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# REVIEWS

The Association for Symbolic Logic reviews selected books and articles in the field of symbolic logic. The reviews were published in ***The journal of symbolic logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Council of the Association decided that beginning in 2000, the Reviews Section would move to this BULLETIN.

The Reviews Section is edited by Herbert Enderton (Coordinating Editor), Matthew Foreman, Gerhard Jäger, Penelope Maddy, and Philip Scowcroft. Authors and publishers are requested to send, for review, copies of books to *Herbert Enderton, Association for Symbolic Logic, Mathematical Sciences Bldg. 7332, UCLA, Los Angeles, California 90095-1566, USA*.

In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the JOURNAL, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VI 94" refers to the review beginning on page 94 in volume 6 of this BULLETIN, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the JOURNAL, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the JOURNAL, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the JOURNAL, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

SAHARON SHELAH. ***Proper and improper forcing.*** Second edition of JSL L 237. Perspectives in mathematical logic. Springer, Berlin, Heidelberg, New York, etc., 1998, xlvii + 1020 pp.

At the dawn of the (first) golden age of the applications of forcing, Martin and Solovay introduced the so-called Martin axiom. Its consistency was based on the fact that finite support iterations preserve the countable chain condition (ccc). It proved to be a simple yet powerful tool for solving various problems inside and outside set theory. Typically one could solve a problem in one direction from the continuum hypothesis (usually the "negative" way), in the other from Martin's axiom (the "right" way). Literally hundreds of problems in set theory, combinatorics, set theoretical topology, algebra, and real analysis have been settled by devising a ccc poset and $\aleph_1$ dense sets, whose intersection gave the required object (establishing that some set is, say, meager). (In 1984 D. H. Fremlin published a thick volume, JSL LIII 650, describing several of these applications.)

As time went by, more and more problems accumulated that required different methods and soon it was clear that although the Martin axiom was still useful, something more general needed to be found. An important step was Laver's proof of the consistency of the Borel conjecture (JSL XLVIII 882). One of his tricks was using certain trees to add some reals whose connection to the problem was not a priori clear; the other—more important—innovation was using countable supports, despite the fact that at every step reals were added. A handsome argument shows that $\omega_1$ is not collapsed under the iteration.

When a handful of constructions like this were already present, Baumgartner devised his Axiom A describing a property general enough to include ccc forcings and those similar to

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Laver's, yet iterable with countable supports without causing much harm (that is, collapsing cardinals). One interesting feature of his axiom was that among some combinatorial properties, one semantical property (i.e., appealing to what the forcing did) appeared. Martin's axiom is purely combinatorial; one can learn and use it without knowing anything of the theory of forcing.

Immediately after this, Shelah isolated a more general notion which he aptly called "proper" forcing. Its definition is entirely semantical (it refers to the effect of the forcing rather than to the combinatorial properties) and to investigate its properties he incorporated two notions from model theory. One is that of the "monster model" (a set model large enough to contain everything of interest and modeling enough set theory), the other is the extensive use of countable elementary submodels (which are, as he showed, the natural tool for proving preservation theorems for countable support iterations). This notion was not just a lucky definition but a sample element of an endless army of variants and generalizations, and with this proper world-view Shelah started his massive attack on hard independence problems. (I think it is not just coincidental that his first published use of proper forcing is a consistency result concerning his classification of models.)

With a slight but essential change Shelah got the notion of semiproper forcing which is a notion as decent as proper forcing concerning subsets of $\omega_1$ but it includes forcings that change the cofinality of some ordinal to $\omega$ (and are, therefore, improper). It necessitates generalizing the notion of countable support iteration to the technically delicate notion of revised countable support iteration, but the upshot is several consistency results where a large cardinal is collapsed to $\omega_2$ (and the results imply that large cardinals are needed).

Using semiproper forcing, Foreman, Magidor, and Shelah (JSL LVII 1131) established the consistency (from a supercompact cardinal) of Martin's Maximum (MM), which is, in a well-defined sense, the ultimate generalization of Martin's axiom. Their paper is one of the most important papers in modern set theory. It contains several striking results, such as the consistency of the saturatedness of the stationary ideal on $\omega_1$ when a supercompact is Levy collapsed to $\omega_2$, from which Shelah and Woodin proved that if a supercompact exists then every set in $L(R)$ is measurable, so no canonical inner model for a supercompact can exist. It also led Woodin to the discovery of the stationary tower forcing, with which he proved the consistency of AD from a supercompact, and then . . . , well, the rest is history.

Most of what Shelah proved in the late seventies using his ideas was eventually published in his almost 500-page long volume ***Proper forcing*** (JSL L 237), and here is the substantially reworked and expanded second edition.

This monograph not only gives a careful introduction to the theory of proper forcing with a long list of examples but one can find several axioms ready for applications. Understandably—apart from a few cases—the author gives his version of the theory, but the reader must be aware that some parts have other, different descriptions, as well. I recommend parallel reading.

What follows is a short overview of the contents of the work. (There is also a very good *Annotated content* section at the beginning of the book.)

In Chapter I, after a short introduction to forcing, some very important examples, like adding or destroying CH, Cohen and random reals, cardinal collapsing, and adding diamond are carefully presented, with all the dirty details. Finally the remarkable proof of the consistency of "club does not imply CH" is given (originally the proof was hidden deep in a Whitehead group paper).

Chapter II serves as an introduction to iterated forcing. After the proof of the consistency of Martin's axiom and some immediate applications to uniformization, we see two more interesting applications. One concerns the cardinality of maximal almost disjoint families of a countable set (it can be any regular cardinal between $\aleph_1$ and the continuum), the other shows that one can can have $\aleph_1$ branches of a certain finitary tree with the uniformization property.

In Chapter III, our journey begins. We learn the basic definitions, and the properties of proper forcing. The proof of the countable-support-preservation theorem is given (actually, two). A nice, natural proof of Mitchell's result on $\aleph_2$-Aronszajn-trees is presented.

Chapter IV treats the oracle chain condition, a variant of ccc. Two applications are given: the solution of the lifting problem for the measure algebra and the proof that there can be only trivial automorphisms on $\mathcal{P}(\omega)/\mathrm{Fin}$.

In Chapter V various preservation theorems and applications are considered. (A general form of) S-properness is shown to be preserved and not to add new reals. Another property investigated is ${}^{\omega}\omega$-boundedness. Applications show that it is consistent with GCH that every Aronszajn-tree is special (so the Suslin hypothesis holds), and that CH + SH + KH is also consistent.

