a_1\,dx_1 + \cdots + a_n\,dx_n$ is a 1-form on U with coefficients a_i of class C^1 and definable in $P(R)$, and L is a Rolle leaf of the equation $w = 0$, then L is definable in $P(R)$. In particular, if U is as above and f is a C^1 function on U such that $d_i f(x) = F_i(x, f(x))$ for all x in U and $i = 1, \dots, n$, where $d_i f$ denotes the partial derivative of f with respect to x_i and each F_i is a C^1 function definable in $P(R)$, then f is definable in $P(R)$. I will give a short introduction to the way o-minimality of $P(R)$ is proved. The corresponding construction also turns out to preserve other nice properties the o-minimal structure R might have; in particular, analytic cell decomposition and (under certain conditions) exponential bounds.

► JOHAN VAN BENTHEM, *Guarded quantifiers: A new look at decidable fragments*.
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Logicians have long studied decidable fragments of first-order logic, searching for broad areas of reasoning that are exempt from 'Church's Curse'. These fragments should preferably be well-behaved in other respects too, in particular, having a good model theory and proof theory. We propose a new direction of search here, inspired by modal logic, using a systematic analysis of quantifier bounds. The results are large 'guarded fragments', orthogonal to most existing decidable fragments, that turn out to combine decidability with a nice meta-theory based on suitable notions of bisimulation. We will show how guarded analysis works, discuss some recent results—and state further questions. A useful intuition there is that of 'logical dynamics': quantifiers stand for actions that jump to new states, subject to certain constraints. Undecidability then lurks once we move from purely sequential to parallel quantifier actions.

► ANDREAS WEIERMANN, *Proof theory and subrecursive hierarchies*.
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In the first part we survey how local predicativity style methods can be applied to theories like PA and Gödel's T. The aim is to give illuminating classifications of the computational complexity of these theories. In the second (more advanced) part we survey recent results on the comparison between the slow and fast growing hierarchies.

Abstracts of contributed talks

► PATRICK BELLOT, *Logic can handle dynamicity*.
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When logic wants to deal with dynamic components, it has to be perverted with extra-logical features, e.g., a time parameter. This is because logic has been developed in the actual and infinite mathematical world of Plato-Cantor. Intuitionistic and constructivist logics have incorporated the notion of effective computability although the first one did not manage the practical realizability of the computing process. Linear logic (LL) introduced the real world notion of resource by suppressing the structural rules of contraction and weakening. Linear logic allows us to describe state changes as sequents.

We designed a linear logic, the Omega logic, which handles actions as first class citizens. We have two projects. The first is the synthesis of imperative programs from a logical specification. This is already done. The second is to prove usual properties of specifications such as consistency, termination, liveness, etc. In both cases, we do not want to limit specifications to state descriptions because we prefer specifications to be real descriptions instead of some kinds of modelization.

Formulae can be situations as in LL such as "the door is open". Mathematical formulae are modalized situation formulae as in LL. But formulae can be actions such as "to open the door". A situation formula is interpreted as the action of doing things in such a way that the situation holds. Therefore, everything is action.

Connectives are similar to the LL connectives but they have an additional interpretation in terms of action. For instance, the first linear conjunction is also interpreted as the concurrent execution of its two operands. We introduced a noncommutative version of this connective to express sequentiality.

Axiomatization is unusual because actions do not have the same properties as situations.

E.g., if one can deduce a conclusion C from two hypotheses A and B, then one cannot deduce C from the conjunction of A and B. This is mainly because one cannot formally split a concurrent execution of A and B into two separate executions. When writing an axiom, one has to verify that if we assume programs for doing the hypotheses, then one is able to build programs for doing the conclusion.

Omega logic has an operational semantics. We designed and implemented the Omega calculus. It is based on the same principles as the Combinators theory but it is designed for imperative programs and supports concurrency, sequentiality, deterministic and non-deterministic choice. We use the realizability technique to attribute a term of Omega calculus, the program semantics, to a proved formula. When executed, this term does the formula action.

Omega logic can be used for imperative program synthesis. One designs the axiomatization of a problem and gives a program semantics to these axioms. Then one proves the specification of a program and gets the program as an Omega calculus term. This has been applied to data computing and to planning problems such as the World of Blocks and Towers of Hanoi.

► WOLFGANG BURR, *Fragments of Heyting arithmetic*.
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It is well known that the prenex normal-form theorem does not hold for intuitionistic logic, i.e., there are formulae that are not equivalent to any prenex formula. Thus the fragments $i\Pi_n$ of Heyting arithmetic (HA) do not *a priori* form a hierarchy that exhausts HA. Indeed, prenex induction within an intuitionistic framework is remarkably weak. Visser proved (we cite a sharpened version by Wehmeier): $i\Delta_0$ + prenex induction (and hence each $i\Pi_{n+2}$) is Π_2-conservative over $i\Pi_2$. Since HA and PA are Π_2-equivalent, this implies that a measure for the complexity of a formula should not merely count quantifier alternations. Within intuitionistic arithmetic nested implication and quantification are responsible for the complexity of a formula. This leads to the subject of this talk: we define classes $\mathbf{\Phi}_n$ of formulae with the following properties:

(i) $I\Pi_n = I\mathbf{\Phi}_n$.
(ii) $I\Pi_n$ and $i\mathbf{\Phi}_n$ prove the same Π_2-formulae.

Hence $i\mathbf{\Phi}_n$ are reasonable counterparts to the classical fragments $I\Pi_n$. The definition of $\mathbf{\Phi}_n$ provides a normal-form for formulae of arithmetic in the intuitionistic context and gives rise to truth-predicates for these classes of formulae. We also generalize the result by Visser and Wehmeier and show that it holds at any level of our hierarchy: if we close $\mathbf{\Phi}_n$ (for $n \geq 2$) under quantification (we call these new classes Θ_n) the new fragments $i\Theta_n$ still prove the same Π_2-theorems as $i\Phi_n$.

Furthermore, we examine principles such as collection within the intuitionistic framework. We show, for example, that collection within intuitionistic arithmetic is rather weak: all provable total functions of $i\Delta_0$ plus arbitrary collection are bounded by polynomials.

[1] Wolfgang Burr, *Fragments of Heyting arithmetic*, to appear in ***Journal of Symbolic Logic***.

► DUMITRU BUSNEAG, *Extensions of pseudo-valuations on Hilbert algebras*.
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Following A. Diego by *Hilbert algebra* (Ha) we mean an algebra $(A, \rightarrow, 1)$ of type $(2, 0)$ satisfying the following identities:

(a_1) $x \to x = 1$,
(a_2) $1 \to x = x$,
(a_3) $x \to (y \to z) = (x \to y) \to (x \to z)$,
(a_4) $(x \to y) \to ((y \to x) \to x) = (y \to x) \to ((x \to y) \to y)$.

If A is Ha, then the relation $x \leq y$ if and only if $x \to y = 1$ is a partial order on A (which will be called *natural ordering* on A); with respect to this ordering 1 is the largest element of A. A *bounded* Ha is a Ha with a smallest element 0 relative to the natural ordering.

A *Hertz algebra* is a Ha H with the property that for every $x, y \in H$, the infimum $x \wedge y$ (relative to the natural ordering) exists in H.

If $(A, \to, 1)$ is a Hilbert algebra, I defined *pseudo-valuation* on A as a real-valued function v on A satisfying $v(1) = 0$, and $v(x \to y) \geq v(y) - v(x)$ for every $x, y \in A$ (v is valled a *valuation* if $x = 1$ whenever $v(x) = 0$).

Every pseudo-valuation (valuation) v induces a pseudo-metric (metric) on A defined by $d_v(x, y) = v(x \to y) + v(y \to x)$ for every $x, y \in A$, under which $\to$ is uniformly continuous in both variables.

The scope of my communication is to provide many theoremes of extensions for pseudo-valuation (valuation):

> *If A is a valuated Ha, then the Hertz algebra H_A free over A, the maximal Ha of quotients $Q(A)$ of A (with A bounded) and the Ha of fractions $A[S]$ relative to a closed system $S \subseteq A$ are valuated Ha.*

► RICCARDO CAMERLO AND SU GAO, *The completeness of the isomorphism relation of countable Boolean algebras*.
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We show that the isomorphism relation between countable Boolean algebras is Borel complete; this means that the isomorphism relation of any countable models is Borel reducible to that of countable Boolean algebras. This implies that the Ketonen's classification of countable Boolean algebras is nearly optimal in the sense that the kind of objects used for the complete invariants cannot be improved in an essential way. The result is obtained by proving its topological counterpart, the Borel completeness of the homeomorphism relation between separable Boolean spaces, and then using the Stone duality. We also give a stronger form of the Vaught conjecture for Boolean algebras which states that, for any complete first-order theory of Boolean algebras that has more than one countable model up to isomorphism, the class of models for the theory is Borel complete. The results are applied to settle a lot of other classification problems, arising in algebra and topology, related to countable Boolean algebras and separable Boolean spaces. Among the equivalence relations we show Borel complete are the translation equivalence between closed subsets of the Cantor space, the isomorphism between ideals of the unique (up to isomorphism) countable atomless Boolean algebra, the conjugacy equivalence between homeomorphisms of the Cantor space. Another corollary of our results is the Borel completeness of the commutative AF C^*-algebras, which in turn gives rise to similar results for Bratteli diagrams and dimension groups. We list some of the main problems related to the subject that are still open.

► F. COLLOT, *An explicit construction of a well-ordering on the continuum*.
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E-mail: `unavailable`.

The author proposes in ZFC a process which well-orders the whole set $P(\mathbf{N})$.

A partition uses the natural numbers which are missing into each member (i.e., between the least and the last natural number of the member). Those are named "*lacunae*", (noted "O") and permit us to introduce the concept of 0-lacunary set, 1-lacunary set, ..., ω-lacunary set.

Every n-lacunary subset is constituted with infinite sequences of members. Every sequence is well-ordered with the *lexicographic order*, (L). But the sequences of every n-lacunary subset are well-ordered if their first members are ordered according to the *inverse lexicographic order*, L_{-1}. In a same n-lacunary set the order L_{-1} induces a "*backing*" of the second natural number which is present in every member. Every n-lacunary set has the cardinal $\mathbf{N}^{n+1}$. (Proposition 1.)

For the ω-lacunary set (named **G**), it is necessary to change the infinite open parts of **N** in bounded and closed parts. The solution consists to start from another ordering on **N**. So the author uses the ordered union of some infinite subsets of **N**, but these subsets are not well-ordered because the necessary *backing* of the 2nd present natural number which is induced by the order L_{-1}: ex. the prime, odd (but not prime), and even numbers. So instead to start from **N**, we start from $\mathbf{N}'$ which is equal to **N**:

$$\mathbf{N}' = \{2, \dots, 11, 7, 5, 3, \dots, 9, 7, 5, 1, 0, 4, 6, 8, \dots\}$$

the order type of which is $1 + \star\omega + \star\omega + \omega$.

