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1998–99 ANNUAL MEETING
OF THE ASSOCIATION FOR SYMBOLIC LOGIC

University of California, San Diego, March 20–23, 1999

The Annual Meeting of the Association was held on the campus of the University of California, San Diego, March 20–23, 1999. The program included eight invited talks, six special sessions, and six contributed talk sessions in covering a broad range of mathematical logic, including philosophical logic. The program also included the Tenth Annual Gödel lecture, a Retiring Presidential Address, a mini-symposium on Mathematical Intuition, and two tutorial lectures on Finite Model Theory. The program committee members were Sam Buss (Chair), Bradd Hart, Carl G. Jockusch, Jr., Vann McGee, and W. Hugh Woodin, and the local organizing committee consisted of Sam Buss, Jeff Remmel, and Gila Sher (Chair).

There were 126 registered participants at the meeting, including 36 graduate students. Additional financial support was provided by the University of California, San Diego.

The hour-long talks at the meeting included the Tenth Annual Gödel Lecture, *Logic and computational complexity*, presented by Stephen Cook, the retiring Presidential Address, by Menachem Magidor, and the following invited talks:

Sy Friedman (MIT), *Genericity and large cardinals*.
Julia F. Knight (Notre Dame), *Complexity and arithmetic*.
Steffen Lempp (Wisconsin-Madison), *The structure of the Σ_2 enumeration degrees*.
Toniann Pitassi (Arizona), *Algebraic proof systems and binomial ideals*.
Hans Schoutens (Wesleyan), *Asymptotically definable structures*.
Simon Thomas (Rutgers), *The automorphism tower problem*.
Victor Vianu (UC San Diego), *Topological queries in spatial databases*.
Jindrich Zapletal (Dartmouth), *Terminal notions in set theory*.

The mini-symposium on Mathematical Logic featured talks by Charles Parsons, *Two concepts of intuition*, and Michael Friedman, *Geometry, construction, and intuition in Kant and his successors*. Ronald Fagin presented two tutorial talks on finite model theory, *Finite-model theory – a personal perspective* and *Easier ways to win logical games*.

There were parallel special sessions on Computability Theory, Finite Model Theory and Stability Theory, Nominalism and Realism, Proof Theory and Complexity, Second-Order Logic, and Set Theory, with a total of 26 speakers. There were six contributed talk sessions with a total of 23 speakers. Six papers were contributed by title.

Abstracts of talks given in person or by title by members of the Association for Symbolic Logic follow.

For the Program Committee
SAM BUSS

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Abstracts of invited tutorials

► RONALD FAGIN, *Finite-model theory – a personal perspective*.
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Finite-model theory is a study of the logical properties of finite mathematical structures. This talk gives an overview, including:

(1) Differences between the model theory of finite structures and infinite structures. Most of the classical theorems of logic fail for finite structures, which gives us a challenge to develop new concepts and tools, appropriate for finite structures.

(2) The relationship between finite-model theory and complexity theory. Surprisingly enough, it turns out that in some cases, we can characterize complexity classes (such as NP) in terms of logic, without using any notion of machine, computation, or time.

(3) Zero-one laws. There is a remarkable phenomenon, which says that certain properties (such as those expressible in first-order logic) are either almost surely true or almost surely false.

(4) Descriptive complexity. Here we consider how complex a formula must be to express a given property.

In recent years, there has been a re-awakening of interest in finite-model theory. One goal of this talk is to help "fan the flames" of interest, by introducing the audience to this fascinating area.

► RONALD FAGIN, *Easier ways to win logical games*.
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The computational complexity of a problem is the amount of resources, such as time or space, required by a machine that solves the problem. The descriptive complexity of a problem is the complexity of describing the problem in some logical formalism. There is an intimate connection between the descriptive complexity and the computational complexity. This connection was first discovered by the speaker, who showed that the complexity class NP coincides with the class of properties of finite structures expressible in existential second-order logic (where we are allowed to existentially quantify over not just points, as in first-order logic, but also over relations). The equivalence of questions in computational and descriptive complexity holds the promise that techniques from one domain can be brought to bear on questions in the other domain.