Chapter VI is devoted to some very general preservation theorems. As an application, a proof is given of the author's theorem on the consistency of non-existence of P-points. It is also shown that there may be a unique Ramsey filter.

In Chapter VII the Proper Forcing Axiom (PFA) and some variants (S-proper, etc.) are formulated, and their consistency is deduced from the consistency of a supercompact cardinal. A long list of the best known applications (trees, topological statements, etc.) is given. The chapter concludes with an important counter-example: a forcing, that does not destroy any stationary subset of $\omega_1$ yet collapses $\omega_1$ when iterated $\omega$ times.

In Chapter VIII, after a result on mixed iterations of $\omega_1$ and $\omega_2$-posets, a strengthening of chain condition is investigated. Then, an iteration theorem is proved for a variant of $D$-completeness.

Chapter IX contains results on free limits. The main application is to show that the Suslin hypothesis does not imply that every Aronszajn-tree is special.

In Chapter X the author starts investigating semi-proper forcing and proves the natural but technical theorem that (under suitable conditions) it is preserved under revised countable supports. As first applications some problems of Abraham and Friedman are solved.

In Chapter XI the technically delicate notion of I-condition is introduced and studied. This notion makes it possible to scale down several of the earlier consistency results to equi-consistency. To give but one example, the author proves, from a 2-Mahlo, the consistency of the existence of a club set in $\omega_2$ of $L$-inaccessible cardinals.

In the short, curiously titled Chapter XII the author settles a problem that troubled him for a while. He shows that the Namba forcing is semiproper iff there is a winning strategy in a certain game iff there is a semiproper forcing that changes the cofinality of $\aleph_2$ to $\aleph_0$. This holds in some models and it also implies Chang's conjecture so it can be regarded as a solution to the problem of semistationarity of the Namba forcing. As a by-product, a nice short proof is given to the following surprising statement: if a measurable cardinal is Levy-collapsed to $\aleph_2$ then, in the resulting model, Chang's conjecture holds. (This was later extended, via a deep argument, to $\omega_1$-Erdős, by Baumgartner and independently by Donder and Levinski; see ***Annals of pure and applied logic***, vol. 54 (1991), p. 217.)

Chapter XIII, extending the methods of the Foreman–Magidor–Shelah papers on Martin's Maximum, shows that (from some suitable large cardinal) it is consistent that on some stationary set of $\omega_1$, the non-stationary ideal can be layered, Ulam, or Levy.

Chapter XIV considers a generalization of revised countable supports to the case when the (revised) supports are of cardinality $< \kappa$ for some regular uncountable cardinal $\kappa$.

Some very technical conditions are introduced and studied in Chapter XV. They are useful in iterations that must add reals.

Chapter XVI provides the proof that the consistency of a Woodin cardinal implies the consistency of $2^{\aleph_0} = \aleph_2$ plus the stationary ideal on $\omega_1$ is $\aleph_2$ saturated (and variants).

Chapter XVII contains various results in different fields. It is shown that SPFA = MM yet does not imply PFA$^+$. A nice observation is that in ZFC there are two proper forcings with a product that is not proper. The other topic in this chapter is canonical functions. It is shown that there may exist a function on $\omega_1$ bigger (modulo stationary) than the first $\omega_2$ canonical functions plus PFA plus the negation of PFA$^+$. Another theorem is that the constant-$\omega_1$ function can be a canonical function but $0^\sharp$ will fail.

In Chapter XVIII, first an example shows that some conditions cannot be omitted from an earlier theorem on not adding reals. Then a condition ensuring no real is added is given which is particularly useful for posets of cardinality $\aleph_1$. Other preservation theorems are also presented, which are similar to those in Chapter VI, but more general. Finally, the consistency of "there is a unique P-point" is given.

The *Appendix* covers the author's work on some principles which he regards as very weak forms of the diamond, uniformization properties (they can be regarded as very weak forms of Martin's axiom), and applications to the Whitehead problem. It contains, among many nice results, the proof of the Devlin–Shelah $\Phi$, which is one of my favorite Shelah proofs.

In comparison with the first edition, Chapters XIII–XVIII are entirely new, and new material is also added to the old chapters at several points. Old Chapter XIII (on the strong covering lemma and the pre-pcf proof of $2^{\aleph_\omega} < \aleph_{(2^\omega)^+}$) is left out completely; the original Chapter XIV is now the *Appendix*.

Several parts of the book are more or less revised versions of papers that already appeared; we can discover papers numbered 65, 100, 119, 120, 177, 185, 207, 250, 253, 263, and probably more here.

I would like to emphasize strongly that the author (and several of his disciples, e.g. Martin Goldstern, to name but one) spent a lot of energy and innumerable long hours with writing, organizing, re-writing, re-organizing, checking, and proof-reading this work. It shows. The book is readable, with nicely elaborated proofs, with several comments that clarify what is going on.

Alas, still there are a fair number of typographical errors, misprints, unclosed parentheses, inverted question marks before Froms, etc. By the way, Facts III.8.4 (on page 139) and V.8.2 (on page 242) are identical.

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## Ten papers by Arthur Apter on large cardinals

ARTHUR W. APTER. *On the least strongly compact cardinal*. ***Israel journal of mathematics***, vol. 35 (1980), pp. 225–233.

ARTHUR W. APTER. *Measurability and degrees of strong compactness*. ***The journal of symbolic logic***, vol. 46 (1981), pp. 249–254.

ARTHUR W. APTER. *A note on strong compactness and supercompactness*. ***Bulletin of the London Mathematical Society***, vol. 23 (1991), pp. 113–115.

ARTHUR W. APTER. *On the first n strongly compact cardinals*. ***Proceedings of the American Mathematical Society***, vol. 123 (1995), pp. 2229–2235.

ARTHUR W. APTER and SAHARON SHELAH. *On the strong equality between supercompactness and strong compactness.*. ***Transactions of the American Mathematical Society***, vol. 349 (1997), pp. 103–128.

ARTHUR W. APTER and SAHARON SHELAH. *Menas' result is best possible*. Ibid., pp. 2007–2034.

ARTHUR W. APTER. *More on the least strongly compact cardinal*. ***Mathematical logic quarterly***, vol. 43 (1997), pp. 427–430.