Then if we choose any not prime number (for example 0 noted "v") we obtain an infinity of lacunae between 2 and 0 and the least element of **G**: $\{2, \dots, v\}$. Henceforth **G** will be considered only and the propositions 2, 3, and 4 prove that **G** is not countable, has the power of the continuum, and is well-ordered.

► JOHN CORCORAN, *Self-defeating and self-fulfilling propositions*.
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The universal proposition "Every proposition is true" is *self-defeating* (*SD*) in the sense that, being a proposition which is not true, it is a *counterexample* for itself. Other examples of SD propositions come readily to mind: every proposition is tautological (contradictory, negative, existential, conjunctive, atomic, etc.). Every SD proposition is false and every universal proposition that is about propositions and that has a contradictory predicate is SD. Nevertheless it is not the case that every SD proposition is contradictory, nor is it the case that every contradictory universal proposition about propositions is SD. The existential proposition "Some proposition is true" is *Self-fulfilling* (*SF*) in the sense that, being a proposition which is true, it is a *proexample* for itself. Other examples of SF propositions come readily to mind: some proposition is existential (non-tautological quantificational, non-atomic, etc.). Every SF proposition is true and every existential proposition that is about propositions and that has a tautological predicate is SF. Nevertheless it is not the case that every SF proposition is tautological, nor is it the case that every tautological existential proposition about propositions is SF. Is "Every proposition is SD" itself SD? Is "Some proposition is not SF" itself SF? The singular proposition that "Every proposition is SD" is SD leads easily to a paradox as does the singular that "Some proposition is not SF" is SF. This paper analyses the concepts of SD propositions and SF propositions as well as analogous properties applicable to non-proposition "truth-bearers" such as sentences (construed as strings of characters) and statements (construed as linguistic performances).

► ALEXANDER DENISOV, *The most left branch in computable tree*.
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A tree with nodes of finite type represents many mathematical objects: propositions, derivations [3] and others.

Let $\mathcal{M}_T^{(d_2,d_3)}$ be a structural semiordering of the theory T [1].

DEFINITION 1. If a model $\mathcal{M}$ of T is homogeneous (decidable in $\mathcal{O}'$), then $\mathcal{M}$ will be called a homogeneous ($\mathcal{O}'$-)element of the $\mathcal{M}_T^{(d_2,d_3)}$.

THEOREM. *Let $\mathcal{M}$ be an arbitrary $\mathcal{O}$-element of the signature σ. Then the diagram*

$$\frac{\mathcal{M}}{\mathcal{O}} \quad \underline{\mathcal{O}} \quad \underset{hom}{\overset{\mathcal{O}'}{\longrightarrow}}$$

can always be complemented by indicated way.

PROOF. All n-diagrams [2], $n < \omega$ of an arbitrary decidable theory T form a computable tree with nodes of finite type (diagram omitted).

The most left branch, defined by homogeneous or prime ω-diagram, is decidable in $\mathcal{O}'$ (dots denote false steps of the computing). For $T = \mathrm{Th}(\mathcal{M}, >)$ it gives necessary complementation. The theorem is proved. ⊣

[1] ———, *On symbolism of double structural diagrams* (abstract), this BULLETIN, vol. 4 (1998), no. 1, pp. 110–111.

[2] A. S. DENISOV, *Homogeneous $\mathcal{O}'$-elements in structural semiorderings*, ***Algebra i Logika***, vol. 28 (1989), no. 6, pp. 619–642, in Russian.

[3] D. VAN DALEN, ***Logic and structure***, Springer-Verlag, Berlin, Heidelberg, New York, and Tokyo, 1983.

► MICHAEL DIERKES, *Transformation of guarded Horn clauses for model building*.
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The guarded fragment of first order logic has been defined in [1]. It is interesting due to the fact that it is decidable and several modal logics can be translated into it. Guarded clauses, defined by de Nivelle in [2], are a generalization of guarded formulas in clausal form. In [2], it is shown that the class of guarded clause sets is decidable by saturation under ordered resolution.

By now, no method exists that allows us to build models for satisfiable sets of guarded clauses automatically. Such a method should be useful to build models for formulas of modal logics that can be translated into the guarded fragment, thus providing the same advantages to those logics as model building for first order logic has (see [3] and references in it). In this work, we deal with the problem of model building for satisfiable sets of guarded Horn clauses.

We define the following notion: a guarded Horn clause is *primitive* if and only if it is positive and ground (in this case, it contains only one literal), or its body contains only a flat and linear literal (which is the guard), and its head literal has at least one non-ground functional term as argument.

Then, we show that for every satisfiable and finite set of guarded Horn clauses $\mathcal{S}$, there is a finite set of primitive guarded Horn clauses $\mathcal{S}'$ such that a predicate symbol that occurs in $\mathcal{S}$ is interpreted in the least Herbrand model of $\mathcal{S}$ in the same way as in the least Herbrand model of $\mathcal{S}'$. We show that $\mathcal{S}'$ can be obtained from $\mathcal{S}$ as follows: we first apply de Nivelle's saturation procedure. From the resulting set, we extract certain clauses, which are then transformed to primitive guarded Horn clauses using a resolution-based technique that introduces new predicate symbols.

We conjecture that primitive guarded Horn clauses are a suitable mechanism to represent models of guarded clauses.

[1] HAJNAL ANDRÉKA, JOHAN VAN BENTHEM, and ISVÁN NEMETI, *Modal languages and bounded fragments of predicate logic*, Res. Rep. ML-96-03, ILLC, 1996.

[2] HANS DE NIVELLE, *Resolution decides the guarded fragment*, Research Report Ct-1998-01, ILLC, 1998.

[3] NICOLAS PELTIER, *Nouvelles techniques pour la construction de modèles finis ou infinis en déduction automatique*, ***Ph.D. thesis***, INPG, 1997.

► MARIO J. EDMUNDO, *Abelian groups definable in o-minimal structures*.
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We use group cohomology and group extension theory over an o-minimal structure $\mathcal{N}$ with definable Skolem functions to describe the $\mathcal{N}$-definable abelian groups: Let U be an $\mathcal{N}$-definable abelian group. Then U has an $\mathcal{N}$-definable subgroup V such that U/V is $\mathcal{N}$-definably compact, and V has the form $K \times W_1 \times \cdots \times W_l \times V_1 \times \cdots \times V_k$ where K is an $\mathcal{N}$-definable compact subgroup of U, for each $i \in \{1, \ldots, l\}$, W_i is a one-dimensional torsion-free $\mathcal{N}$-definable group, and for each $j \in \{1, \ldots, k\}$ there is an o-minimal expansion of a real closed field $\mathcal{I}_j$ interpretable in $\mathcal{N}$ such that V_j is an $\mathcal{I}_j$-definable group and $\forall j' \in \{1, \ldots, k\} \setminus \{j\}$, $\mathcal{I}_j$ is orthogonal to $\mathcal{I}_{j'}$.

Supported by JNICT grant PRAXIS XXI/BD/5915/95.

[1] M. EDMUNDO, *Abelian groups definable in o-minimal structures*, preprint, Oxford, February 1999.

[2] A. PILLAY, *on groups and fields definable in o-minimal structures*, ***Journal of Pure and Applied Algebra***, vol. 53 (1988), pp. 239–255.

[3] Y. PETERZIL and C. STEINHORN, *Definable compactness and definable subgroups of o-minimal groups*, preprint, November 1997.

► SILVIO GHILARDI, *Best solving modal equations*.
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Classical propositional calculus enjoys the following remarkable property (corresponding to the unitarity of E-unification type for Boolean algebras): for every formula A, if there is a substitution σ such that $\sigma(A)$ is provable (with a slight abuse of terminology, we say in this case that the formula A is unifiable and that σ is a unifier for it), then there is also 'the best' substitution with this property, i.e., there is a substitution μ such that $\mu(A)$ is provable and any τ such that $\tau(A)$ is provable is, up to provable equivalence, an instantiation of μ.

We wonder whether the same property holds for other logical calculi, for instance for some modal calculi. It is easily seen that the answer is negative for all modal logics enjoying the disjunction property. However we prove that unification is rather nice in many common systems like $K4$, $S4$, $S4Grz$, GL, etc., because it is finitary. This means that there are finitely many 'best unifiers' for any unifiable formula. Otherwise said, if an equation is solvable in the free algebra (on countably many generators), then there are finitely many 'best solutions' for it. We show that such solutions can be effectively computed, thus obtaining as an immediate corollary a new proof of Rybakov theorems about the effective recognizability of admissible inference rules.

► VAL GORANKO, *Logical topologies and semantic completeness*.
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This work is motivated by the following, often arising in logical studies, generic problem.

A deductive system **L** (of any nature) in a certain logical language is intended to axiomatize a class of *standard models* **SM**. A completeness theorem is proved with respect to a larger class of *general models* **GM**, i.e., it is proved that

$$\mathbf{L} \vdash \phi \qquad \text{if and only if} \qquad \mathbf{L} \models_{GM} \phi.$$

The goal is *to prove completeness of* **L** *with respect to the standard models*, i.e.,

$$\mathbf{L} \vdash \phi \qquad \text{if and only if} \qquad \mathbf{L} \models_{SM} \phi.$$

Typical examples of such *relative completeness problem* include, e.g., proving *finite model property* (**GM** consists of all models of the logical theory in question, while **SM** is the class of its *finite* models), and proving *Kripke-frame completeness* in modal logic (**GM** consists of *all* Kripke models for the logic, while **SM** consists of those based on frames in which the logic is valid).

Usually, such a relative completeness proof goes through specific model theoretic arguments and constructions which produce standard models from general ones.

I propose a more general approach to this problem, based on the notion of *logical topology* and the related concept of *logical approximation*.

In this talk I will present some general results regarding these concepts and will discuss them in the framework of first-order logic. Then I will give an example of a particular application of the ideas developed here, and will discuss some directions for further research.

► KEJIA HO AND FRANK STEPHAN, *Simple sets and Σ_3 ideals under m-reducibility*.
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We study nontrivial upper bounds for simple sets with respect to m-reducibility. We show that below some *btt*-complete set there is no simple set; but below every Q-complete set there exists a hypersimple set. We show that no m-incomplete set is an upper bound for all hypersimple sets and no effectively simple set is an upper bound for all strongly effectively simple sets; but there is a simple set such that every effectively simple set is 1-reducible to it. We show that if $\mathcal{S}$ is a Σ_3 class of computably enumerable sets containing only simple sets and confinite sets, then the m-ideal generated by $\mathcal{S}$ has an exact pair of simple sets and every set in $\mathcal{S}$ is 1-reducible to any one half of this exact pair. We study also m-ideals generated by Σ_3 index sets in general and show that there is an m-incomplete set A such that every computably enumerable tt-degree contains a set $B \leq_m A$. We show that given a computably enumerable set A, the class of all computably enumerable sets Turing reducible to A is bounded if and only if A is low$_2$.

[1] K. Ambos-Spies, A. Nies, and R. A. Shore, *The theory of the recursively enumerable weak truth-table degrees is undecidable*, ***Journal of Symbolic Logic***, vol. 57 (1992), pp. 864–874.