Essentially the only known technique for proving hardness (that is, inexpressibility) results in descriptive complexity is to make use of games on graphs played between two players. The purpose of this talk is to discuss some results and techniques that assist in the use of these games, and in particular that make the task of proving inexpressibility results easier. Thereby, we can prove new and deeper inexpressibility results. Our hope is that we can develop such a powerful toolkit that we can eventually make a serious assault on such fundamental problems as the question of whether NP = co-NP.

Abstacts of invited talks in the Mini-symposium on Mathematical Intuition

► MICHAEL FRIEDMAN, *Geometry, construction, and intuition in Kant and his successors*.
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The logical interpretation of Kant's theory of the role of intuition in geometry originally articulated by Evert Beth and Jaakko Hintikka conceives this role as primarily formal or inferential. Intuition serves to generate singular terms in mathematical reasoning in inferences such as we would represent today by existential instantiation. On the approach defended by Charles Parsons, by contrast, the primary role of intuition is to acquaint us with certain phenomenological or perceptual spatial facts which can then be taken to provide evidence for or to verify the axioms of geometry. In this paper I try to build a bridge between the logical and phenomenological interpretations by developing an account of what Kant might mean by the a priori structure of perceptual space. This a priori structure is constituted by the subject's ability imaginatively to locate itself in space at a particular point of view and with a particular orientation or perspective, and then to change this point of view and orientation by translation and rotation respectively. This interpretation then allows us to connect Kant's doctrine with Hermann von Helmholtz's work on space perception and the foundations of geometry, as well as with the closely related work of Henri Poincaré. I conclude with a few remarks on Hermann Weyl's generalized solution to the "space-problem," which, in a certain sense, completes the story about space as a form of intuition begun by Kant.

► CHARLES PARSONS, *Two conceptions of intuition*.
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The philosophical conception of intuition most often appealed to in earlier studies in the foundations of mathematics is derived from Kant's and shares some basic features with his. The primary notion of intuition is intuition of objects; intuitive knowledge rests on such intuition. It is restricted in its application roughly to space and time. This has had the consequence that the demand that mathematics have an intuitive foundation led to severe restrictions on mathematics, as in Brouwer's original version of intuitionism and Hilbert's finitary method for metamathematics.

A more generally applicable conception of intuition might be called that of rational intuition. In contrast to Kantian intuition, rational intuition is primarily and in some versions even purely propositional. The conception begins with the tendency in inquiry to take certain propositions as evident, or at least plausible, without observation or deriving them from others by means of argument. The term "intuition" is often used in philosophy for instances of this tendency of quite varying epistemic weight. It can be argued that it is central to the exercise of reason and thus not restricted as to content as Kantian intuition is. This is in agreement with the views of Gödel.

Some comments will be made on the relation of these two conceptions to each other and to the use of the term "intuition" by mathematicians.

Abstracts of invited plenary talks

► JULIA F. KNIGHT, *Complexity and arithmetic*.
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We identify a model A of arithmetic with its atomic diagram, and we measure the complexity of A by its Turing degree. The standard model has copies in all degrees. By a result of Solovay [5], simplified by Marker [3], the degrees of non-standard models of TA (true arithmetic) are the degrees of "enumerations" of "Scott sets" containing the arithmetical sets. Solovay [6] also characterized the degrees of models of other completions of PA (first order Peano arithmetic). (See [1].)

There were some natural questions on complexity of models of arithmetic asked before

[5] and [6]. Lerman [2] asked whether there is a non-standard model A of TA with no enumeration of just the arithmetical sets computable in A. Lachlan and Soare showed that there is such a model (see also McAllister [4]). Abramson and Knight asked whether there is a non-standard model A of TA whose degree is minimal over the arithmetical degrees. A new result implies that there is no such model.

[1] J. F. KNIGHT, *True approximations and models of arithmetic*, to appear in ***Colloquium '97*** (Cooper and Truss, editors).

[2] M. LERMAN, *Upper bounds for the arithmetical degrees*, ***Annals of Pure and Applied Logic***, vol. 29 (1985), pp. 225–254.

[3] A. MACINTYRE and D. MARKER, *Degrees of recursively saturated models*, ***Transactions of the American Mathematical Society***, vol. 282 (1984), pp. 539–554.

[4] A. M. MCALLISTER, *Completions of PA: models and enumerations of representable sets*, ***Journal of Symbolic Logic***, vol. 63 (1998), pp. 1063–1082.