Arthur W. Apter. *Laver indestructibility and the class of compact cardinals*. ***The journal of symbolic logic***, vol. 63 (1998), pp. 149–157.

Arthur W. Apter. *Patterns of compact cardinals*. ***Annals of pure and applied logic***, vol. 89 (1997), pp. 101–115.

Arthur W. Apter and Moti Gitik. *The least measurable can be strongly compact and indestructible*. ***The journal of symbolic logic***, vol. 63 (1998), pp. 1404–1412.

The papers under review represent a selection of Apter's results on the following old problem in set theory: What is the relation between the concepts of strong compactness, supercompactness, and measurability?

We start by recalling some important early results. Solovay, Reinhardt, and Kanamori (*Strong axioms of infinity and elementary embeddings*, JSL LI 1066(1)) showed that if $\kappa$ is $2^\kappa$-supercompact then $\kappa$ is not the least measurable cardinal. Menas (*On strong compactness and supercompactness*, ***Annals of mathematical logic***, vol. 7 no. 4 (1975), pp. 327–359) showed two results: (1) If $\kappa$ is $2^\kappa$-supercompact and $C$ is any class of cardinals, then $\kappa$ is not the least measurable $\alpha$ with $\alpha = \sup(C \cap \alpha)$. (2) If $\kappa$ is a measurable limit of $\lambda$-strongly compact cardinals then $\kappa$ is $\lambda$-strongly compact. Putting these two facts together, if $\kappa$ is the least measurable limit of strongly compact cardinals then $\kappa$ is fully strongly compact but not even $2^\kappa$-supercompact.

Magidor (*How large is the first strongly compact cardinal?* JSL LI 1066(2)) invented a technique for iterating Prikry forcing and used this together with some delicate preservation arguments to show the following: (1) If $\kappa$ is supercompact, then in some forcing extension $\kappa$ is still supercompact and is the least strongly compact cardinal. (2) If $\kappa$ is strongly compact, then in some forcing extension $\kappa$ is still strongly compact and is the least measurable cardinal.

*On the least strongly compact cardinal* is concerned with large cardinal properties intermediate between measurability and supercompactness. For a large class of reasonable $\Sigma_2$ functions $\varphi : ON \longrightarrow ON$ (for example $\varphi(\delta) =$ the least inaccessible greater than $\delta$), it is shown that if $\kappa$ is a supercompact limit of supercompact cardinals then the following hold: (1) Many $\delta < \kappa$ are strongly compact, $\varphi(\delta)$-supercompact, but not fully supercompact. (2) In some forcing extension the least strongly compact cardinal $\kappa_0$ is $\varphi(\kappa_0)$-supercompact but not fully supercompact. The second result is complementary to the forcing results of Magidor and is proved by similar techniques.

*Measurability and degrees of strong compactness* addresses a question arising from the results of Magidor: To what extent can the classes of measurable and strongly compact cardinals coincide? Using iterated Prikry forcing the paper proves the results: (1) There can be unboundedly many measurable cardinals with each one enjoying some degree of strong compactness, for example each measurable $\kappa$ can be $\lambda$-strongly compact for $\lambda$ the least inaccessible cardinal greater than $\kappa$. (2) If $\kappa_0$ and $\kappa_1$ are the first two measurable cardinals, then $\kappa_0$ can enjoy some degree of strong compactness below $\kappa_1$ while $\kappa_1$ is fully strongly compact. The difficulty with getting $\kappa_0$ to be fully strongly compact is that Prikry forcing at a measurable cardinal $\delta$ adds a weak square sequence at $\delta$ and makes $\delta$ into a cardinal of cofinality $\omega$; by a theorem of Shelah (*On successors of singular cardinals*, ***Logic Colloquium '78***, North-Holland, 1979, pp. 357–380) this destroys the $\delta^+$-strong compactness of all cardinals $\rho < \delta$. The strategy for making $\kappa_1$ the second measurable cardinal is to kill measurable cardinals between $\kappa_0$ and $\kappa_1$ by Prikry forcing, and this limits the degree of strong compactness enjoyed by $\kappa_0$.

*A note on strong compactness and supercompactness* presents two short proofs using Radin forcing of Magidor's result (JSL LI 1066(2)) that the least strongly compact cardinal may be supercompact. The more general of these proofs goes as follows: If $\kappa$ is supercompact then by a suitable Radin forcing we may preserve the supercompactness of $\kappa$ while adding Prikry sequences to unboundedly many cardinals below $\kappa$. By arguments as in the preceding paragraph this destroys all instances of strong compactness below $\kappa$.

So far the consistency results discussed were proved by variations on Prikry forcing. In an unpublished manuscript, Kimchi and Magidor had used Easton iteration of Jensen's forcing for adding a non-reflecting stationary set to prove the following: (1) The classes of strongly compact and supercompact cardinals can coincide except possibly at measurable limit points. (2) For any $n < \omega$, the first $n$ measurable cardinals can be strongly compact. A key point is that, in contrast with Prikry forcing, adding a non-reflecting stationary set to a measurable cardinal $\delta$ need not destroy the strong compactness of every cardinal $\rho < \delta$.

*On the first n strongly compact cardinals* uses the methods of Kimchi and Magidor's manuscript to generalise the consistency result from *On the least strongly compact cardinal*. For reasonable $\Sigma_2$ functions $\varphi$ it is consistent for any $n < \omega$ that, if $\delta$ is among the first $n$ strongly compact cardinals, then $\delta$ is $\varphi(\delta)$-supercompact but not fully supercompact.

*On the strong equality between supercompactness and strong compactness* takes the methods of Kimchi and Magidor to their logical conclusion, and proves that any model of ZFC + GCH has a generic extension in which for every $\lambda$ we have the properties: (1) Instances of $\lambda$-supercompactness from the ground model are preserved. (2) The classes of $\lambda$-strongly compact and $\lambda$-supercompact cardinals coincide except possibly at measurable limit points. A key point is that (sometimes) for $\kappa$ that is $\lambda$-supercompact we can destroy the $\lambda$-strong compactness of $\kappa$ by a forcing which is "mild," so that the $\lambda$-supercompactness of $\kappa$ can be restored by a poset that adds no new $\lambda$-sequences.