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[5] P. Odifreddi, ***Classical recursion theory***, North-Holland, Amsterdam and New York, 1989.

[6] R. I. Soare, ***Recursively enumerable set and degrees***, Perspectives in Mathematical Logic, Omega Series, Springer-Verlag, Berlin, Heidelberg, New York, London, Paris, and Tokya, 1987.

► RAMON JANSANA, *The strong version of a protoalgebraic logic*.
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It has often been observed that some sentential logics come in pairs, one stronger than the other but with the same theorems while the theories of the stronger one are exactly the theories of the weaker one that are closed under some additional inference rule, with respect to which the set of theorems is closed. This phenomenon occurs in Modal Logic. For instance, to each normal modal logic L two consequence relations with the same theorems can be associated, one axiomatized by the theorems of L as axioms and with Modus Ponens as the only rule of the inference and the other one axiomatized in the same way but with the Necessity rule as an additional rule.

We study the phenomenon from the point of view of abstract algebraic logic which allows us to explain it from the behavior of the so-called Leibniz filters. A filter of a matrix is Leibniz if it is minimum with respect to the property of having the same Leibniz congruence. Given a protoalgebraic logic $\mathcal{S}$ we consider the extension $\mathcal{S}^+$ defined by the matrices of $\mathcal{S}$ whose filter is Leibniz. $\mathcal{S}^+$ is *the strong version* of $\mathcal{S}$. When the construction is applied to modal logics the usual pairs of logics are obtained.

We will present some results in [1], in particular the definability properties of the Leibniz filters, an explicit characterization of when $\mathcal{S}^+$ is weakly algebraizable, the study of how some of the possible properties of $\mathcal{S}$ transfer to its strong version and an application to Łukasiewicz many-valued logics and quantum logics.

[1] J. M. Font and R. Jansana, *Leibniz filters and the strong version of a protoalgebraic logic*, to appear in ***Archive for Mathematical Logic***.

► REINHARD KAHLE, *A proof-theoretic view of necessity*.
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We discuss a new account to necessity by proof-theoretic means. The rough idea is to model a binary necessity statement "A is necessary for B" by: Every proof of B uses A. Depending on a proper definition of "use" we show several advantages of such a reading compared with the usual one in terms of modal logic [4], [1], [3]. For instance, $\Box(B \to A)$ implies that tautologies are necessary for everything, regardless whether they were used or not. After presenting some examples, we investigate possible definitions of "use".

[1] G. Hughes and M. Cresswell, ***An introduction to modal logic***, Methuen, London, 1968.

[2] R. Kahle, *Towards a proof thoeretic treatment of necessity*, Technical Report 14, DFG-Forschergruppe *Logik in der Philosophie*, Universitäten Konstanz und Tübingen, 1998.

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[4] C. Lewis and C. Langford, ***Symbolic logic***, The Century Comp., New York and London, 1932.

► ISKANDER SH. KALIMULLIN, *Downey's conjecture for enumeration degrees*.
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For every $n > 1$ let Φ_n be the following formula in the signature of the upper semilattice:

$$(\forall a)\,(\forall c > 0)\,(\exists b_1)\,(\exists b_2)\ldots(\exists b_n)\; [a = b_1 \cup b_2 \cup \cdots \cup b_n \;\&\; c \not\leq b_1 \;\&\; c \not\leq b_2 \;\&\ldots\&\; c \not\leq b_n].$$

Let for all $m > 1$ D_m denote the semilattice of m-c.e. e-degrees.

We prove the following

THEOREM. *For every pair m, n $(m > 1, n > 1)$ $D_m \vDash \Phi_n$ if and only if $m < 2n$.*

COROLLARY. *For every pair m, n $(m > 1, n > 1)$ $D_m \vDash \Phi_{n+1}$ & $\neg\Phi_n$ if and only if $m = 2n$ or $m = 2n + 1$.*

In Turing degree theory so-called Downey's conjecture on elementarily equivalency of n-c.e. and m-c.e. degree structures for $n \neq m$, $n, m > 1$, is still open. It follows from our Theorem that the analogue of this conjecture for enumeration degree theory in general fails: if $1 < m < 2p < n + 1$ for some integer p, then structures D_n and D_m are not elementarily equivalent (since $D_n \vDash \Phi_p$ and $D_m \vDash \neg\Phi_p$). The problem of elementarily equivalency of D_{2p+1} and D_{2p} $(p > 0)$ remains open.

► VLADIMIR KANOVEI, *On Borel transformations associated with the additive group of the reals.*
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$\mathbb{R}$ means the (ordered) additive group of the reals below.

THEOREM 1 (In collaboration with J. R. P. Christensen and M. Reeken). *Suppose that B is a Borel abelian ordered group, having a countable subgroup G as the largest convex subgroup, and that the quotient B/G is order isomorphic to $\mathbb{R}$, the reals. Then B is Borel isomorphic, as an ordered group, to the product $\mathbb{R} \times G$, ordered lexicographically.*

As the main ingredient, we show, answering a question of D. Marker, that all Borel cocycles in $\mathbb{R} \times G$ are Borel coboundaries. An example by G. Hjorth shows that the theorem may be wrong for *un*countable (Borel) subgroups G. Another example shows that the theorem becomes wrong if one changes $\mathbb{R}$ to $\mathbb{Q}$, the rationals, even for $G = \mathbb{Z}$.

THEOREM 2. *Let $D, G \subseteq \mathbb{R}$ be countable groups. Then any Borel homomorphism*

$$H : \mathbb{R}/D \to \mathbb{R}/G$$

is generated by the multiplication by a constant $c \in \mathbb{R}$ such that $c \cdot D \subseteq G$.

In particular any Borel homomorphism $H : \mathbb{R}/\mathbb{Q} \to \mathbb{R}/\mathbb{Q}$ is generated by the multiplication by a rational constant. Theorem 2 resembles some theorems of Velickovic and Farah (see Farah, this BULLETIN, vol. 4 (1998), pp. 37–54) on homomorphisms of quotients of the Boolean algebra $\mathcal{P}(\omega)$.

It is clear that one can define a Borel group operation $\oplus$ on the set $\mathbb{R}$ such that $\langle \mathbb{R}; \oplus \rangle$ is *not* Borel isomorphic to $\langle \mathbb{R}; + \rangle$: for instance, define $\oplus$ so that $\langle \mathbb{R}; \oplus \rangle$ is Borel isomorphic to $\langle \mathbb{C}; + \rangle$ (the additive group of complex numbers), using a Borel bijection $\mathbb{R}$ onto $\mathbb{C}$. However any operation $\oplus$ of this kind cannot be very close to the real number addition.

THEOREM 3. *Let $\oplus$ be a Borel abelian group operation on $\mathbb{R}$ such that the function $f(x, y) = (x \oplus y) - (x + y)$ takes only countably many values. Then $\langle \mathbb{R}; \oplus \rangle$ is Borel isomorphic to $\langle \mathbb{R}; + \rangle$.*

Theorems 1–3 admit suitable generalizations on the Baire measurable case.

Support of NSF grant DMS 96-19880 and DFG grant Wu 101/9-1 acknowledged.

► PETER KOEPKE, *Fine structure with restricted Skolem hulls.*
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Gödel's inner model L is constructed by a transfinite iteration of the *definability operator*:

$$\mathrm{Def}(X) = \{\, \{\, x \in X \mid (X, \in) \vDash \phi(x, \vec{p}) \,\} \mid \phi \text{ is an } \in\text{-formula}, \vec{p} \in X \,\}.$$

Fine-structure theory dissects the Def-operator: the formulae ϕ are arranged in a hierarchy, the parameter sequences $\vec{p}$ are well-ordered, and witnesses for existential quantifiers are chosen minimally. If this is done recursively in a careful way, the transfinite definability process exhibits strong self-similarities by which certain Skolem-hulls are isomorphic to initial parts of the hierarchy. Important events in the model L like the singularization of an ordinal can be localized adequately and the self-similarities yield coherency principles useful to organize transfinite constructions. Principles like SQUARE are an embodiment of such coherencies.

After R. B. Jensen's ground-breaking insights and results [1] there have been several attempts at simplifying the delicate theory. In my talk I shall outline the basics of the approach of [2] in which the formation of hulls uses only Skolem functions for finitely many formulae and for arguments restricted by some bound in the canonical well-ordering of L. I shall give an indication of a simplified proof of the fine-structural combinatorial principle MORASS.

[1] R. B. JENSEN, *The fine structure of the constructible hierarchy*, ***Annals of Mathematical Logic***, vol. 4 (1972), pp. 229–308.

[2] S. D. FRIEDMAN and P. KOEPKE, *An elementary approach to the fine structure of L*, this BULLETIN, vol. 3 (1997), pp. 453–468.

► ULRICH KOHLENBACH, *On extensions of weak König's lemma*.
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The binary ('weak') König's lemma WKL plays an important role in the study of fragments of mathematics which on the one hand are mathematically strong but on the other hand have a low proof-theoretic and computational strength.

In addition to the restriction to binary trees (or equivalently trees with bounded labels), WKL is also 'weak' in that the tree predicate is quantifier-free. In general, the computational and proof-theoretic strength increases when logically more complex trees are allowed. We show that this is not the case for trees which are given by formulas in a class Φ_∞ where we allow an arbitrary function quantifier prefix over bounded functions in front of a Π^0_1-formula. This results in a scheme Φ_∞-WKL.

In the absence of functional parameters (so in particular in a second-order context), the corresponding version of Φ_∞-WKL is equivalent to WKL. In the presence of functional variables of type 2, however, we get a proper hierarchy of principles Φ_n-WKL. Variables of type 2 are necessary for a direct representation of analytical objects and—sometimes—for a faithful representation of such objects at all as we will indicate exhibiting the greater mathematical strength of Φ_∞-WKL compared to WKL. Nevertheless, Φ_∞-WKL does not contribute to the provably recursive functionals (of type ≤ 2) of the underlying fragments of arithmetic in all finite types.

Finally, we also consider a uniform version UWKL of WKL which asserts the existence of a functional Φ which selects uniformly in a given infinite binary tree f an infinite path Φf. The computational strength of UWKL depends on the amount of extensionality available. In the context of C. Spector's quantifier-free rule of extensionality, UWKL has the same strength as WKL. However, in the presence of the full axiom of extensionality, UWKL is much stronger than WKL.

► OLIVER KULLMANN, *On polynomial time decidable hierarchies of SAT instances*.
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In [5] a new hierarchy $G_k(\mathcal{U}_0, \mathcal{S}_1)$ of poly-time SAT decidable (and recognizable) classes of conjunctive normal forms is introduced, substantially extending the hierarchy of generalized

Horn clause-sets in [3]. With respect to *unsatisfiability*, the hierarchy (G_k) is based on a natural concept of nested input-resolution proofs, yielding quasi-automatizability of tree-like resolution and also "quasi-polynomially" tight lower bounds for tree-like resolution (cf. [2] and [1]). With respect to *satisfiability*, the notion of "linearly satisfiable clause-sets" ([4]), using linear programming, is exploited.