[5] R. SOLOVAY, preprint circulated in 1982.

[6] ———, personal correspondence, 1991.

► STEFFEN LEMPP, *The structure of the Σ_2 enumeration degrees*.
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I will report on progress on deciding the $\forall\exists$ theory of the Σ_2 enumeration degrees (under partial ordering), in particular, that all finite lattices are embeddable into the Σ_2 enumeration degrees preserving 0 and 1 (joint work with A. Sorbi), and on progress toward a decision procedure for the extension of embeddings problem (joint work with T. A. Slaman and A. Sorbi).

► TONIANN PITASSI, *Algebraic proof systems and binomial ideals*.
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The polynomial calculus (PC) is a natural, algebraic proof system for proving that a system of polynomial equations are unsolvable. In this system, one starts with an initial family of polynomial equations, $P_1 = 0, \dots, P_m = 0$, (typically over a field), and the goal is to derive $1 = 0$. The allowable operations are: (1) adding linear combinations of previously derived equations, and (2) multiplying a previously derived equation by a variable. The degree of a PC refutation is the maximal degree of any intermediate polynomial in the derivation. This proof system and its variants have been well-studied over the last several years due to its fundamental nature, and also because of its connections with classical proof complexity.

In this talk, we first present a brief survey of algebraic proof systems: what they are, their connections to classical proofs, and the state-of-the-art in lower and upper bounds.

We will then focus on proving good lower bounds on PC degree, a problem that has received much attention recently. We will present nearly optimal lower bounds on the minimal degree of PC refutations of Tseitin's graph tautologies and the mod p counting principles. The lower bounds apply to the PC over rings or fields. These are the first linear lower bounds for PC; and distinguish linearly the rings Z_q and Z_r where q and r do not have identical prime factors. Moreover, our proof is quite simple, and the method applies to any initial system of binomial ideals.

This is joint work with Sam Buss, Dima Grigoriev, and Russell Impagliazzo.

► HANS SCHOUTENS, *Asymptotically definable structures*.
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Many geometrically meaningful sets are clearly not first-order definable: for instance, the collection of prime ideals of an affine ring; the set of (isomorphism classes) of finitely generated modules over an affine ring, etc., are not first-order definable in the field K. However, we can slice up these sets, such that each slice becomes definable. The basic idea is that we look at a slice consisting of data which can be described with a fixed number of polynomials of a fixed degree and the encoding is then given by a tuple listing all the coefficients of the polynomials involved. Sets of this form will be termed *asymptotically definable*. In an asymptotically definable structure, one can make first-order statements about its elements and about constructions applied to its elements. The prerequisite is that such a construction should be *bounded* in the following sense: if we start with an object described by polynomials of degree at most d, then the result after applying the construction is again an object described by polynomials of degree at most d', where d' only depends on d and not on the objects nor the base field.

We briefly discuss two applications of this technique: transfer of results from one characteristic to the other via the Lefschetz Principle and uniform constructibility results of certain numerical invariants. Among the former we will discuss the Bass Conjecture, the Zariski-Lipman Conjecture, and the New Intersection Theorem; among the latter we will discuss Betti and Bass numbers, Cohen-Macaulay and Gorenstein loci, and Hilbert series.

► SIMON THOMAS, *The automorphism tower problem*.
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If G is a centreless group, then there is a natural embedding of G into its automorphism group $\mathrm{Aut}(G)$, obtained by sending each $g \in G$ to the corresponding inner automorphism $i_g \in \mathrm{Aut}(G)$. It turns out $\mathrm{Aut}(G)$ is also centreless, and this enables us to define the automorphism tower of G to be the ascending chain of groups

$$G = G_0 \leqslant G_1 \leqslant G_2 \leqslant \cdots G_\alpha \leqslant G_{\alpha+1} \leqslant \cdots$$

such that for each ordinal α

(a) $G_{\alpha+1} = \mathrm{Aut}(G_\alpha)$; and
(b) if α is a limit ordinal, then $G_\alpha = \bigcup_{\beta<\alpha} G_\beta$.

The automorphism tower is said to terminate if there exists an ordinal α such that $G_{\alpha+1} = G_\alpha$. In 1939, Wielandt proved that if G is finite, then its automorphism tower terminates after finitely many steps. In 1984, I showed that the automorphism tower of an arbitrary centreless group eventually terminates; and that for each ordinal α, there exists a group whose automorphism tower terminates after exactly α steps.