*Menas' result is best possible* is concerned with the result by Menas mentioned at the start of this review. This paper uses methods related to those of *On the strong equality between supercompactness and strong compactness* to produce a model with a cardinal $\kappa$ such that $\kappa$ is the least measurable limit of strongly compact cardinals, the classes of strongly compact and supercompact cardinals coincide below $\kappa$, and $\kappa$ is $< 2^\kappa$-supercompact with $2^\kappa$ large. Along the way another proof is given of an unpublished result by Woodin that the least measurable $\kappa$ can be $< 2^\kappa$-supercompact with $2^\kappa$ large.

*More on the least strongly compact cardinal* combines the ideas of *Menas' result is best possible* and Magidor's JSL LI 1066(2) to produce a model where the least measurable cardinal $\kappa$ is both strongly compact and $< 2^\kappa$-supercompact. The key point is that in the model of *Menas' result is best possible*, the $< 2^\kappa$-supercompactness of $\kappa$ is witnessed by an embedding into a model in which $\kappa$ is not measurable, which then makes possible a proof in the style of Magidor that iterating Prikry forcing to kill all measurable cardinals below $\kappa$ preserves the $< 2^\kappa$-supercompactness of $\kappa$.

The last three papers under review involve the notion of "indestructibility" introduced by Laver (*Making the supercompactness of $\kappa$ indestructible under $\kappa$-directed closed forcing*, ***Israel journal of mathematics***, vol. 29 (1978), pp. 385–388). Let us say for a large cardinal property P and a cardinal $\kappa$ that "$\kappa$ is indestructibly P" if and only if $\kappa$ is P in $V$ and in every extension by $\kappa$-directed closed forcing; we say "$\kappa$ is weakly indestructibly P" if and only if $\kappa$ is P in $V$ and in every extension by $\kappa$-directed forcing that adds no subsets of $\kappa$. The principal fact about indestructibility is that by Laver's paper, if $\kappa$ is supercompact then there is a poset that renders $\kappa$ indestructibly supercompact.

*Laver indestructibility and the class of compact cardinals* combines ideas from Kimchi and Magidor's manuscript and *Menas' result is best possible* to produce a generic extension of $V$ in which the following (among other properties) hold: (1) All instances of supercompactness from $V$ are preserved. (2) The classes of strongly compact and supercompact cardinals coincide, except possibly at measurable limit points. (3) Every supercompact cardinal is indestructibly supercompact. (4) Every strongly compact cardinal is weakly indestructibly strongly compact.

*Patterns of compact cardinals* shows that ZFC gives very little information about which members of the class of strongly compact cardinals are supercompact. The idea is to start with the model produced in *Laver indestructibility and the class of compact cardinals* and then to destroy instances of supercompactness more or less at will, by using the indestructibility promised there to show that those supercompact cardinals not marked for death will survive into the generic extension.

*The least measurable can be strongly compact and indestructible* answers a question left open in *Patterns of compact cardinals*, using a technique of Gitik (*Changing cofinalities and the nonstationary ideal*, ***Israel journal of mathematics***, vol. 56 (1986), pp. 280–314) for iterating Prikry forcing so as to get better chain condition and large cardinal preservation properties than is possible with the technique of Magidor. It is shown that the least measurable cardinal can be indestructibly strongly compact, and also (in the spirit of *On the least strongly compact cardinal* and *On the first n strongly compact cardinals*) that for reasonable $\Sigma_2$ functions $\varphi$, the least strongly compact cardinal $\kappa$ can be indestructibly strongly compact and indestructibly $\varphi(\kappa)$-supercompact without being fully supercompact.

Strongly compact cardinals still remain rather mysterious. Among the outstanding open questions are the following: (1) (Solovay) Is the consistency strength of strong compactness the same as that of supercompactness? (2) (Magidor) Can the first $\omega$ measurable cardinals be strongly compact? (3) (Woodin) If GCH holds below a strongly compact cardinal, does it hold everywhere?

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***Defeasible deontic logic***, edited by Donald Nute, Synthese library, vol. 263, Kluwer Academic Publishers, Dordrecht, Boston, and London, 1997, x + 354 pp.—therein:

DONALD NUTE and XIAOCHANG YU. *Introduction*. Pp. 1–16.

JOHN F. HORTY. *Nonmonotonic foundations for deontic logic*. Pp. 17–44.

LENNART ÅQVIST. *Systematic frame constants in defeasible deontic logic. A new form of Andersonian reduction*. Pp. 59–77.

LEENDERT W. N. VAN DER TORRE and YAO-HUA TAN. *The many faces of defeasibility in defeasible deontic logic*. Pp. 79–121.

Y. U. RYU and R. M. LEE. *Deontic logic viewed as defeasible reasoning*. Pp. 123–137.

MICHAEL MORREAU. *Reasons to think and act*. Pp. 139–158.

NICHOLAS ASHER and DANIEL BONEVAC. *Common sense obligation*. Pp. 159–203.

MARVIN BELZER and BARRY LOEWER. *Deontic logics of defeasibility*. Pp. 45–57.

ROBERT C. KOONS and T. K. SEUNG. *Defeasible reasoning and moral dilemmas. The divorce of ethics from classical logic*. Pp. 205–222.

HENRY PRAKKEN and MAREK SERGOT. *Dyadic deontic logic and contrary-to-duty obligations*. Pp. 223–262.

LAMBÈR ROYAKKERS and FRANK DIGNUM. *Defeasible reasoning with legal rules*. Pp. 263–286.

DONALD NUTE. *Apparent obligation*. Pp. 287–315.

JOSÉ CARMO and ANDREW J. I. JONES. *A new approach to contrary-to-duty obligations*. Pp. 317–344.

R. P. LOUI. *Alchourrón and von Wright on conflict among norms*. Pp. 345–351.

**Overview.** The volume collects fourteen chapters that have as a common objective to relate *normative reasoning* to *defeasible reasoning*. Normative, or deontic logic, owes its formal foundation to the early work (1951) by von Wright (JSL XVII 140), who proposed a reasoning system based on obligations and permissions. Defeasible logic studies the kind of reasoning one encounters when using rules that compete or conflict with each other. Often, this gives rise to non-monotonicity of the underlying inference mechanism, since the choice for which rules to apply may depend on the given theory or context. Non-monotonic reasoning was studied by researchers in artificial intelligence since the eighties, as a means to analyze common-sense reasoning.