[1] ELI BEN-SASSON and AVI WIGDERSON, *Short proofs are narrow—resolution made simple*.

[2] MATTHEW CLEGG, JEFFERY EDMONDS, and RUSSELL IMPAGLIAZZO, *Using the Groebner basis algorithm to find proofs of unsatisfiability*, in ***Proceedings of the 28th ACM symposium on theory of computation***, 1996, pp. 174–183.

[3] GIORGIO GALLO and MARIA GRAZIA SCUTELLÀ, *Polynomially solvable satisfiability problems*, ***Information Processing Letters***, vol. 29 (1988), pp. 221–227.

[4] OLIVER KULLMANN, *Investigations on autark assignments*, October 1998, 19 pages, submitted to ***Discrete Applied Mathematics***.

[5] ———, *A general hierarchy of polynomially decidable classes of CNFs based on short tree-like resolution proofs*, July 1999, 81 pages.

► HANNES LEITGEB, *The possible worlds of semantically closed languages*.
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Let L be a first order language with a truth predicate True, W a set of possible worlds, V an evaluation of formulas such that $V(p)$ is a proposition (= the set of worlds where p is true).

Let T, S be binary relations of worlds such that

(i) $T(w_1, w_2)$ if and only if for every p: if p is true in w_1 then True('p') is true in w_2 (call w_2 a 'metaworld' of w_1),

(ii) $S(w_2, w_1)$ if and only if for every p: if True('p') is true in w_2 then p is true in w_1.

In his classical papers on truth Tarski has shown that in the presence of self-reference it is impossible that T, S are inverse, and reflexive for at least some world w; in particular, they cannot be identical to the identity relation.

In my talk I want to give some answers to the following questions:

(i) Which other intuitively plausible sets of properties could we postulate for T, S?

(ii) For which of these sets can we prove existence results?

(iii) What are the necessary laws of truth (= which sentences are true in every world)?

The answers will include various modern results on truth theories (for an excellent overview see Visser [1]) in a restated form.

Moreover, I will study properties of the induced mapping $f(X) = Y$ where X, Y are propositions such that $Y = \{ w_2 : T(w_1, w_2) \text{ with } w_1 \text{ in } X \}$; under certain circumstances, f may be shown to preserve the underlying Boolean structure on W. Some new results will be presented on this topic.

[1] A. VISSER, *Semantics and the liar paradox*, in ***Handbook of Philosophical Logic IV*** (D. Gabbay, F. Guenther, and D. Reidel, editors), Dordrecht, 1989, pp. 617–706.

► LARISA MAKSIMOVA AND ANDREI VORONKOV, *Complexity of some problems in modal and superintuitionistic logics*.
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A logic is called *tabular* if it can be characterized by finitely many finite models; and *pretabular* if it is maximal among non-tabular logics. A logic L is said to have *Craig's interpolation property* (*CIP*), if for every formula $(A \to B) \in L$ there exists a formula C such that (i) both $A \to C$ and $C \to B$ belong to L, and (ii) every variable of C occurs in both A and B. A logic L is said to have *interpolation property IPD* if $A \vdash_L B$ implies that there exists a formula C such that (i) $A \vdash_L C$, $C \vdash_L B$, and (ii) every variable of C occurs in both A and B.

Denote by $L + A$ the extension of a logic L by an extra axiom scheme A. By the *tabularity* (*pretabularity*, etc.) *problem over* L we mean the problem of determining for arbitrary A, whether $L + A$ is tabular (pretabular, etc.), and consider its complexity with respect to the size of A over intuitionistic logic Int and the modal S4.

THEOREM 1.

(i) *The tabularity problems over both* Int *and* S4 *are* NP-*complete.*
(ii) *The pretabularity problems over both* Int *and* S4 *are in* Δ_2^p *and* coNP-*hard.*

THEOREM 2.

(i) *The interpolation problem over* Int *is* PSPACE-*complete.*
(ii) *The problem of determining whether* $\mathrm{Int} + A$ *is a tabular logic with CIP is* NP-*complete.*
(iii) *Both CIP and IPD problems over* S4 *are in* coNEXP *and* PSPACE-*hard.*
(iv) *The problems of determining whether* $\mathrm{S4} + A$ *is a tabular* (*pretabular or locally tabular*) *logic with CIP* (*or IPD*) *are in* Δ_2^p *and* coNP-*hard.*

By rewriting these results in the algebraic language, we get

THEOREM 3.

(i) *The amalgamability problem for finitely based varieties of Heyting algebras is* PSPACE-*complete.*
(ii) *The amalgamability and super-amalgamability problems for finitely based varieties of closure algebras are in* coNEXP *and* PSPACE-*hard.*

► G. A. MARCO, *Incomplete objects and descriptive set theory*.
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A. Mathias asked when the going forth condition suffices for the usual back and forth argument with respect to a closed permutation group on ω.

P. J. Cameron gave an example of a countably categorical and homogeneous structure for which the forth condition doesn't suffice for its automorphism group.

In this paper we call the sufficiency of the going forth for the automorphism group of a structure M the "asymmetry principle". We give a descriptive set theoretical characterization of it.

Let X_L be the space of the countable structures for a countable language L. It is a topological space universal for $\mathrm{Perm}(\omega)$.

We give a "local" analysis of types conceived as descriptive set theoretical objects in the $\mathrm{Perm}(\omega)$ X_L space.

As a result of this analysis we show that the collection of X_L types with "local selectors" that doesn't admit Borel Uniformization is precisely the collection of the types for which the principle of asymmetry fails to hold.

We conceive the above collection of types as a collection of incomplete objects whose meaning is "possibly" given by games of "imperfect information".

[1] P. J. CAMERON, ***Oligomorphic groups***, London Mathematical Society Lecture Note Series, vol. 152, Cambridge University Press, Cambridge, 1990.

[2] H. BECKER and A. S. KECHRIS, ***The descriptive set theory of Polish group actions***,

London Mathematical Society Lecture Note Series, vol. 232, Cambridge University Press, Cambridge, 1996.

► SERGEJ MARDAEV, *Least fixed points in temporal logic*.
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We study the definability of least fixed points of propositional bimodal Π-formulas in temporal models.

Bimodal formulas are assumed to be constructed from propositional variables and the constant $\perp$ via connectives $\wedge$, $\vee$, $\neg$, $\Box_L$, $\Box_R$.

A bimodal formula φ is called Π-formula when φ is constructed via connectives $\wedge$, $\vee$, $\Box_L$, $\Box_R$ from formulas without modalities.

A formula φ is called positive in p when each occurrence of p in φ is positive.

THEOREM 1. *For each positive in p bimodal Π-formula $\varphi(p, q_1, \dots, q_n)$ there exists a bimodal Π-formula $\omega(q_1, \dots, q_n)$ which defines the least fixed point of the formula φ in each finite linearly ordered model $\langle W, \leq, v\rangle$.*

THEOREM 2. *For each positive in p bimodal Π-formula $\varphi(p, q_1, \dots, q_n)$ there exists a bimodal Π-formula $\omega(q_1, \dots, q_n)$ which defines the least fixed point of the formula φ in each finite strictly linearly ordered model $\langle W, <, v\rangle$.*

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We consider the following notions of forcing $(P, \leq)$, where the stronger object is the smaller one:

DEFINITION 1.
(1) Forcing with summable filters: $\mathbf{P}_1 = (\mathbb{R}_+^\omega \setminus \ell^1, \leq^*)$, where $\ell^1 = \{f \in \mathbb{R}_+^\omega \mid \sum_{n\in\omega} f(n) < \infty\}$.
(2) We equip 2^ω with the Cantor space topology. A filter in $[\omega]^\omega$ is a subset of $[\omega]^\omega$ that is closed under supersets and finite intersections. An F_σ-filter is a filter in $[\omega]^\omega$ whose set of characteristic functions is an F_σ-set in the Cantor space topology. The partial order $\mathbf{P}_2 = (\{F_\sigma\text{-filters on } \omega\}, \supseteq)$ is called forcing with F_σ-filters.

Forcing with F_σ-filters and forcing with summable filters both add a P-point with no rapid ultrafilter $\leq_{RK}$ below it [2]. This is a hint that the two forcings might be equivalent as forcings or at least might be related as complete suborders (see [3, Chapter 7]). Using techniques from [5] and [1] we show

THEOREM. *It is consistent relative to* ZFC *that the regular open algebras of* $\mathbf{P}_1$ *and of* $\mathbf{P}_2$ *are isomorphic.*

DEFINITION 2. The mapping id "identity" is the following

$$\mathrm{id}\colon P_1 \to P_2, \qquad f \mapsto \left\{ X \subset \omega \;\middle|\; \sum_{n\in\omega\setminus X} f(n) < \infty \right\}.$$

Vojtáš' work [6] shows that id respects the relations in both partial orders. However, using results of Mazur [4], we show:

THEOREM. *The mapping* id *is not a complete embedding from* $\mathbf{P}_1$ *to* $\mathbf{P}_2$.

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[4] Krzysztof Mazur, *F_σ-ideals and ω_1, ω_1^*-gaps in $\mathcal{P}(\omega)/I$*, ***Fundamenta Mathematicae***, vol. 138 (1991), pp. 103–111.

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► MORTEZA MONIRI, *Classical equivalence of $\mathcal{H}(T)$ and T.*

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For a classical theory T, $\mathcal{H}(T)$ denotes the intuitionistic theory of T-normal (i.e., locally T) Kripke structures. S. Buss has asked whether HA belongs to the range of $\mathcal{H}$. We provide a negative answer to this question by showing that any intuitionistic axiomatization of $\mathcal{H}(T)$ is classically equivalent with T. Indeed, we prove that no fragment of HA extending $i\Pi_1$ belongs to the range of $\mathcal{H}$.

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[2] S. Buss, *Intuitionistic validity in T-normal Kripke structures*, ***Annals of Pure and Applied Logic***, vol. 59 (1993), pp. 159–173.

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In this paper I will deliver soundness and completeness theorems for a free logic outfitted with an absolute probabilistic semantics. I will also argue that some of the philosophical problems free logic was designed to solve are better handled by free logic with a probabilistic semantics as opposed to free logic with other alternative semantical treatments. The paper will offer a brief history of the semantics for free logic and special attention will be given to the period marking the beginning of the development of probabilistic semantics.

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In our days several authors are studying the progressive evolution in K. Gödel's philosophical thought from the platonism, that the author defended since 1925 and which remained reflected in important works like The Gibbs Conference (1951) and What is Cantor's continuum problem? (1964), to a position close to phenomenology, mainly expressed in The modern development of the foundations of mathematics in light of philosophy (1961). The K. Gödel's phenomenological thought allows us to bring light on the limitation results, especially the incompleteness thoerems. This new research may consist in a clarification of the meaning of the mathematical concepts by directing our attention in a certain way, namely, onto our own acts in the use of these concepts and onto our powers in carrying out our acts.