In this talk, I will survey some more recent developments in this area and discuss some of the many open problems.

► VICTOR VIANU, *Topological queries in spatial databases*.
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Handling spatial information is required by many database applications, and each poses different requirements on query languages. In many cases the precise size of the regions is important, while in other applications we may only be interested in the *topological* properties of regions, which are invariant under homeomorphisms. Such differences in scope and emphasis are crucial, as they affect the data model, the query language, and performance. This talk focuses on logics targeted towards topological properties of two-dimensional regions specified by polynomial inequalities with integer coefficients. We focus on two main aspects: (i) languages for expressing topological queries, and (ii) the representation of topological information. In regard to (i), we study several region-based variations of first-order logic, which use as basic predicates well-known topological relationships between pairs of planar regions. In regard to (ii), we show that the topological information in a spatial database can be precisely summarized by a finite structure which can be viewed as a topological annotation to the raw spatial data. All topological queries can be answered using this annotation, called topological invariant. This yields a potentially more economical evaluation strategy for such queries, since the topological invariant is generally much smaller than the raw data. We examine in detail the problem of translating topological queries against the spatial database into queries against the topological invariant. The languages considered are first-order on the spatial database side, and fixpoint and first-order on the topological invariant side. In particular, fixpoint turns out to express precisely the PTIME queries on topological invariants. This suggests that topological invariants are particularly well-behaved with respect to descriptive complexity. These results parallel recent results in finite-model theory on the expressiveness of fixpoint logic over planar graphs.

► JINDRICH ZAPLETAL, *Terminal notions in set theory*.
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For certain notions currently in use in mathematics a thesis can be exactly formulated and proved saying that these notions cannot be split into finer subnotions. This involves proving some absoluteness theorems and a construction of some optimal models of set theory. It is a formal explanation of the fact that mathematicians intuitively consider some of these notions as optimal in a certain type of arguments.

I will give several examples.

Abstracts of invited talks in the Special Session on Computability Theory

► DOUGLAS CENZER AND ANDRE NIES, *Initial segments of the lattice of* Π^0_1 *classes*.
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Department of Mathematics, University of Chicago, Chicago, IL 60637, USA.

The study of the lattice $\mathcal{E}$ of computably enumerable sets under inclusion has been one of the central tasks of computability theory since the 1960s. We investigate initial segments of the lattice $\mathcal{L}$ of Π^0_1 classes (of sets) under inclusion and we compare this lattice with $\mathcal{E}$. It was recently proved by Nies that the theory of each interval of the lattice $\mathcal{E}$ which is not a Boolean algebra interprets true arithmetic (and is therefore certainly undecidable).

However, we will show that in $\mathcal{L}$ there are initial segments $[\emptyset, P] = \mathcal{L}(P)$ which are not Boolean algebras, but which have a decidable theory. In fact, we will construct, for any finite distributive lattice L which satisfies the dual of the usual reduction property, a Π^0_1 class P such that L is isomorphic to the lattice $\mathcal{L}(P)$ modulo finite differences.

The construction of the Π^0_1 class corresponding to a given lattice builds on the construction of a *minimal* Π^0_1 class by Cenzer, Downey, Jockusch and Shore in 1993. The simplest minimal Π^0_1 class P has a single limit point together with countably many isolated points. P has the property that every Π^0_1 subclass Q of P is either finite or is cofinite in P—furthermore, Q is the intersection of P with a clopen set. Thus the lattice $[0, P]$ of Π^0_1 subclasses of P is isomorphic to the class of finite/cofinite subsets of ω and is a Boolean algebra. Such a class plays a role in the lattice $\mathcal{L}$ corresponding to the dual of the role played by a maximal c.e. set in the lattice $\mathcal{E}$.

For the simplest new Π^0_1 class P constructed, P has a single, non-computable limit point and $\mathcal{L}(P)^*$ has three elements, corresponding to $\emptyset$, P and a minimal class $P_0 \subset P$. The third type of subclass has no complement in the lattice, which is why the lattice is not a Boolean algebra. On the other hand, the theory of L is shown to be decidable, so that this lattice may not be realized as the class of c.e. subsets of any c.e. set.