This gives us a first link between normative and defeasible reasoning: normative actors do not always behave according to dictates of moral theory, but by rules of thumb, like 'do not cause harm,' 'return what you borrow,' 'do not steal,' ... Such rules are obviously defeasible and may be overruled by others, like 'feed your children.' As Nute and Yu observe in their introductory chapter, "The very expression '*prima facie* obligation' cries out for a defeasible or nonmonotonic analysis" (p. 1). Such obligations may be overridden, just as defeasible rules in non-monotonic logic may be overruled, or as defeasible arguments in argumentation may be defeated.

A second link between normative and defeasible reasoning is provided by the presumption of innocence in legal systems, which expresses that, unless proved otherwise, a person is innocent. Using $i$ for 'innocent'—all defeasible systems encountered in the book are based on propositional logic, except for the approach of Asher and Bonevac—we can translate almost directly such an assumption into one of the most prominent systems dealing with non-monotonic reasoning, i.e., default logic, as a default rule $\top : i/i$, expressing that, if it is consistent to assume that $i$, conclude that $i$.

There is a third reason why defeasible systems are of interest for deontic logicians. Since von Wright in 1951, the first approaches to a logic of obligation and permission gave rise to a number of so-called deontic paradoxes (to be addressed below), which deal with incompatible obligations, conditionals, and contrary-to-duty obligations. It is exactly these kinds of paradoxes that are addressed in this volume, and to which techniques of defeasible logic are applied in order to resolve them.

**Contents.** In the introductory chapter, Nute and Yu give a brief description of "standard deontic logic." Starting with von Wright's definition of 1951, this standard logic is a modal logic, in which the primitive $\mathcal{P}$ (for permission) is defined as a diamond, and where $\bigcirc$ (for obligation) is its dual.

Then, some of the so-called deontic paradoxes march in review. These paradoxes have to be understood as either logically paradoxical, moral dilemmas, or counter-intuitive results. This treatment fits in place here, since, as mentioned above, those paradoxes were for many researchers the motivation to look at defeasible formalizations. Some of the paradoxes immediately trace back to using standard modal logic, like the paradox that deals with material implication and closure under consequence, such as $(\bigcirc A \rightarrow \bigcirc(A \vee B))$ (the Ross-paradox, expressing "If I ought to mail a letter, I ought to mail or burn it"), or with the inheritance of obligations (from $\vdash A \rightarrow B$ derive $\vdash \bigcirc A \rightarrow \bigcirc B$). Here, problems with *incompatible* obligations also come to surface: suppose two sentences $A$ and $B$ are logically inconsistent, then standard modal logics would derive $\bigcirc(A \wedge B)$, $\bigcirc C$, and hence $\bigcirc\bot$ from $(\bigcirc A \wedge \bigcirc B)$.

A problem in deontic logic that is related to both reasoning from inconsistent premisses and material implication is that of contrary-to-duty obligations (CTD's). It addresses the question which obligations persist, if already some violations to the rules are made. A prominent example here is provided by the so-called Chisholm set, saying that one should go to help $(h)$ his neighbors, and one should tell $(t)$ them when coming to help. Moreover, one should not tell them if one does not go to help. Finally, it is a fact that one does not help. Writing $\top$ for an arbitrary tautology, and $\bigcirc(A/B)$ for the conditional '$A$ ought to be the case, given $B$,' the Chisholm set has $(\star)$ as one of its formalizations:

$$(\star) \quad \text{(a): } \bigcirc(h/\top)\ (\equiv \bigcirc(h)), \qquad \text{(b): } \bigcirc(t/h), \qquad \text{(c): } \bigcirc(\neg t/\neg h), \qquad \text{(d): } \neg h$$

Now, one question is how the strengthening of the antecedent of the derivable obligation $\bigcirc(t/\top)$ to $\bigcirc(t/\neg h)$ can be blocked, or, in terms of defeasible reasoning, although we would conclude $\bigcirc(t)$ from the premisses (a), (b), and (c), we would rather not derive this obligation from the complete Chisholm set—expressing the need for a non-monotonic entailment. Of course, defeasible techniques are not the only way out: For the Chisholm paradox alone, I counted seven different formal *representations* throughout the book!

Nute and Yu close their chapter with a two-page discussion on recurring themes in defeasible reasoning. Syntactically, they emphasize notions such as extensions and fixed-point solutions, taking default logic as a most typical defeasible logic. Semantically, approaches to defeasible reasoning typically utilize the notion of a preference on the possible worlds. For moral reasoners, this preference has a natural meaning: it is based on the deontic principles that a world validates.

Horty's chapter (*Nonmonotonic foundations for deontic logic*) is completely inspired by the deontic paradoxes. The chapter provides both a logic of conflicting ought statements and a logic governing conditionals expressing *prima facie* oughts. Horty argues why we want a logic consistently to represent conflicting oughts like $\bigcirc A \wedge \bigcirc \neg A$: Such properties model imperfect rules of thumb for moral reasoning, and, moreover, a general "ought" represents norms of morals, etiquette, aesthetics, fun, etc., which need not be consistent all together. The third argument is a typical AI-argument: any automatic intelligent system that takes a user's goals as its obligations, must be prepared to encounter such inconsistent oughts. His line of reasoning suggests that we should represent an obligation $\bigcirc A$ as a default rule $\top : A/A$. Then, we can fine-tune the kind of conclusions we want to derive from the premisses $\bigcirc A, \bigcirc \neg A$. Modal logic would derive any obligation $\bigcirc B$, but with the default rules, using a credulous approach, one would derive both $\bigcirc A$ and $\bigcirc \neg A$ (but not $\bigcirc(A \wedge \neg A)$), whereas a skeptical approach would yield only $\bigcirc(A \vee \neg A)$.

Åqvist (*Systematic frame constants in defeasible deontic logic*) shows how dyadic deontic logic can be represented as a "pure" alethic logic, in which the operator $\bigcirc(A/B)$ is only a defined operator. Although taking this route is interesting in itself, and although the author proves representation results of dyadic deontic logics in his alethic system, he does not provide grounds for his claim that "There are, we take it, definite advantages in doing so" (p. 60). Precise theorems accompanied by proofs show that there is an axiomatization for his system, and that it is equivalent to a system in which the operators for ought and permitted are primitive.