The research that we carried out studies the possibility of supplementing the basic intuition of the order in the natural numbers, with new intuitions based on the distinguishability of the elements that constitute the set of the natural numbers. To carry this out, a phenomenological analysis of the act of enumerating the elements of a set will be presented. As a consequence of this analysis, several classical results will be exposed in a simpler form, especially the existence of non-standard models and their importance in order to explain the incompleteness theorems.

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► ANDREJA PRIJATELJ, *From bounded structural rules towards linear logic modalities*.
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In this paper, we shall present propositional classical logics with bounded structural rules. To be specific, for any $n \geq 2$, we shall fix an axiomatic system, CLL_n^n, where $2n$ copies of a formula may be contracted to n copies (i.e., n-contraction) and n copies of a formula may be weakened (i.e., n-weakening). Due to the restriction on structural rules we are, in fact, dealing with extensions of classical linear logic without the modalities. But can these logics be of any use? Our central goal is to define linear modalities in these systems. Let us recall that in classical linear logic a formula $!A$ (and, dually, $?A$) means a potential availability of a formula A to be used an arbitrary number of times (including zero). It turns out that taking $!A := A^n$ and $?A := nA$, with n copies of A being tensored and paired respectively, in CLL_n^n amounts to the fact that as soon as n copies of A are produced within a derivation any number of copies of A greater than n is available within the same derivation. Moreover, our definition of the two modalities is well-behaved also in the following sense. All the classical linear logic rules involving '!' and '?' (i.e., dereliction, promotion, contraction and weakening) are derivable in CLL_n^n with the two modalities being defined as above. Finally, our systems do enjoy the cut-elimination property. To achieve that, a more subtle concept of a sequent is to be taken as a basis of the calculus, the so-called classical-linear sequent with communication possible in both ways. To be specific, these are sequents of the form $\Pi|\Gamma \rightarrow \Delta|\Sigma$, where Π, Σ denote finite multisets of n copies each of formulas, and Γ, Δ stand for finite multisets of formulas in the usual language of linear logic without the modalities. The idea of such a sequent has its origin in Girard's unified logic. In this work, however, the outer classical part of the sequent (i.e., with n-copies of formulas) can just be viewed as an auxiliary reservoir of formulas that may be weakened, contracted and de-tensored and de-paired in order to switch (back) to the linear part (derivation) as modalized formulas.

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We say that a pair $\langle I, J\rangle$ of σ-ideals with Borel basis satisfies the *Fubini Property* (FP) if for every Borel subset D of the plane $X \times Y$, if all its vertical sections $D_x = \{ y : \langle x, y\rangle \in D \}$ are in J, then its horizontal sections $D^y = \{ x : \langle x, y\rangle \in D \}$ are in I, for every y outside a set from J. Let $F \subset P(\omega)$ be a proper filter containing all cofinite subsets. For $s \in [\omega]^{<\omega}$

and $A \in \mathcal{F}$ where $s < A$ we define

$$[s, A] = \{ X \in [\omega]^{\omega} : s \subset X \subset s \cup A \}.$$

Let

$$O_{\mathcal{F}} = \{ [s, A] : s \in [\omega]^{<\omega} \text{ and } A \in \mathcal{F} \}.$$

This is a base for a σ-centered (so CCC) topology $\tau_{\mathcal{F}}$ and let $\mathcal{M}_{\mathcal{F}}$ denote meager sets in $\tau_{\mathcal{F}}$. $\mathcal{M}_{\mathcal{F}}$ is a σ-centered (so CCC) σ-ideal with standard Borel basis on $[\omega]^{\omega}$.

We prove several results on Fubini properties for such ideals.

THEOREM 1. *Assume that* $\mathrm{cov}(\mathcal{M}) = \mathbf{c}$. *Then there is a non-countably generated filter* $\mathcal{F}$ *such that the pair* $(\mathcal{M}, \mathcal{M}_{\mathcal{F}})$ *has the Fubini property.*

THEOREM 2. *Assume Martin's Axiom and* $\mathbf{c} > \omega_2$. *There are non-countably generated filters* $\mathcal{G}, \mathcal{F}$ *such that the pair* $(\mathcal{M}_{\mathcal{G}}, \mathcal{M}_{\mathcal{F}})$ *has FP.*

Let $\mathcal{F}_0$ denote the filter on $\omega \times \omega$ generated by

$$\{ \omega \times \omega \setminus \{n\} \times \omega : n \in \omega \}$$

and by

$$\{ \{ (k, l) \in \omega \times \omega : f(k) \leq l \} : f \in \omega^{\omega} \}.$$

THEOREM 3. *Suppose that a filter* $\mathcal{G}$ *has the Baire property and a filter* $\mathcal{F}$ *contains the filter* $\mathcal{F}_0$. *Then there is a Borel subset D of the plane* $[\omega]^{\omega} \times [\omega]^{\omega}$ *with all its vertical sections in* $\mathcal{M}_{\mathcal{F}}$ *and with complements of all its horizontal sections in* $\mathcal{M}_{\mathcal{G}}$.

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The basic considerations touch on *categorical semantics* and its notions: truth and denotation, about which historical remarks will be given at the beginning.

The formalized theory *TCG* of categorical grammar enriched by a denotation operation as a semantic component (see [1], [2], and [3]) is the starting point in the present analysis. In *TCG*, every well-formed expression of language generated by categorical grammar and belonging to the syntactic category *CAT* with an index—either of names (proper or predicate ones) or name functions, sentences or sentence functions, name-forming functors, sentence-forming functors, or even quantifiers—has its denotation belonging to a respective ontological category *ONT* with the same index. In accordance with Frege's principle of categorical agreement (CA) the authors see the *adequacy of syntax with respect to semantics* which allows the identification of syntactic categories with semantic ones, understood as sets of expressions whose denotations belong to suitable ontological categories. In the semantic description of any categorical language, the authors use a referential model which, like the language itself, is treated as a partial algebra.

The definition of a true sentence appears as a crucial topic of the theory *TCG*. It is possible to formulate within it both kinds of definitions of a true sentence: *syntactic*, corresponding to Tarski's famous convention (T), and *semantic*, referring to the notion of denotation. It is also possible to show their adequacy on the basis of *TCG*. The authors also foresee the possibility of replacing the notion of satisfaction in Tarski's sense with the notion of denotation.

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Let $G(x)$ be a formula. A rooted Kripke model with root k is a G-*expansion* if for all $k' \succeq k$ and $d \in Dk'$ we have $d \in Dk$ or $k' \Vdash G(d)$.

We present a theory with completeness theorem for the class of G-expansion models. This result generalizes our results on end-extension Kripke models, and also generalizes well-known results on constant domain models, see [1] and [2].

We discuss applications to cofinal-extension models.

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► V. V. RYBAKOV, *Kripke completeness with respect to admissible rules*.
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It has been proved in [1] that all modal logics of width greater than 3 and possessing the co-cover property do not have the finite model property with respect to admissible rules. Therefore the majority of well-recognized modal logics fail to have the finite model property with respect to admissibility. Here we would like to look at Kripke completeness with respect to admissible rules.

Definition 3. Given a logic L, we say L is Kripke complete by admissibility with respect to a class of frames $\mathcal{K}$ if for any rule r which is not admissible in L, there is a frame $F \in \mathcal{K}$ such that all rules admissible in L are valid in F but $F \nvDash r$.

For other definitions we need we refer to [2].

Theorem 1.

(i) *If a modal logic L extending $K4$ has the co-cover property and the width at least* 4 *then L is Kripke incomplete by admissibility with respect to any class of frames without infinite ascending chains and without infinite antichains.*

(ii) *Any superintuitionistic logic L of width at least* 4 *with the co-cover property is not Kripke complete by admissibility with respect to any class of posets without infinite ascending chains and without infinite antichains L.*

It is reasonable to ask whether Kripke completeness holds at all even for certain well-recognized logics. As an answer we have

Theorem 2.

(i) *Modal logics $K4$, $S4$, $S4.1$, $S4.2$, Grz, GL are Kripke complete by admissibility with respect to certain classes of rooted countable Kripke frames without infinite ascending* *chains.*

(ii) *Superintuitionistic logic IPC, KC are Kripke complete by admissibility with respect to certain classes of rooted countable posets without infinite ascending chains.*

THEOREM 3.

(i) *Any modal logic above $K4$ of finite depth and of width at least* 3 *is Kripke incomplete by admissibility with respect to any class of rooted frames.*

(ii) *Any superintuitionistic logic with finite depth with width at least e is Kripke incomplete by admissibility with respect to any class of rooted posets.*

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We axiomatize two important intermediate logics, classifying end-extension Kripke models and cofinal-extension Kripke models. As applications, we show that Heyting Arithmetic, HA, is complete with respect to the class of its end-extension Kripke models and every cofinal-extension Kripke model of HA is PA-normal.

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The logic treated here is the intuitionistic modal logic obtained from the smallest intuitionistic modal logic **IntK** by adding the axioms T_c: $a \supset \Box a$ and 4_c: $\Box\Box a \supset \Box a$. This logic was considered in Benton, Bierman and de Paiva [1], Fairtlough and Mendler [3] and Goldblatt [4]. [1] described that the logic corresponds to the computational typed lambda calculus introduced in Moggi [6] by the Curry-Howard isomorphism. They gave a natural deduction system for the logic and proved the strong normalization theorem. [3] treated it as the logic with applications to the formal verification of hardware. In [4], the logic was introduced as the logic having the interpretation "*locality*".

Here we discuss the set of formulas constructed from the propositional variables $p_1, \ldots, p_n$ and the constant $\perp$ using $\supset$, $\wedge$ and $\Box$ in the intuitionistic modal logic. The set of these non-modal formulas was already considered in Diego [2]. He showed that the set of such non-modal formulas contains only finitely many pairwise non-equivalent in intuitionistic propositional logic. Urquhart [7] and Hendriks [5] gave more precise descriptions about such sets of non-modal formulas. We extend their results into the intuitionistic modal logic. Normal forms of disjunction-free formulas with finite fixed propositional variables are also given.

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► PAVEL SCHREINER, *Fragment of predicate intuitionistic logic complete under Kripke frames with finite domains*.
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It is well known that neither classical predicate logic nor intuitionistic predicate logic are complete under Kripke frames with finite domains. Moreover, by transition to models with finite domains, these logics lose many of the "good" properties, for example, such as interpolation and the Beth property ([1] and [3]).

In this paper, we prove that the fragment of predicate intuitionistic logic without disjunction and existential quantifier is complete under Kripke frames with finite domains.

THEOREM 1. *If a formula F contains no disjunction and existential quantifier then the following are equivalent*:

(1) *F is provable in intuitionistic predicate logic,*
(2) *F is valid in all Kripke frames with finite domains.*

For proving Theorem 1, some ideas of Komori [2] are applied to standard Kripke frames for predicate intuitionistic logic.

We say that a logic L has the *weak interpolation property* if, for any formula $(A \to B)$ in L, there exists a formula C such that formulas $(A \to C)$ and $(C \to B)$ belong to L, and C contains only those predicate symbols which occur both in A and B.