A Π^0_1 class P is said to be decidable if it is the set of paths through a computable tree T with no dead ends. We show that if P is decidable and has only finitely many limit points, then $\mathcal{L}(P)^*$ is always a Boolean algebra. We show that if P is a decidable Π^0_1 class and $\mathcal{L}(P)$ is not a Boolean algebra, then the theory of $\mathcal{L}(P)$ interprets the theory of arithmetic and is therefore undecidable.

► ROD DOWNEY, *Automorphisms of the lattice of* Π^0_1 *classes*.
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Π^0_1 classes are important to the logical analysis of many parts of mathematics. They are important not only to computability theory but also to proof theory via systems such as WKL_0. In this talk I will report on some work looking at definability in the lattice of Π^0_1 classes and degrees of unsolvability. In particular we demonstrate that there is an analog of the Martin-Soare work on maximal sets and high degrees available in this setting, via thin classes and anr degrees. This is joint work with Richard Coles, Peter Cholak, and Eberhard Herrmann.

► LEO HARRINGTON, *Invariant jump classes*.
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This talk will report on joint work with Peter Cholak. A class of r.e. sets is called invariant with respect to the structure of all r.e. sets if it is fixed by all automorphisms of that structure. A class of Turing degrees of r.e. sets is called invariant if it is the class of degrees of members of an invariant set. It is shown that for $n > 1$, the high-n classes and the complements of the low-n classes are all invariant.

► GEOFFREY LAFORTE, *Isolation in higher degree types*.
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Isolation arises when there is some pair of degrees $\mathbf{a} < \mathbf{b}$ such that there do not exist any degrees of some specified class $\mathcal{C}$ between the two. Isolation is interesting as a weaker, and therefore more widespread, version of nondensity. Additionally, isolation often provides an example of a maximal $\mathcal{C}$-degree $\mathbf{a}$ for which $\mathbf{b}$ is CEA. The simplest case of any interest arises in the context of the difference hierarchy when $\mathbf{a}$ is c.e., $\mathbf{b}$ is d.c.e., and $\mathcal{C}$ is the class of c.e. degrees. In this case, $\mathbf{a}$ is called an isolated d.c.e. degree. Recent work by Cooper and Yi [2], Arslanov, Lempp and Shore [1], and LaForte [3] has focused on these isolated d.c.e. degrees. We give some results about the situation at higher levels in the difference hierarchy. We also examine the more general situation of isolation in the CEA-hierarchy, where things are more complex because the sets involved are not Δ^0_2.

[1] M. M. Arslanov, S. Lempp, and R. A. Shore, *On isolating r.e. and isolated d.r.e. degrees*, in ***Computability, enumerability, unsolvability*** (S. B. Cooper et al., editors), London Mathematical Society Lecture Notes, no. 224, Cambridge University Press, 1996, pp. 61–80.

[2] S. B. Cooper and X. Yi, *Isolated d.r.e. degrees*, ***Mathematical Logic Quarterly***, vol. 42 (1996), no. 2.

[3] G. LaForte, *The isolated d.r.e. degrees are dense in the r.e. degrees*, ***Mathematical Logic Quarterly***, vol. 42 (1996), no. 2, pp. 83–103.

► MANUEL LERMAN, *The decidability of the existential theory of the poset of degrees with jump and least element*.
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We present a decision procedure for the existential theory of the poset of degrees with jump and least element; a sentence in this language is true if it is consistent with the axioms for posets with least element, and the axioms stating that the jump operator $'$ is an order-preserving function which also satisfies $x < x'$ for all x. The proof technique is an iterated trees of strategies priority argument of level n, where n is effectively determined from the sentence.

► RICHARD A. SHORE, *Computable dimension, degree spectra, and expansions by constants*.
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The *computable dimension* of a structure $\mathcal{A}$ is the maximal cardinality of a set of computable structures $\mathcal{A}_i$ each isomorphic to $\mathcal{A}$ but pairwise not computably isomorphic. The *degree spectrum* of a relation R on a computable structure $\mathcal{A}$ is the set of the Turing degrees of all images of R under isomorphisms from $\mathcal{A}$ to a computable structure $\mathcal{B}$.