Torre and Tan (*The many faces of defeasibility in defeasible deontic logic*) also take dyadic deontic logic as their starting point, by distinguishing between conditional obligations—allowing for unrestricted strengthening of the antecedent—and contextual obligations, where strengthening is not always allowed. They provide a variety of examples, focusing on the Chisholm paradox (they claim that "the example cannot be represented in a temporal deontic logic," p. 86, although the introductory chapter provides a temporal representation of Vorobej, p. 10). The authors distinguish "factual defeasibility," i.e., when there is a conflict between an obligation and an actual fact from "overridden defeasibility," where a dyadic obligation $\bigcirc(A/B)$ may be overridden by $\bigcirc(A'/B')$ if $B'$ is "logically stronger" than $B$ and if $A$ and $A'$ are "contradictory." Thus, it seems that the authors want to capture specificity here. The authors systematically study a number of inference rules, and also provide a preference semantics for contextual obligation, which is sound with respect to a number of these rules. Since in many examples the authors claim to have blocked an unwanted derivation, however, there is obviously a need for a completeness theorem as well.

Ryu and Lee (*Deontic logic viewed as defeasible reasoning*) are also interested in proposals to resolve conflicting obligations or permissions, where they typically want to allow that such a conflict arises when more than two rules are considered. They thus come up with a theory for competing obligations, and, in a default-logic-like style, they define the deontic extension of a deontic theory—a closed theory to which some maximal set of conditionals (the deontic base) is applied. (As acknowledged by Ryu in a private communication, their main definition 2.2 on p. 125 is a bit loose, and they leave some constraints implicit on how to use their *deontic theory*, p. 124.) They give comparisons of their approach to other ones.

Morreau (*Reasons to think and act*) has an analogue to Ryu and Lee's deontic bases: he also takes maximal sets of "schemes" that are consistent with a given theory. His "reason sentence" $A \rhd B$ has to be read as "if $A$, then, other things being equal, $B$." One of his main concerns is to characterize when we can infer $B$ from $A$ and $(A \rhd B)$; such an inference should be *invalid* (in the sense that $(A \rhd B) \to (A \to B)$ is not valid), *defeasible*, and *perplexing* (in the sense that incompatible things can seem to be the case). In order to resolve possible conflicts, he suggests use of a priority order on reasons.

The main point of Asher and Bonevac's chapter (*Common sense obligation*) is to make a distinction between *prima facie* obligations as holding under normal circumstances, and as holding *ceteris paribus*. The latter is similar to what Morreau calls "other things being equal" obligations. They also agree with Morreau that those are *constitutive*: a *ceteris paribus* reading of $\bigcirc(A/B)$ implies that the truth of $B$ is a *reason* for doing $A$. The "normal circumstances" reading of $\bigcirc(A/B)$, however, has an *epistemic* flavor: here, the truth of $B$ is a good indicator that $A$ is obligatory. Hence, "If other things are normally not equal, then sentences of these forms convey different information" (p. 166). The authors provide several examples for the different readings.

The chapter of Belzer and Loewer (*Deontic logics of defeasibility*) starts from recognizing that normative rules are defeasible, but their interest concerns what we ought to do, and are permitted, given a normative system, and all things considered. Clearly, this kind of reasoning is monotonic: if we conclude that we ought to do $A$, all things considered, then no matter what kind of abnormalities we are presented, we must, by definition, stick to our conclusion. They present a semantics for a dyadic operator, and show that it is weak enough to bypass all the (conditional) deontic paradoxes. Starting from such a weak system, they show how several properties of $\bigcirc$ can be incorporated by putting additional conditions on their semantics.

Koons and Seung (*Defeasible reasoning and moral dilemmas*) defend a model of ethics in which all principles are defeasible and in which the totality of ethical truths is not logically consistent. They claim that default logic does not handle exceptions well, and that "Circumscription does much better" (p. 212). (Their argument in favor of circumscription, however, in which they represent "moral rules" as $\varphi \mathbin{\&} \neg ab_{37} \to \psi$, p. 212, straightforwardly translates to default theories. Moreover, their own approach and examples are soaked with default rules.) They extend an idea of van Fraassen and assign a non-empty subset $V$ of truth-values $\{u, f, t\}$ to a formula $\varphi$, given a (deontic) default theory $T$, depending on whether there are extensions $\mathcal{A}$ of $T$ that verify $\varphi$. If both $t$ and $f$ are in $V$, we are facing a moral dilemma, and the values $V = \{t, u\}$ and $V = \{f, u\}$ are called "residual values," which give rise to guilt, blame, or secondary obligations. Although this basic idea is simple and appealing, the authors unfortunately do not analyze this any further; the chapter is concluded with a number of examples which cannot or cannot easily be matched with their set-up. (For instance, in their first example, p. 214, they assume two extensions for the theory, but there seems to be no standard way to derive them.)

Prakken and Sergot (*Dyadic deontic logic and contrary-to-duty obligations*) also study a dyadic obligation $\bigcirc(B/A)$, to be read as "$B$ is obligatory given, or pre-supposing, or from the point of view of, the context $A$." They argue that such a contextual obligation differs from conditional obligation as ordinarily understood, in the sense that regarding something $(A)$ as settled, does not automatically block obligations (for instance $\bigcirc\neg B$) from outside the context, so that, in particular, we cannot derive $\bigcirc B$ from $\bigcirc(B/A)$ and the mere fact $A$. They then look for conditions under which "inheritance" between contexts hold: conditions that allow one to derive $\bigcirc(B/C)$ from $\bigcirc(B/A)$, if the context $C$ is weaker ("down inheritance") or stronger ("up inheritance") than $B$. They come up with a Hansson–Lewis kind of semantics, with a pre-order (ordering "goodness") on worlds. Roughly, the contextual obligation $\bigcirc(B/A)$ is true then if there is a $B \wedge A$ world that is better than any $B \wedge \neg A$

world. Interestingly enough, in order to find the inheritance conditions that the authors find intuitively valid, they adapt a technique that Ryan used to order models according to how well they satisfy a given set of defaults.

The approach of Royakkers and Dignum (*Defeasible reasoning with legal rules*) is purely syntactical: rather than considering orders on worlds, they put an order on rules and norms. Their system is basically a dialectic one, with the central notion of an *argument*, which is a set of rules. Arguments can then be *defeated*, *justified*, *defensible*, or *overruled*. (Their notion of a justified argument, p. 268, is rather strong: in dialectic terms, their proponent has a justified argument if *every* defeating move he uses as a reply to the opponent can be successfully completed, rather than requiring that the proponent has *one* move that leads to a successful argument for every reply of the opponent.) Various examples show how they can allow for permissions in a default-like setting, how they can deal with arbitrarily many competing rules, and that they can handle both defeasible rules and norms in one system.