THEOREM 2. *The fragment of predicate intuitionistic logic without disjunction and existential quantifier has the weak interpolation property and the Beth property.*

We note that it contrasts with failure of the interpolation and the Beth properties for the predicate logic characterized by all intuitionistic Kripke frames, all of whose domains are finite, in the language with disjunction and existential quantifier [4].

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Some hierarchies of regular star-free languages have been studied by other authors. E.g., J. Brzozowski and R. Knast have shown that the dot-depth hierarchy (based on counting nests of concatenations and Boolean operations) is infinite. W. Thomas [2] found out the connection of this hierarchy with the so-called *quantifier hierarchy* $\{S_k\}_{(k<\omega)}$ (based on counting the number of alternations of quantifiers $\exists$ and $\forall$ in related prenex form formulas). In particular, the fact that one of these hierarchies is infinite implies that another one is infinite too. Later on, W. Thomas applied the Ehrenfeucht-Fraïssé games method to prove directly that the quantifier hierarchy is infinite and gave a new proof of the infinity of the dot-depth hierarchy.

We study a hierarchy $\{D_\alpha\}_{(\alpha<\omega^2)}$ of regular star-free languages that refines the quantifier hierarchy. This refined hierarchy consists of levels of difference hierarchies [1] over each S_k $(k < \omega)$. Let $D_{\omega k+p}$ $(k, p < \omega)$ be the set of all languages $\bigcup_i (L_{2i} \setminus L_{2i+1})$ where $L_i \in S_k$ satisfy $L_0 \supseteq L_1 \supseteq \dots$ and $L_{p+1} = \emptyset$. The sequence $\{D_{\omega k+p}\}_{p<\omega}$ is the difference hierarchy over S_k. For a class of languages S, let $\check{S} = \{ \bar{A} \mid A \in S \}$ denote the dual class for S. Our main result is that this hierarchy is nontrivial, namely

The classes D_α and $\check{D}_\alpha$ do not coincide for all $a < \omega^2$.

It follows immediately that the difference hierarchies over each level of the quantifier hierarchy are infinite, generalizing the infinite of the quantifier hierarchy, namely, inclusions $D_\alpha \cup \check{D}_\alpha \subset D_\beta$ are strict for all $\alpha < \beta < \omega^2$.

The analog of this result for ω-languages holds.

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In [1] Petr Hajek urges us to ‘strongly distinguish fuzziness from uncertainty as a degree of belief (e.g., probability)’ (p. 2). Drawing upon my work in [2], I will examine the distinction as it is formalized in the semantics of Fuzzy Logic and Probability Logic.

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I show in this paper that there is a uniform solution of the paradoxes of Self-Reference, but what has to be given up is not the Law of Non-Contradiction, as some believe; instead it is the Tractarian demand for determinateness of sense which must go. The point is illustrated with the Heterologicality Paradox (see [1, p. 347]). Copi derives the contradiction, but he makes the one telling assumption—that "Het" is univocal—without proceeding to derive its negation using the Reduction which is available. A similar need just to allow for ambiguity, to solve the Liar and related Paradoxes, was exposed in [2]. And that also solves Curry's Paradox, which some classify differently. With Berry's Paradox one must remember non-attributive referential terms, so "the least number definable in less than 19 words" does not need to refer to a least number definable in less than 19 words, making its reference indeterminate. And the presupposition omitted from before the Abstraction Scheme, whose denial provides an escape from the set-theoretic paradoxes, is that the ensemble determined by "P" has a determinate number of elements, as it doesn't if "P" is a mass term [4, pp. 68 and 151]. Tiles realises, with Cantor, that this solves Cantor's Paradox, but she does not look at Cantor's diagonal procedure in the same light [4, pp. 115–116]. Properly seen, however, that involves just another self-referential paradox, to be eliminated in the same way as above [3, p. 7]

[1] I. M. Copi, ***Symbolic logic***, 3rd ed., Macmillan, New York, 1967.

[2] C. Sayward, *Prior's theory of truth*, ***Analysis***, vol. 47 (1987), pp. 83–87.

[3] B. H. Slater, *Some Wittgensteinian theses proved*, ***Wittgenstein Studien***, vol. 2-94 (1994), pp. 1–11.

[4] M. Tiles, ***The philosophy of set theory***, Blackwell, Oxford, 1989.

► VLADIMIR SOTIROV, *Monadic predicate calculus arithmetized à la Leibniz*.
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A formula of the monadic predicate calculus without free variables is called a *monadic proposition*. For such formulae we adopt Theorem 25.4 from [1]:

Lemma. *Any monadic proposition is equivalent to a monadic proposition with the same predicate symbols and one variable only.*

Let the sole variable be x. We may suppose that no formula is a Boolean combination of two subformulae $A(x)$ and B, one of them containing a *free* x and the other containing x *bound*; in $(Qx)\ A(x)$, a free x does occur in A.

Let $u > 1$ be an arbitrary integer *without multiple factors*, and let its divisor d_i be associated with the predicate $P_i(x)$. Given a monadic proposition, *divisors* of u are associated with subformulae containing a free x, and *statements* about divisors are associated with subformulae which do not contain x free: if a and b are associated with $A(x)$ and $B(x)$, then g. c. d.(a, b) is associated with $A(x)\ \&\ B(x)$ and u/a with $\neg A(x)$. The statements $a = u$ and $a > 1$ are associated with $(\forall x)\ A(x)$ and $(\exists x)\ A(x)$. If statements p and q are associated with subformulae A and B, then "p and q" and "not p" are associated with $A\ \&\ B$ and $\neg A$. If the final statement is true for an arbitrary integer u, the initial proposition is called *arithmetically true*.

Theorem. *Any monadic formula is a predicate tautology if and only if its closure is arithmetically true.*

[1] G. Boolos and R. Jeffrey, ***Computability and logic***, 3rd ed., Cambridge University Press, 1989.

► KATRIN TENT, *BN-pairs of finite Morley rank*.
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The Cherlin-Zil'ber Conjecture, viz. that a simple group of finite Morley rank is an algebraic group over an algebraically closed field, has been the starting point for much research in model theory. We report on recent progress made on groups with BN-pairs using their connections to generalized polygons.

THEOREM 1 ([1]). *There are many almost strongly minimal n-gons for all n not interpreting an infinite group whose automorphism group acts transitively on the set of ordered ordinary* $(n + 1)$*-gons*.

The automorphism group of such a strongly homogeneous polygon thus has a BN-pair, which, of course, cannot be definable. The construction is similar to that of the non-desarguesian projective planes constructed by Baldwin. In contrast to these examples we have the following result:

THEOREM 2 ([2]). *Let G be a group of finite Morley rank acting definably and effectively on a generalized n-gon $\mathcal{P}$ whose point rows and line pencils are strongly minimal. If G acts transitively on the set of ordered ordinary n-gons contained in $\mathcal{P}$, then G is isomorphic to* $PSL_3(K)$, $PSp_4(K)$ *or* $G_2(K)$ *for some algebraically closed field* K.

This implies the Cerlin-Zil'ber Conjecture for buildings with panels of Morley rank 1.

[1] K. TENT, *Very homogeneous generalized n-gons of finite Morley rank*, to appear in the ***Journal of the London Mathematical Society***.

[2] K. TENT and H. VAN MALDEGHEM, *Cherlin's conjecture for buildings with panels of Morley rank 1*, preprint.

► DMITRY TISHKOVSKY, *Interpolation and Beth properties of superintuitionistic predicate and intuitionistic polymodal logics*.
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As it was shown in [1], for each superintuitionistic predicate logic L there exists an intuitionistic polymodal propositional logic ML such that every theorem of ML is a metatheorem of L.

The question arises: is there any correlation of well-known properties of predicate logics with the corresponding properties of polymodal logics?

In the paper we discuss the deducibility interpolation property (IPD), implicative interpolation property (IP), the Beth property (B2) and the projective Beth property (PB2) (see [2]) of the logics mentioned above.

The main theorem of the paper is

THEOREM 1. *Let L be a superintuitionistic predicate logic, ML be a polymodal propositional logic corresponding to L. If ML has* IPD (IP, B2, PB2) *then L possesses the same property.*

Using known results on superintuitionistic predicate logics we get the following corollaries:

COROLLARY 1. *The family of intuitionistic polymodal logics without the Beth property has the cardinality of the continuum.*

COROLLARY 2. *For each superintuitionistic propositional logic L having the interpolation property there exist infinitely many conservative polymodal extensions of L without* IPD.

[1] D. E. TISHKOVSKY, *On algebraic semantics for superintuitionistic predicate logics*, ***Algebra i Logika***, vol. 38 (1999), no. 1, pp. 68–95, in Russian.

[2] L. MAKSIMOVA, *Explicit and implicit definability in modal and related logics*, ***Bulletin of the Section of Logic***, vol. 27 (1998), no. 1/2, pp. 36–39.

► SERGEI TUPAILO, *Realizability interpretation into explicit mathematics*.
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Systems of Explicit Mathematics were introduced by S. Feferman in the 1970s as a foundation for constructive mathematics (see [1] and [2]). In [2] he gave an embedding of the theory T_0 into a subsystem of second order arithmetic $\Delta^1_2 - CA + BI$. In [3] G. Jäger established proof-theoretic equivalence of T_0 with $\Delta^1_2 - CA + BI$. However, a question about *direct embeddings of strong conventional theories* (e.g., analysis or set theory) into Explicit Mathematics was not answered.

We show that *this can be solved in the intuitionistic setting by an appropriate realization interpretation*.

For each formula F of an intuitionistic theory T in question we define a realization formula $x \underline{\mathrm{rn}} F$ in the language $\mathcal{L}_{\mathrm{EM}}$ of Explicit Mathematics, and also a translation $\mathcal{I}\colon T \mapsto \mathcal{T}$, $\mathcal{I}(F) := \exists x\ (x \underline{\mathrm{rn}} F)$, where $\mathcal{T}$ is a theory in $\mathcal{L}_{\mathrm{EM}}$. We call a formula F *realizable in* $\mathcal{T}$ if and only if there exsits a term t with $\mathrm{FV}(t) \subset \mathrm{FV}(F)$ such that $\mathcal{T} \vdash t \underline{\mathrm{rn}} F$. Then we prove the following theorems.

THEOREM 1. *Axiom of Choice of second order arithmetic*

$$\forall x\ \exists Y\ F[x, Y] \rightarrow \exists Y\ \forall x\ F[x, Y_x]$$

is realizable in EETJ.

THEOREM 2. *Each theorem of the subsystem* IARI *of second order arithmetic is realizable in* T_0.

(In [4] it's shown that IARI has the same proof-theoretic strength as T_0.)

THEOREM 3. *Each theorem of intuitionistic Constructive Set Theory* CZF+REA *is realizable in* $T_0 + \mathrm{Lim}$.

THEOREM 4. *If F is a first-order negative formula, then* $T \vdash F \Longrightarrow \mathcal{T} \vdash \mathrm{EM}(F)$, *where* $\mathrm{EM}(F)$ *is a natural translation of F into the language of Explicit Mathematics.*

[1] S. FEFERMAN, *A language and axioms for explicit mathematics*, in ***Algebra and logic***, Lecture Notes in Mathematics, no. 450, Springer-Verlag, Berlin, 1975, pp. 87–139.