This talk will discuss recent results on constructing computable structures $\mathcal{A}$ and relations R on $\mathcal{A}$ while simultaneously controlling both the computable dimension of $\mathcal{A}$ and the degree spectrum of R. The results represent work of Denis Hirschfeldt, Bakhadyr Khoussainov and the speaker. They answer questions from [2] and [4]. The methods developed by Hirschfeldt [3] to solve these problems also provide the tools to settle a problem left open in [1] about the possible effects of expansions of a computably categorical structure $\mathcal{A}$ by constants on its computable dimension. All of the phenomena discussed occur in some common algebraic structures such as nilpotent groups, rings, and even integral domains.

[1] P. CHOLAK, S. S. GONCHAROV, B. KHOUSSAINOV, and R. A. SHORE, *Computably categorical structures and expansions by constants*, ***Journal of Symbolic Logic***, to appear.

[2] S. S. GONCHAROV and B. KHOUSSAINOV, *Degree spectra of decidable relations*, ***Doklady Akademii Nauk SSSR***, vol. 352 (1997), pp. 301–303.

[3] D. HIRSCHFELDT, ***Ph.D. dissertation***, Cornell University, 1999.

[4] B. KHOUSSAINOV and R. A. SHORE, *Computable isomorphisms, degree spectra of relations and Scott families*, ***Annals of Pure and Applied Logic***, vol. 93 (1998), pp. 153–193.

Abstracts of invited talks in the Special Session on Finite Model and Stability Theory

► MICHAEL BENEDIKT, *Some extensions of finite model theory*.
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In this talk I'll overview several extensions of finite model theory that combine an uninterpreted component with some fixed, interpreted structure. I'll concentrate on two extensions: the Metafinite model theory of Gurevich and Gradel, and the Embedded Finite Model Theory that comes out of the constraint database community.

► MARKO DJORDJEVIĆ, *Finite variable logic, stability, and finite models*.
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L^n is first-order logic with the restriction that at most n distinct variables may occur in a formula. Analogues of concepts from stability theory will be introduced for L^n-theories. It will follow that for L^n-theories with finitely many L^n-types (so in particular for L^n-theories with finite models) stability and ω-stability are equivalent. We also consider the problem of when an L^n-theory T with infinite models also has finite models. It is easy to see that if T has infinitely many L^n-types then T has no finite models. Since the L^3-theory of dense linear order has finitely many L^3-types but has no finite models it follows that a finite bound on the number of L^n-types is not enough to guarantee the existence of finite models. However, the L^3-theory of dense linear order is unstable. We will see that if an L^n-theory T has a natural amalgamation property and is stable and has only finitely many L^n-types then T can be extended to an L-theory which is ω-categorical and ω-stable, and from this it follows that T has arbitrarily large finite models.

► LEO HARRINGTON, *Finite ultrahomogeneous structures*.
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This will be a presentation of the Cherlin-Lachlan classification of finite ultrahomogeneous structures in a fixed finite relational language. (Here ultrahomogeneous means: quantifier-free homogeneous.) This result relies on the classification of finite simple groups to show that the theory (of all finite ultrahomogeneous structures in a fixed finite relational language) is omega-stable of finite rank. This presentation will not deal with that part of the argument, but will rather assume it and then present how Lachlan's shrinking technique leads to a classification.

► TAPANI HYTTINEN, *Stability in finite models*.
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We assume that τ is a finite similarity type, k is greater or equal to the arity of τ and in any case ≥ 2, $\mathcal{L} = L^k$, T is a complete $\mathcal{L}$-theory and

$$\mathbf{K} = \{\, \mathcal{A} \mid \mathcal{A} \vDash T \text{ is a finite } \tau\text{-model} \,\}.$$

In addition we assume that $\{\, \mathcal{A}/\cong \mid \mathcal{A} \in \mathbf{K} \,\}$ is infinite and that the following amalgamation property holds: Let $\mathcal{A}$ and $\mathcal{B}$ be $\mathbf{K}$-models (i.e., $\in \mathbf{K}$) and $t_{\mathcal{A}}(\mathcal{A} \cap \mathcal{B}, \emptyset) = t_{\mathcal{B}}(\mathcal{A} \cap \mathcal{B}, \emptyset)$. Then there is a $\mathbf{K}$-model $\mathcal{C}$ and a function $f : \mathcal{A} \cup \mathcal{B} \to \mathcal{C}$ such that $f \restriction \mathcal{A}$ and $f \restriction \mathcal{B}$ are $\mathcal{L}$-elementary.

