THE BULLETIN OF SYMBOLIC LOGIC
Volume 5, Number 4, Dec. 1999

1999 SPRING MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

New Orleans, Louisiana, May 6–7, 1999

The Spring Meeting of the Association for Symbolic Logic was held in New Orleans, Louisiana, on May 6–7, 1999, in conjunction with the annual meeting of the Central Division of the American Philosophical Association. The Program Committee consisted of Graeme Forbes and Charles Parsons (co-chairs) and Philip Kremer. The program consisted of a symposium on Applications of Non-Standard Consequence Relations, with two invited talks and a commentator, two other invited talks, five contributed papers, and five papers presented by title only.

The speakers in the symposium were Ken Gemes (Yale University) and Gerhard Schurz (University of Salzburg); the commentator was Arnold Koslow (Graduate Center, City University of New York).

The other invited talks were:

Kit Fine (New York University), *Abstraction*.

Neil Tennant (Ohio State University), *Logicism and molecularism*: *Considerations of conservative extension*.

Abstracts of invited and contributed talks (given in person or by title) follow.

For the Program Committee
CHARLES PARSONS

Abstracts of contributed talks

► G. ALDO ANTONELLI, *Free set algebras satisfying systems of equations*.
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In this talk we introduce the notion of a set algebra $\mathcal{S}$ satisfying a system $\mathcal{E}$ of equations. After defining a notion of freeness for such algebras, we show that, for any system $\mathcal{E}$ of equations, set algebras that are free in the class of structures satisfying $\mathcal{E}$ exist and are unique up to a bisimulation. Along the way, we mention analogues of classical set-theoretic and algebraic properties as well as connections between the algebraic and set-theoretic viewpoints.

► PHILIP EHRLICH, *The reals and the surreals*.
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In his monograph *On numbers and games* [1], J. H. Conway introduced an ordered field *No* which contains (in a suitable sense that can be made precise) "all numbers great and small." However, in addition to its distinguished structure as an ordered field *No* has a rich

1079-8986/99/0504-0004/$1.60

algebraico-tree-theoretic structure, or *simplicity hierarchy*, that emerges from the recursive clauses in terms of which it is defined. In a recent paper [2], we introduced a class of ordered fields with simplicity hierarchies whose properties generalize those of *No* and drew attention to some of the important relations that exist between *No* and this more general class of *s-hierarchical ordered fields* as we call them. Among a host of other results, we showed that an ordered field together with a tree structure is an s-hierarchical ordered field if and only if it is isomorphic to an ordered field that is an initial subtree of *No*, and that every real-closed ordered field is a reduct of some s-hierarchical ordered field. In the present paper, we examine the relation between *No* and its unique *s-hierarchical ordered subfield of reals* and establish a variety of theorems which naturally suggest that whereas the s-hierarchical ordered field of reals should merely be regarded as constituting an *Archimedean s-hierarchical arithmetic continuum*, *No* may be regarded as a sort of *absolute s-hierarchical arithmetic continuum* (*modulo von Neumann-Bernays-Gödel set theory*). For proofs of these and related results, see [3].

[1] J. H. CONWAY, ***On numbers and games***, Academic Press, 1976.

[2] P. EHRLICH, *Number systems with simplicity hierarchies*: *A generalization of Conway's theory of surreal numbers*, ***Bulletin of Symbolic Logic***, vol. 4 (1998), no. 2, p. 212.

[3] ———, *Number systems with simplicity hierarchies*: *A generalization of Conway's theory of surreal numbers*, to be published in ***The Journal of Symbolic Logic***.

► PAOLO MANCOSU, *Between Russell and Hilbert: Behmann on the foundations of mathematics*.
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After giving a brief overview of the renewal of interest in logic and the foundations of mathematics in Göttingen in the period 1914–1921, I give a detailed presentation of the approach to the foundations of mathematics found in Behmann's doctoral dissertation of 1918, *Die Antinomie der transfiniten Zahl und ihre Auflösung durch die Theorie von Russell und Whitehead*. The dissertation was written under the guidance of David Hilbert and was primarily intended to give a clear exposition of the solution to the antinomies as found in *Principia Mathematica*. In the process of explaining the theory of *Principia*, Behmann also presented an original approach to the foundations of mathematics which saw in sense perception of concrete individuals the Archimedian point for a secure foundation of mathematical knowledge. The last part of the paper argues for the importance of Behmann's investigations for the development of Hilbert's approach to the foundations of mathematics in the early twenties.