[2] ———, *Constructive theories of functions and classes*, in ***Logic colloquium '78***, 1979, pp. 159–224.

[3] G. JÄGER, *A well-ordering proof for Feferman's theory* T_0, ***Archive for Mathematical Logic***, vol. 23 (1983), pp. 65–77.

[4] E. GRIFFOR and M. RATHJEN, *The strength of some Martin-Löf type theories*, ***Archive for Mathematical Logic***, vol. 33 (1994), pp. 347–385.

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George Boolos ([1] and [2]) developed a plural interpretation of second-order set theory, which interpreted the second-order quantifier "$\exists X$" as a plural quantifier over the first-order domain of sets.

In this paper, we investigate the possibility of using the resources of plural quantification and plural predication to interpret both two- and one-sorted, first-order theories of classes. A case study is Kelley's original formalization of the theory of classes in a first-order language containing only variables for classes in [3]. Its plural interpretation regards its variables as plural variables which range over a domain of sets, the quantifier "$\exists x$" as a plural quantifier, and, finally, the predicate "$\in$" as a plural predicate which denotes a relation some sets bear to some other sets just in case the former are all and only those sets which are members of one of the latter. We can then define a new predicate: "*x is represented*" by : $\exists y\ (x \in y)$—very roughly, some sets are represented just in case there is a set which contains all and only those sets as members. And we can show that this interpretation verifies all of the axioms of Kelley's theory of classes, when the predicate "*is a set*" is replaced by "*is represented*."

We will argue that the plural interpretation of impredicative theories of classes has a number of advantages over other, more traditional accounts of the set-class distinction, which is often motivated by a distinction between a "logical" and a "combinatorial" conception of collection. We will see, in particular, that plurals enable us to simulate "logical" collections without threatening the universality of set theory, which we can still regard as the most comprehensive theory of collections.

[1] G. Boolos, *To be is to be the value of a variable (or some values of some variables)*, ***Journal of Philosophy***, vol. 81 (1984), pp. 430–450.

[2] ———, *Nominalist Platonism*, ***The Philosophical Review***, vol. 94 (1985), pp. 327–344.

[3] J. L. Kelley, ***General topology***, Van Nostrand, London, 1955.

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Interpretability logic IL is a natural extension of provability logic GL (Gödel-Löb). It is (non-standard) propositional modal logic with two modal operators: $\Box$ (unary) and $\rhd$ (binary). A. Visser gave axioms for IL which express the basic property of interpretability relation.

We are only interested in IL as a system of modal logic. We study generalized Veltman models as defined by D. de Jongh.

S. Feferman proved the generalization of Gödel's second incompleteness theorem, i.e., the formula Cons (which expresses the consistency of Peano arithmetic) is not interpretable in PA. Feferman's principle F, i.e., $(A \rhd \Diamond A) \to \Box(\neg A)$ is a modal description of Feferman's theorem.

We determined the characteristic class of generalized Veltman frames of the principle F. Let $(W, R, \{ S_w : w \in W \})$ be $\mathrm{IL_{set}}$-frame and $w \in W$. We denote with $\overline{S_w}$ and $\overline{R_w}$ the following relations:

- for $\emptyset \neq A \subseteq W[w]$ and $\mathcal{B} \subseteq \mathcal{P}(W[w]) \setminus \{\emptyset\}$ is valid
$$A\, \overline{S_w}\, \mathcal{B} \iff (\forall a \in A)\,(\exists B \in \mathcal{B})\,(a\, S_w\, B);$$
- for $\mathcal{C} \subseteq \mathcal{P}(W[w]) \setminus \{\emptyset\}$ and $\emptyset \neq D \subseteq W[w]$ is valid
$$\mathcal{C}\, \overline{R_w}\, D \iff (\forall C \in \mathcal{C})\,(\forall c \in C)\,(\exists d \in D)\,(c\, R\, d).$$

We denote by $(F)^*$ the following property of $\mathrm{IL_{set}}$-frame:

the relation $\overline{S_w} \circ \overline{R_w}$ is reverse well-founded for all $w \in W$.

Proposition. *Let* $\mathbf{W}$ *be an* $\mathrm{IL_{set}}$*-frame. We have*

$$\mathbf{W} \vDash F \qquad \text{if and only if} \qquad \mathbf{W} \textit{ satisfies } (F)^*.$$

The principles F and W have the same characteristic class of (ordinary) Veltman frames. We proved that the principle F and W have different characteristic classes of $\mathrm{IL_{set}}$-frames.

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(Joint work with Ludomir Newelski.)

O-minimal and quasi o-minimal Boolean algebras were introduced by C. Toffalori in [1]. He raised two questions:

1. Is every quasi o-minimal Boolean algebra o-minimal?
2. Are prime models unique up to isomorphism in a complete theory T of o-minimal Boolean algebras?

We show that every atomless Boolean algebra has weak elimination of imaginaries. We give a description of prime models of T over arbitrary sets or parameters. We show also that the algebraic closure has some exchange properties. As a consequence we answer positively both questions of C. Toffalori.

[1] CARLO TOFFALORI, *Lattice ordered o-minimal structures*, ***Notre Dame Journal of Formal Logic***, to appear.

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The idea of describing the intended Brouwer-Heyting-Kolmogorov semantics for intuitionistic logic by means of the modal logic of classical provability was suggested by Gödel. He introduced the modal logic of provability $S4$ and defined intuitionistic logic inside $S4$. However, the problem of finding the provability interpretation for $S4$ itself was left open. In particular, the studies of the modal logic complete under the straightforward interpretation of the modality $\Box$ as the provability formula in arithmetic led to the logic GL (see [3]) incompatible with $S4$.

The above problems were solved in [1], where Artemov found the complete axiom system for the logic of proofs $\mathcal{LP}$ in the format "t is a proof of F" and showed that $\mathcal{LP}$ was sufficient to emulate the whole $S4$. Proof terms in $\mathcal{LP}$ are constructed by three elementary operations on proofs.

Our goal was to develop the joint logic that incorporates both the modality "F is provable" and the proof operator "t is a proof of F".

We find the natural axiomatization $\mathcal{MLP}$ of the minimal complete system that includes both GL and $\mathcal{LP}$. The presence of $\Box$ in the language of $\mathcal{MLP}$ requires two new elementary operations on proofs. We show that $\mathcal{MLP}$ suffices to realize all operations on proofs admitting description in the modal propositional language. We also find the complete answer to the question of what versions of the interpolation property hold in $\mathcal{LP}$.

[1] S. ARTEMOV, *Operational modal logic*, Technical Report no. 95–29, Mathematical Science Institute, Cornell University, 1995.
[2] T. SIDON, *Provability logic with operations on proofs*, Lecture Notes in Computer Science, vol. 1234, Springer-Verlag, 1997, pp. 342–353.
[3] R. SOLOVAY, *Provability interpretation of modal logic*, ***Israel Journal of Mathematics***, vol. 25 (1976), pp. 287–304.

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Let T be a first order theory. A first order formula A is called *T-valid* if for every interpretation f of the predicate language into the language of T the formula $f(A)$ is provable in T. *The first order logic $\mathcal{L}(T)$ of a theory T* is the set of all T-valid formulas.

The logics of intuitionistic theories were studied in [2]. Our goal was to study logics $\mathcal{L}(T)$ for different classical first order theories T.

THEOREM 1. *Let PC denote the predicate calculus. Then $\mathcal{L}(T) = PC$ for the following theories: the group theory, the theory of rings, the theory of fields, and all arithmetically correct theories in the entire arithmetical language.*

Our proof in the case of the group theory makes use of recent results by O. V. Belegradek concerning the unitriangular groups of matrices [1].

Let *Pre* stand for Presburger's arithmetic of addition, let *Sko* be Skolem's arithmetic of multiplication, let DO be the theory of the order type of natural numbers and let $\mathcal{L}fin$ be the logic of finite models.

THEOREM 2. *The following proper inclusions hold*:

$$PC \subset \mathcal{L}(Sko) \subset \mathcal{L}(Pre) \subset \mathcal{L}(DO) \subset \mathcal{L}fin.$$

THEOREM 3. *The first order logic of the theory of equality is decidable but cannot be axiomatized by any set of schemes of the restricted arity. The same holds for the logic of the theory of dense linear order without minimal and maximal elements.*

[1] O. V. BELEGRADEK, *Model theory for the unitriangular and existentially closed groups*, ***Doctoral Thesis***, Novosibirsk, 1995, in Russian.

[2] V. E. PLISKO, *Constructive formalization of the Tennenbaum theorem and its applications*, ***Matematicheskie zametki***, vol. 48 (1994), no. 4, pp. 108–118, in Russian.

[3] R. E. YAVORSKY, *Logical schemes for first order theories*, Lecture Notes in Computer Science, vol. 1234, Springer-Verlag, 1997, pp. 410–418.

Abstracts presented by title

► V. N. GLUSHKOVA, *Logical specifications as productions for transformation of program graphs*.
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In Sigma-programming concept had been introduced many-sorted models with list superstructures and Δ_0-formulas that use only bounded-quantifier variables whose sorts belong to sets I and {list} [2]. We have suggested to treat both I as alphabet of CF-grammar, and superstructure as CF-list set. In [1] the $\Delta_0 T$-formulas had been distinguished whose prefix has "tree" structure:

$$(\forall x_1 \mathrel{\dot\in} t_1)\ldots(\forall x_m \mathrel{\dot\in} t_m)\,\varphi(\bar{x},\bar{t}) \to \psi(\bar{x},\bar{t}). \tag{1}$$

Here $\dot\in$ is relation $\in$ or $\overset{\star}{\in}$—reflexive transitive closure of $\in$; φ, ψ are the conjunctions of atomic formulas r, $t_1 = t_2$ (r—predicate; t_1, t_2—terms of the certain signature Σ) or their negations.

The parsing tree is identified with its root and can be presented as some CF-list in superstructure. The functions are attribute defined on tree nodes. The predicates are edges' labels which connect the suitable nodes. Treating (1) as productions, it is possible to apply (1) to obtain a graph of context relations of a program from its parsing tree. A similar method for description of context syntax has been offered in [3].

A choice of suitable computable model semantics is the key point for this formula's class. Keeping the specific strategy for a constant's substitution into productions (1) and the conception of weak equality one can construct the initial inductive model from signature's constants for those formulas, which possess of confluentness and Noetherian property. Taking into account the closeness of attributive grammars and $\Delta_0 T$-formulas it is possible to formulate the syntactical conditions for finite completeness of the computations that can be expressed in terms of various strategies of tree's circuits.

The $\Delta_0 T$-formulas are realized not worse than attribute grammars, in particular with respect to context analysis' task of programming languages. Moreover their application permits us to estimate the time complexity of context analysis.

[1] V. N. GLUSHKOVA, *Estimating the execution complexity of logical specifications based on context-free grammars*, ***Kipernetika i Sistemnyi Analiz***, vol. 4 (1996), pp. 50–58.