Nute's chapter (*Apparent obligation*) also focuses on syntax, and he also assumes an order on the rules. He distinguishes three types of rules: strict rules (material implication), defeasible rules (default-like rules), and undercutting defeaters (which may block inferences using defeasible rules). With this apparatus, one can define *prima facie* obligations (that bind, other things being equal), actual obligations (binding, when all relevant circumstances are, or *would be*, considered) and apparent obligations (binding, given all *known* morally relevant circumstances considered). Unlike the second kind of obligation (see the comments on the chapter of Belzer and Loewer), the first and third kind should be represented by using defeasible rules. Several examples demonstrate that part of the solution to many puzzles and paradoxes lies in their proper formalization. In Nute's system, for instance, one must explicitly determine relevant minimal sets of conflicting formulas.

The chapter of Carmo and Jones (*A new approach to contrary-to-duty obligations*) is motivated by providing a formalization of the Chisholm set ($\star$) that fulfills a number of requirements: consistency, logical independence, analogous structures of the sentences (b) and (c) from this set, and the four capacities to derive the actual obligation $\bigcirc(\neg t)$, to derive the ideal obligation $\bigcirc(h)$, to represent the violation of the obligation $\bigcirc(h)$, and, finally, to avoid what they call "pragmatic oddity," that is, to avoid the derivation of $\bigcirc(h \wedge \neg t)$ from the set ($\star$). Then, by fine-tuning the kind of possibility ("degree of alterability") in the premisses, they show how their requirements are met for a variety of examples.

The last chapter, by R. Loui (*Alchourrón and von Wright on conflict among norms*) is the shortest contribution to the book, and the least technical one. According to Loui, von Wright and Alchourrón had in common that they somehow reason incompatible or conflicting norms away (from $\bigcirc(A/B)$ and $\bigcirc(\neg A/C)$ conclude $\neg(B \wedge C)$). The problem, says Loui, is, however, that in real normative systems conflicting norms *do* occur, and, to resolve them, we should have a theory that helps us to discriminate, or to choose, between such conflicts. Von Wright ("Such contradictions cannot be 'solved' in logic, only in the practice of norm-giving," p. 349) and Alchourrón ("The particular details of revisions (and the choice functions) are never analyzed by a logician," p. 351) seemed both to be convinced that logicians never can solve problems of this kind, where a deference, derogation, or a defeat occurs. Loui counter-argues: "The defeasible logics do not need to pretend that there are no conflicts, or that someone else will solve the problem somehow. The defeasible logics provide a place for computation" (p. 351). A nicer recommendation for the book and its theme is hard to imagine.

**Audience.** The book can be used in an advanced graduate course in (philosophical) logic. Students need to know only a little about default theory, preference semantics, and non-monotonic entailment to appreciate it. The book is very valuable on the shelf of researchers in moral theory, philosophical logic, philosophy of law, and legal reasoning, as well as for those interested in AI-like decision support and expert systems. The chapters are coherently written around the theme of defeasible deontic logic: researchers may even find a challenge to unify different approaches in one general theory, like general possible world semantics to give an account for all the different conditional obligations that appear throughout the book.

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JEAN GOUBAULT-LARRECQ and IAN MACKIE. ***Proof theory and automated deduction.*** Applied logic series, vol. 6. Kluwer Academic Publishers, Dordrecht, Boston, and London, 1997, xv + 424 pp.

The book under review gives an introduction to a broad spectrum of classical and non-classical *logics*, i.e., formalisms consisting of a syntax, a semantics, and a deduction system (p. 11), ranging from the fundamental to the ephemeral, and including various systems for classical logic, intuitionistic logic, modal logic, and linear logic.

For classical propositional logic (Chapter 2) in particular, the authors give an almost encyclopedic overview of different deduction systems starting with truth tables and a "very unnatural" (p. 21) Hilbert-type system attributed to P. Andrews. Although the proof of completeness for this system is omitted, important consequences are obtained from it, viz. the deduction theorem, the compactness theorem, and the decidability of classical propositional logic. All three theorems are again obtained from the natural deduction system presented next. For this system the authors give an autonomous completeness proof based on so-called decision trees. Completeness of the sequent calculus, however, is shown by direct construction of a deduction using backwards application of the rules. These basic systems are followed by a number of "automated proof methods," including tableaux, resolution, the Davis–Putnam procedure (unfortunately not in the original formulation which is much closer to resolution), two variants of binary decision diagrams, and Hsiang's rewrite procedure (which is actually a disguised form of a certain resolution procedure). Here the authors give detailed information on implementational issues and on heuristics. Many readers will probably appreciate this kind of information more than the lengthy discussion of theoretical questions which is neither flawless nor elegant (many simplifications in the completeness proofs made possible by obvious dependencies among the deduction systems remain unexploited.)

Surprisingly all this is developed without ever mentioning why one should be interested in classical tautologies. The only application the authors hint at is coding Turing machines (p. 66 f), whereas readers coming from linguistics will probably be discouraged when learning that sentences such as "someone likes chocolate" are called atomic sentences or that "if $A$ and $B$ are true, we can deduce that $A$ is true" (p. 1).

For intuitionistic propositional logic, Chapter 3 gives Hilbert-style, natural deduction, sequent calculus, and tableaux formalizations together with a complete list of intertranslations. Moreover the notion of Kripke semantics is introduced here, but a completeness proof with respect to Kripke semantics is not given. Besides a number of useful observations on the structure of derivations, this chapter also contains a number of serious misunderstandings concerning the equivalence of $\Phi \vee \neg\Phi$ and $\neg\neg\Phi \rightarrow \Phi$ for obtaining classical logic (p. 85), the uniqueness of deductions in NJ (p. 86), and the proof that intuitionistic logic is not tabular (p. 102).

Chapters 4 and 5 deal with the Curry–Howard isomorphism and modal and temporal logics.

Chapter 6 and the following chapters are devoted to predicate logic. The theoretical investigations are based mainly on sequent calculus, i.e., on cut elimination. (Unfortunately the cut degree defined on page 211 does not seem to decrease under the constructions on page 211 ff.)