► FRED SEYMOUR MICHAEL, *Aristotelian and Leibnizian worlds*.
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Much recent development in modal logic has been motivated by the desire to find a basis for an adequate intentional logic. That is what motivated Carnap's modal semantics and the work of many prominent successors. It is common among those with this motivation to base a conception of possible worlds on Carnap's conception of a state-description, defining possible worlds as complete, consistent states of affairs, or as linguistic or conceptual counterparts of these states of affairs. These worlds are generally understood as being discrete from one another. Such worlds I call Leibnizian. A state of affairs is said to be *possible* if and only if it holds in some Leibnizian world, *necessary* if and only if it holds in all such worlds. For an intentional logic concerned with relations of meaning, which are, or are assumed to be, permanent, a possible world semantics which accommodates only permanent modalities may certainly be adequate.

Leibnizian modalities are all permanent. States of affairs never come to be necessary or cease to be possible. However, there are transient modalities, modalities that come to be and pass away, modalities of a kind with which a Leibnizian possible world semantics is not equipped to deal. As a holder of a ticket in a certain lottery yet to be held, I am a possible winner of that lottery. At the same time, I am a possible loser. One of these possibilities will cease to be. It is clearly transient. A Leibnizian possible world semantics, however, can deal with transient modalities only by treating them as if they are permanent. Even when I turn out to be a lottery loser, I remain permanently a possible winner of that lottery because there is some Leibnizian world in which I win it. If we are to recognize transient possibilities, we need a different kind of modal semantics, the form of modal semantics I call Aristotelian. These are worlds in which are indeterminate with respect to some states of affairs. I am a possible winner and loser in a lottery only in a world in which I am in fact neither winner nor loser. There are transient possibilities only where worlds are incomplete. An Aristotelian modal semantics is what we need to accommodate transient modalities. Other applications are to future contingents and counterfactuals. My principal objective in this paper is to develop in some detail the logic of Aristotelian modalities and to show that the semantics of this logic cannot be bivalent.

► RICHARD ZACH, *Hilbert, Bernays, and some fundamental advances in logic,* 1918–1923.
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The development of formal logic experienced some revolutionary developments in the 1920s, including the drawing of a distinction between syntax and semantics, and the formulation of questions of completeness and decidability of logical systems. Many of these advances can be traced to unpublished lecture notes and theses by Hilbert and Bernays dating to the period 1917–1923. The role of Hilbert's unpublished lectures has recently been investigated in the literature [1, 2]; it is argued here that Paul Bernays had a very significant part to play. In particular, the *Habilitationsschrift* (1918) of Bernays contains the first precise distinction between syntax and semantics, a formulation and proof of soundness and semantic completeness, and a proof of decidability of the propositional calculus of *Principia*. Furthermore, archival evidence is adduced to show that Bernays deserves at least partial credit for the advances made in Hilbert's 1917–1918 lectures on the *Principles of Mathematics* at Göttingen.

[1] G. H. MOORE, *Hilbert and the emergence of modern mathematical logic*, ***Theoria (Segunda Época)***, vol. 12 (1997), pp. 65–90.
[2] W. SIEG, *Hilbert's programs*: 1917–1922, ***Bulletin of Symbolic Logic***, vol. 5 (1999), pp. 1–44.

Abstract presented by title

► JOHN BYRNES, *A search calculus for normal natural deductions*.
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The common wisdom in automated theorem proving is that natural deduction search is infeasible; if one wishes to find natural deductions, search in a sequent calculus and translate. Work focuses on formulating sequent calculi for which search is efficient and which translate easily to natural deductions. Dyckhoff and Pinto [1] discuss a cut-free permutation-free sequent calculus *LJT* of Herbelin [2] as a particularly good example, as its derivations are in a one-to-one correspondence with normal natural deductions.