[2] S. S. GONCHAROV and D. I. SVIRIDENKO, *Theoretical aspects of Sigma-programming*,

Mathematical Methods of Specification and Synthesis of Software Systems, vol. 85 (1985), pp. 169–179, Proceedings of the International School.

[3] O. Odersky, *Defining context-dependent syntax without using context*, ***Transaction Programming Languages and Systems***, vol. 15 (1993), no. 3, pp. 535–562.

► TIMOTHY J. HERRON, *An analysis of de Finetti's exchangeability theorem in second order arithmetic*.
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We participate in the logical program of classifying theorems of mathematics using canonical subsystems of second-order arithmetic; the program led by S. Simpson and named "reverse math" [1]. However, instead of analyzing core existence results in central branches of mathematics, we propose to apply Simpson's methodology to the mathematics of statistics: both in order to understand the non-constructive content of mathematics and statistics, and as a test of reverse mathematics. Thus, we characterize the logical requirements of a simple version (and some extensions) of de Finetti's exchangeability theorem. We first review X. Yu's coding of finite measure theory into second-order arithmetic; we show that her coding is for the most part a reasonable platform on which to classify de Finetti's theorem. By modifying an old proof of T. H. Hildebrandt, we find that the theorem is provable within the base subsystem RCA_0. This appears surprising because in the practice of ordinary mathematics, de Finetti's theorem is equivalent to a simple version of the Riesz representation theorem. We explain how this latter equivalence also holds, in some sense, in Yu's coding—thus dissolving our surprise. Finally, we discuss why very general de Finetti-style theorems are not like to be proven within Yu's framework for measure theory.

To conclude the talk, we will note that the theorem classifications discussed above are a bit deceptive when interpreted as fixing the roles of de Finetti's theorem and Riesz's theorem within statistics and finite measure theory, respectively. In particular, both theorems are usually endowed with more non-constructive content than they contain within Yu's coding of finite measure theory. We also make some general comments about coding mathematics into second order arithmetic, and how certain features of codings reflect upon the philosophical claims made by Simpson on behalf of the reverse mathematics program.

[1] Stephen Simpson, ***Subsystems of second order arithmetic***, Perspectives in Mathematical Logic, Springer-Verlag, Berlin, 1999.

► ALEXANDRE IVANOV, *Unimodular strongly minimal structures of bounded valency*.
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Let W be an infinite structure of a finite relational language L. We say that W is of bounded valency if there is a natural number n such that for any $r(\bar{x}) \in L$ and $a \in W$ the number of all realisations of $r(\bar{x})$ containing a is bounded by n. It is shown in [2] that non-ω-categorical strongly minimal structures with disintegrated acl arise from transitive, strongly minimal structure of bounded valency, where the algebraic closure of a point is infinite.

We study such strongly minimal structures M with the property that for any $\bar{a}, \bar{b} \in M$ if $\mathrm{acl}(\bar{a}) = \mathrm{acl}(\bar{b})$ then $\mathrm{mult}(\bar{a}/\bar{b}) = \mathrm{mult}(\bar{b}/\bar{a})$. They form a proper subclass of the class of unimodular structures (see [1]).

Any transitive strongly minimal structure of bounded valency M as above is described as follows. *There exists an infinite finitely generated group G and a finite cover $\pi\colon W \to \Gamma_G$ onto the Cayley graph (coloured) of G such that W and M are interdefinable.*

We say that a group G has a *threading tuple* if there are $g_1, \ldots, g_n \in G$ such that for any $g \in G \setminus \{1\}$, $\langle g \rangle \cap g_i^G$ is non-trivial for some $i \leq n$. The results below are motivated by the observation that in the situation above if W satisfies acl $=$ dcl then W is finitely axiomatisable if and only if Aut(W) is a finitely presented group having a threading tuple.

Results. Let $\pi \colon W \to \Gamma_G$ be as above. Then:

- if W is finitely axiomatisable then G is not virtually nilpotent (this is connected with the fact that G is periodic if G has a threading tuple and satisfies the ascending chain condition for subgroups);
- if W has a subexponential growth and G has a threading tuple then G is periodic;
- Aut(W) has fixed points under any action on a tree without inversions if and only if Ker(π) is not the union of a countable increasing chain of proper subgroups;
- if Ker(π) is finite then W does not have the finite model property if and only if there exist a finitely presented group G_1, a surjective homomorphism $\rho \colon G_1 \to G$ and a finite set $\Sigma \subset G_1$ such that Ker$(\rho) \cap \Sigma = \emptyset$ and any normal subgroup of G_1 of finite index intersects Σ.

[1] E. Hrushovski, *Unimodular minimal structures*, ***Journal of the London Mathematical Society***, vol. 46 (1992), pp. 385–396.

[2] A. Ivanov, *Strongly minimal structures with disintegrated algebraic closure and structures of bounded valency*, in ***Proceedings of the tenth Easter conference on model theory*** (M. Weese and H. Wolter, editors), Berlin, 1993, pp. 64–69.

► CYRUS F. NOURANI, *Infinite multiplayer game trees*.
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The 1994 on techniques and theory of computing with intelligent trees, e.g., [1], are applied to define intelligent game trees. The author had presented a chess-playing basis to the AAAI-97. For the intelligent game trees and the problem-solving techniques defined the same model can be applied to the game trees in the sense of two-person games and to the state space from the single-agent or player view. The two-person game tree is obtained from the intelligent tree model, as is the state space tree for agents. To obtain the two-person game tree the cross-board-coboard agent computation is depicted on a tree, whereas the state-space tree for each agent is determined by the computation sequence on its side of the board-coboard. Thus a tree node m might be $f(a_1, a_2, a_3)$ & $g(b_1, b_2)$, where f and g are Boolean functions of three and two variables, respectively, and a_is and b_is are Boolean valued agents satisfying goal formulas for f and g. A tree game degree is the game state a tree is at with respect to a model truth assignment, e.g., to the parameters to the Boolean functions above. Let generic diagrams, or G-diagrams, be diagrams definable by specific functions. Intelligent signatures [1] are signatures with designated multiplayer game tree function symbols.

Theorem 1. *Let T be a theory on an intelligent signature language L. T is*

(a) *a sound logical theory if and only if every axiom or proof rule in T preserves tree game degree*;

(b) *a complete logical theory if and only if there is a* ⟨function, set⟩ *pair defining a canonical structure C and a G-diagram, such that C has a generic diagram definable model.*

Let N be the set of all functions from ω to ω. Let A be a subset of N. Gale-Stewart 1953 associated with A a two-person game of perfect information $G(A)$. Player I begins by choosing n_0 in ω; player II chooses n_1 in ω; then player I chooses n_2 in ω; and so on. Let $a(i) = n_i$. Player I wins $G(A)$ if and only if a is in A. $G(A)$ is determined if one of the players has a winning strategy.

PROPOSITION. *If $G(A)$ is determined, the complexity upper bound on the number of moves to win is A's cardinality.*

THEOREM 2. *For every pair p of opposing agents there is a set $A(p)$ a subset of N. The worst-case bound for the number of moves for a determined game based on the intelligent game tree model is the sum*

$$\sum\{\,|A\langle p\rangle| : p \text{ agent pairs}\,\}.$$

PROOF. Sum over the proposition. $\dashv$

[1] C. F. NOURANI, *Intelligent languages—A preliminary syntactic theory*, ***Mathematical foundations of computer science, Brno, Czech Republic***, Springer-Verlag, Berlin, 1998, p. 1450.

► CARLO TOFFALORI AND STEFANO LEONESI, *Weakly o-minimal expansions of Boolean algebras*.

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In [1], it was discussed a notion of o-minimality for lattice ordered structures, in particular for expansions of Boolean algebras.

A structure $\mathcal{A} = (A, \leq, \dots)$ of this kind is called quasi o-minimal if every definable (1-ary) subset S is a finite Boolean combination of sets defined by conditions $v \leq a$ or $b \leq v$ with $a, b \in A$, and o-minimal if, when S is X-definable, then a, b can be chosen in the algebraic closure of X.

O-minimal expansions of Boolean algebras include some interesting examples (for instance, Boolean algebras with finitely many atoms) and enjoy some nice model theoretic properties (part of them is already proved in [1], and several others were discovered by Wencel and Newelski).

Here we discuss weak o-minimality in this setting. A linearly ordered structure $\mathcal{A} = (A, \leq, \dots)$ is called weak o-minimal if every definable S is a finite Boolean combination of convex subset. This definition can be easily extended to expansions of Boolean algebras. But in the lattice ordered context, there are two possible ways of defining convexity: We might consider convex sets, or prefer to deal with convex sublattices. It turns out that the latter option is the right one. In fact, every Boolean algebra is weakly o-minimal in the former sense, while, according to the latter one, weak o-minimality just equals o-minimality (and quasi o-minimality) in this setting. On the other hand, there do exist weakly o-minimal expansions of Boolean algebras which are not o-minimal (for instance look at a structure $(\mathcal{A}, \mathcal{U})$ where $\mathcal{A}$ is an o-minimal Boolean algebra, and $\mathcal{U}$ is a non-principal ultrafilter). We shall discuss these, and further, properties in this setting.

[1] CARLO TOFFALORI, *Lattice ordered o-minimal structures*, ***Notre Dame Journal of Formal Logic***, to appear.

► ZHOU XUNWEI, *Brief introduction to mutually-inversistic logic*.

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Mutually-inversistic logic is constructed by the author. The reason to construct it is that the author found out that classical logic has a more serious defect than the material implicational paradox. That is, seen from the definition of $\rightarrow$, Table 1, $\rightarrow$ cannot be used to make hypothetical inference.

Table 1

A	B	$A \to B$
F	F	T
F	T	T
T	F	F
T	T	T

Table 2

A	B	$A \overset{-1}{<} B$
F	F	T
F	T	T
T	F	F
T	T	T

Table 3

$A \overset{-1}{<} B$	A	B
F	F	F/T
F	T	F/T
T	F	F/T
T	T	T

Table 4

$A \overset{-1}{<} B$	B	A
F	F	F/T
F	T	F/T
T	F	F
T	T	F/T

Hypothetical inference is from the major premise $A \to B$ being true and the minor premise A being true to infer the consequence B being true. Whereas Table 1 is from the truth values of A and B to determine the truth value of $A \to B$, just converse to the direction of hypothetical inference.

For this reason, $\overset{-1}{<}$ (mutually inverse proper implication) in mutally-inversistic logic is defined as Tables 2, 3, and 4. Table 2 is the same as Table 1, this means mutually-inversistic logic inherits from classical logic. "F/T" in Tables 3 and 4 means "unable to determine whether it is true or false". Table 3 tells us that if the major premise $A \overset{-1}{<} B$ is true and the minor premise A is true then the consequence B is true, otherwise we can't determine the truth value of B. Therefore Table 3 describes the affirmative expression of hypothetical inference. Likewise, Table 4 describes the negative expression of hypothetical inference. Table 2 is mutually inverse with Tables 3 and 4.

[1] A. G. Hamilton, ***Logic for mathematicians***, Cambridge University Press, Cambridge, 1978.