Chapter 7 deals with resolution and contains a number of remarkable observations e.g. on the relation between the cut and resolution rules and on the role of the factoring rule in resolution deductions. (The latter remark, however, is not reflected in the completeness proof for resolution. Reading this proof "we might" in fact "think that factoring is useless," p. 242.) Besides ordinary resolution the authors also present various refinements such as ordering and set-of-support strategies, hyperresolution, and linear resolution. Completeness proofs for all these refinements follow the usual pattern of first eliminating a certain propositional variable, applying the induction hypothesis to generate a resolution deduction, and finally reintroducing the variable. (And the argument on page 262 clearly shows that for linear logic this approach is inferior to the approach using transformations of deductions.)

The book closes with a discussion on the relationship between resolution, tableaux, and the connection method (attributed to Andrews and Bibel, rather than to D. Prawitz, *An improved proof procedure*, JSL XXXI 126), a chapter on *Incorporating knowledge*, one on logic programming and two appendices with (mostly useful) answers to exercises (although answer 3.6 says "We just show one direction," where the other direction is plainly wrong), and *Basics of topology*.

The experienced reader of this book is rewarded with a number of important remarks on the structure of deductions in different logical systems and with information on implementational details. The inexperienced reader, however, will probably be puzzled by a plethora of results of different reliability on a great number of seemingly disparate proof systems.

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Jan Dejnožka. ***Bertrand Russell on modality and logical relevance.*** Avebury series in philosophy. Ashgate, Aldershot, Brookfield, Vt., etc., 1999, ix + 241 pp.

In various works spanning several decades Bertrand Russell remarks that necessity and possibility should be seen as features of propositional functions and, accordingly, that a function is necessary when it is "always" true, possible when "sometimes" true. The observation that there is a logical analogy between quantifiers and modal operators is an important insight which resulted, through Carnap's work, in contemporary possible worlds semantics. Russell, however, introduced this account of necessity and possibility as part of a criticism of the view that modality was a feature of logic. The identification of modality with quantification makes its first appearance in the recently published 1905 paper *Necessity and possibility* in volume 4 of Russell's collected papers (***Foundations of logic, 1903–05***, Routledge, 1994, pp. 507–520). Russell there claims that "the traditional modal distinctions ... which are valid seem, so far as I can discover, to be none of them fundamental, and all of them better described in non-modal terms" (p. 508). He goes on to talk of several sources of the idea of necessity such as the distinction between a priori and empirical knowledge and the notion of that which can be demonstrated from prior truths. Causal laws, he says, are felt to be necessary because they are universal, holding everywhere at all times. Russell concludes, after giving his quantificational account as the best approximation to what people intend, that "the subject of modality ought to be banished from logic, since propositions are merely true or false, and there is no such comparative and superlative of truth as is implied by the notions of contingency and necessity" (p. 520).

Dejnožka will have none of this. He asserts that the account of modality as quantification is indeed a modal "theory" MDL, and that when developed into the view that a proposition fully generalized (FG) with respect to individuals and functions that is true in virtue of its logical form is a necessary proposition, it becomes the "modal logic" FG-MDL*. Readers of this Bulletin will be disappointed to discover that no formal development of this logic is presented. After an investigation of several characteristic axioms of some of the Lewis systems, Dejnožka concludes that FG-MDL* is "close to" S5 in strength, differing by the addition of a thesis he calls "I", namely $\Box(x)(x = x)$. (This principle follows from the logical truth $(x)(x = x)$ by the rule of Necessitation in most quantified modal logics.) No further formal details of the quantified logic, such as a formal possible worlds semantics, are provided. One might think that a thesis of S5 such as the theorem that whatever is necessarily true is necessarily necessary might obtain simply by the fact that quantification with respect to a bound variable is vacuous, thus '$(x)(x)Fx$' is easily treated as vacuously true if the embedded quantification '$(x)Fx$' is already true. Rather, on Dejnožka's account "necessarily" actually means "true when fully generalized using only universal quantifiers and true in virtue of its [logical] form" (p. 63). It is then not because the additional operator is vacuous that necessary truths are themselves necessarily true, but rather because if it is in virtue of its logical form that a fully generalized sentence is true, then that must be true in virtue of its logical form. If "necessarily" is to be a genuine sentential operator rather than simply a meta-linguistic predicate, it appears that Dejnožka is right to claim that there is a modal notion here. The sentential connective "it is true that ... in virtue of its logical form" will be a non-truthfunctional operator which might indeed have its own modal logic. Then, however, the two aspects of universal generalization and truth in virtue of logical form would have to be distinguished and only the latter treated as a modal notion. Dejnožka, however, seems to insist that universal generalization is the heart of Russell's "modal logic."

Russell frequently asserts that logical truths must be true in "all possible worlds," using a phrase that he undoubtedly acquired from his study of Leibniz. In his informal philosophical writing Russell also frequently uses the notions of necessity and possibility, asserts subjunctive conditionals, uses "ought" and "it could be true that" as operators, and so on. It is a long way from this to ascribing to him a combinatorial theory of possible worlds and epistemic, deontic, and causal logics, as Dejnožka proceeds to do. As a survey of Russell's use of modal notions in his philosophical language, this is a valuable project. To impute to Russell a wealth of modal "logics" seems to go far beyond what the texts will bear.

Two final chapters bear on seemingly incidental matters. One argues that Russell was the source, at least in recent logic, of the idea that the truth conditions of a logical consequence are included in those of the premisses. Dejnožka concludes that Russell thereby had a "relevance" logic, and that those who assert that classical logic does not make logical consequence relevant are in fact mistaken. This startling thesis is followed by a chapter arguing that the source of the logical notion of probability in Wittgenstein and later Carnap, is in fact in 19th century legal discussions of relevance in the theory of evidence, by way of Russell and Keynes.

This book will be of interest for gathering together quotations from Russell using various modal locutions and for its speculations about what Russell would say about a number of issues concerning modality. The thesis that Russell's proposal to treat necessity as universal quantification is a full fledged theory of modality is a bit far fetched, however, as many correspondents of Dejnožka have suggested in passages which he quotes in order to controvert them. Indeed W. Quine is quoted as "kindly" writing to the author that "Certainly I have no objection to necessity and possibility when interpreted in Russell's way" (p. 16). One feels that Dejnožka has missed some irony in the response.

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