Sieg's intercalation calculus [3] formalizes a particular search space for finding normal natural deduction proofs. I will present a refinement of this search, *nLIC*, motivated by restricting the search strategies. *nLIC* is very similar to *LJT* (extended to classical logic). I then present a restricted natural deduction calculus, *NIC*, based on intercalation. *NIC* defines exactly the full class of normal classical natural deductions.

I will define when two calculi define isomorphic search spaces and show that *NIC* is isomorphic to *nLIC*. Thus, proof search occurs directly in a natural deduction calculus with exactly the same efficiency as in the sequent calculus.

[1] Dyckhoff and Pinto, *Cut-elimination and a permutation-free sequent calculus for intuitionistic logic*, ***Studia Logica***, vol. 60 (1998), no. 1, pp. 107–118.

[2] Herbelin, *A λ-calculus structure isomorphic to Gentzen-style sequent calculus structure*, in ***Proceedings of the 1994 conference on computer science logic*** (Pacholski and Tiuryn, editors), 1995, pp. 61–75.

[3] Sieg and Byrnes, *Normal natural deduction proofs (in classical logic)*, ***Studia Logica***, vol. 60 (1998), no. 1, pp. 67–106.

► ALBERT A. MULLIN, *Topics on recursive number theory*.
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This note continues the author's developments on consecutive differences for differences of consecutive square-frees, with special emphasis on consecutive bi-composites and consecutive tri-composites. Put $d_n^* = s_{n+1} - s_n$, where s_n is the nth square-free.

Lemma 1. *Then* $d_n^* < d_{n+1}^* < d_{n+2}^*$ *for infinitely many* n.

Problem 1. Does there exist an n for which $d_n^* < d_{n+1}^* < d_{n+2}^* < d_{n+3}^*$?

Problem 2. Prove that $d_n^{(2)} < d_{n+1}^{(2)} < d_{n+2}^{(2)}$ for infinitely many n, where $d_n^{(2)} = b_{n+1} - b_n$, the difference of consecutive bi-composites.

Problem 3. Prove that $d_n^{(3)} < d_{n+1}^{(3)} < d_{n+2}^{(3)}$ for infinitely many n, where $d_n^{(3)} = t_{n+1} - t_n$, the difference of consecutive tri-composites.

Problem 4. Characterize the finite groups with a prime number of Sylow p-subgroups.

Problem 5. Give an example of palindromic elegiac poetry.

Problem 6. Characterize the countable groups with a prime number of solvable Polish subgroups.

Lemma 2. *If* $n \geq 4$ *then the generalized Fermat number* $(2^n + 1)$ *is a sum of a prime and a composite square-free in at least n essentially different ways.*

► CYRUS F. NOURANI, *Functorial syntax and paraconsistent logics*.
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A paraconsistent view to a challenging project [2, 3] which had defined abstract computational agent linguistics with intelligent syntax, model theory, and categories is presented. The paraconsistent functorial syntax and model theory might be defined where, without defying classical or intuitionistic logic, we can sensibly handle inconsistent theories. Without engaging on a fundamental logical and philosophical debate, we offer paraconsistent syntax and structures on a par with a model theory. Inconsistent theories might have a syntactic lift, for which a paraconsistent model is definable. Meta-contextual reasoning, as we have defined in 1994 briefed in [2, 3] is an important application area. Linguistics abstraction had been defined by our [3] in many forms: Intelligent Syntax Abstraction, Abstract Models, and

Language Categories. The reformist's quest for consistency in ontological matters, and the argument to 'thinking necessarily requires inconsistency (and negation-incompleteness)' can be remedied by taking a model-theoretic view to ontologies, where consistency is at models, e.g., model embedding. While 'thinking' can be onto functorial theories on varying syntax and models. The dialectheist's contention that the world is fundamentally inconsistent might be contemplated with worlds defined with functorial syntax. There are possible worlds with their own theories and paraconsistent logics. Our aim is not a recapture of logics in paraconsistent logics, but only to define syntax and model theories with which paraconsistency can be defined by lifts form syntax, context, and models.

[1] J. LAMBEK, *The mathematics of sentence structure*, ***American Mathematical Monthly***, vol. 65, pp. 154–170.

[2] C. F. NOURANI, *Intelligent languages: A preliminary syntactic theory*, in ***Mathematical foundations of computer science, MFCS '98, Brno, Czech Republic, August 1998***, Lecture Notes in Computer Science, no. 1450, Springer-Verlag, 1998.

[3] ———, *Abstract linguistics: A brief overview*, in ***ICML, Mathematical linguistics***, 1996, Tarragona, Catalunia, Spain.

► WILFRIED SIEG, *Turing and Gandy machines*.
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Turing and Gandy provided abstract machines connected by *theses* to deterministic calculations of "human computors", respectively "discrete mechanical devices"; cf. [4], [5], and [1]. Gandy's Thesis is the claim that any such device (can be represented as a discrete dynamical system and its operations) must satisfy the physically motivated restrictive *principles of local causation and unique assembly*; cf. [3]. This is completely parallel to Turing's Thesis expressing the claim that any human computor (can be represented by a discrete dynamical system and his operations) must obey *boundedness and locality conditions*; cf. [2]. The crucial difference between these two models of computation lies in the fact that Turing machines modify in each computation step only *one* bounded part of a state, whereas Gandy machines operate on *arbitrarily many* bounded parts. Gandy machines thus provide a general model of parallel computations. I describe the analyses (leading to the paper machine models), present Gandy machines in detail, and discuss applications.

[1] R. GANDY, *Church's thesis and principles for mechanisms*, in ***The Kleene symposium*** (J. Barwise, H. J. Keisler, and K. Kunen, editors), North-Holland Publishing Company, 1980, pp. 123–148.

[2] W. SIEG, *Step by recursive step: Church's analysis of effective calculability*, ***Bulletin of Symbolic Logic***, vol. 3 (1997), pp. 154–180.

[3] W. SIEG AND J. BYRNES, *An abstract model of parallel computations: Gandy's Thesis*, 1999, to appear in ***The Monist***.

[4] A. M. TURING, *On computable numbers, with an application to the Entscheidungsproblem*, ***Proceedings of the London Mathematical Society***, Series 2, vol. 42 (1936), pp. 239–265.

[5] ———, *Solvable and unsolvable problems*, ***Science News***, vol. 31 (1954), pp. 7–23.

► FRANK WROBLEWSKI, *Resolving Russell's paradox with Quine's definition of class*.
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For Cantor a class was a many-as-one collection. Cantor's fanciful but contradictory metaphor describes a metaphysical construction beyond mortal means. Thus Cantor's class transcends mediation by language, i.e., it transcends definition, and so is solipsistic. With no public handle on Cantor's class, it's not surprising that the axiom legislating class-in-use $\beta \in \{\, \alpha \mid \varphi(\alpha)\,\} \equiv \varphi(\beta)$, where α and β are variables, tripped up on Russell's set. Without knowing the cause of the contradiction, an ad hoc cottage industry grew up to outwit the contradiction by revising the naive axiom, by substituting the part-whole metaphor for Cantor's metaphor, or by rejecting all classes. For the cause of Russell's paradox and its cure, we must turn to Quine.

According to Quine's *Set theory and its logic*, Russell's paradox is the result of substituting classes for variables without the support of comprehension. Here Quine defines the subject-predicate class-in-use notation $\beta \in \{\, \alpha \mid \varphi(\alpha)\,\}$ with a subject-predicate proposition deduced from the class condition $\varphi(\beta)$. To identify the definition of class, it remains to match the terms on either side of the '$\in$' copulas. To resolve Russell's paradox we use Quine's definition to prove that Russell's set cannot be an element of any class, including Russell's set. Quine proves that the characterization of class-in-use requires comprehension:

$$\zeta \in \{\, \alpha \mid \varphi(\alpha)\,\} \equiv \exists\beta\ (\beta = \zeta).\varphi(\zeta),$$

where ζ is a class. When ζ is a variable, the characterization reduces to the naive axiom.

Time permitting, we will discuss the empirical interpretation of mathematics which establishes the mathematics-reality connection and accounts for the indispensability of mathematics to science. Here, variables denote subatomic particles while classes articulate their properties. Comprehension $\exists\beta\ (\beta = \{\, \alpha \mid \varphi\,\})$ expresses the particle-property duality of nature where for any natural property there exists a subatomic particle identifying with that property.