We define a notion of stability for $\mathbf{K}$ and create a theory of forking under the assumption that $\mathbf{K}$ is stable. Finally, we apply the theory to model constructions, e.g., we prove, for all $n \in \mathbb{N}$, the existence of n-saturated $\mathbf{K}$-models by assuming that forking behaves nicely.

► MARTIN GROHE AND PHOKION G. KOLAITIS, *On global closure ordinals*.
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The *closure ordinal* $\mathrm{cl}(\varphi, \mathbf{A})$ of a first-order formula $\varphi(\bar{x}, X)$ on a structure $\mathbf{A}$ is the least ordinal ξ such that $\varphi^{\xi}_{\mathbf{A}} = \varphi^{<\xi}_{\mathbf{A}}$, where

$$\varphi^{\xi}_{\mathbf{A}} = \varphi^{<\xi}_{\mathbf{A}} \cup \{\, \bar{a} \mid \mathbf{A} \vDash \varphi(\bar{a}, \varphi^{<\xi}_{\mathbf{A}}) \,\} \qquad \text{and} \qquad \varphi^{<\xi}_{\mathbf{A}} = \bigcup_{\eta<\xi} \varphi^{\eta}_{\mathbf{A}}.$$

This is a classical *local* notion that has been extensively studied during the 1960s and the 1970s in the context of inductive definability on fixed infinite structures. Since the early 1980s, researchers in finite model theory have carried out a detailed study of *global* inductive definability on classes of finite structures, and have shown it to be intimately connected to polynomial-time computability. By and large, however, the study of global inductive definability on classes of infinite structures has remained unexplored.

We introduce and study the concept of the *global closure ordinal* $\mathrm{gcl}(\varphi)$ of a first-order formula $\varphi(\bar{x}, X)$, where $\mathrm{gcl}(\varphi)$ is the supremum of the closure ordinals $\mathrm{cl}(\varphi, \mathbf{A})$ over all structures **A**. We establish that universal first-order formulas and positive existential-universal first-order formulas can realize every global closure ordinal, a result that sharply contrasts with the state of affairs in local inductive definability. We also characterize the class of global closure ordinals in terms of a certain Π^1_1-describability criterion, a result that makes it possible to derive further structural properties of global closure ordinals.

After this, we investigate the uniform closure ordinals of inductively definable queries, where the *uniform closure ordinal* of a query Q is the infimum over the global closure ordinals of all possible inductive definitions of Q. Our main result here is that a query Q has uniform closure ordinal less than or equal to ω if and only if it is inductively definable by an existential first-order formula. Finally, using results from the model theory of infinitary logic and admissible fragments, we show that a countable admissible ordinal is a global closure ordinal if and only if it is the uniform closure ordinal of a query inductively definable by a universal first-order formula.

► MICHAEL C. LASKOWSKI, *The independence property and finitary analogues*.
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This talk will survey known results about the independence property of families of sets. We discuss both what can be said in full generality and when the sets form a uniformly definable family.

Abstract of invited talk in the Special Session on Second-Order Logic

► MICHAEL D. RESNIK, *The renaissance of second-order logic in the philosophy of mathematics*.
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Early Twentieth Century philosophy of mathematics enjoyed important successes through using second-order logic, but the supposed set theoretic presuppositions of this logic led subsequent philosophers to denigrate its status as logic and the achievements based upon it. Recently, both philosophical and technical considerations have led to a rebirth of its use in the philosophy of mathematics. This paper surveys these developments.

Abstracts of invited talks in the Special Session on Proof Theory and Complexity

► S. BELLANTONI AND MARTIN HOFMANN, *A new "feasible" arithmetic*.
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A classical quantified modal logic is used to define a "feasible" arithmetic $\mathcal{A}^1_2$ whose provable functions are exactly the polynomial-time computable functions. Informally, one understands $\Box\alpha$ as "α is feasibly demonstrable".

$\mathcal{A}^1_2$ differs from a system $\mathcal{A}_2$ that is as powerful as Peano Arithmetic only by the restriction of induction to ontic formulas. (An ontic formula is one whose type is $\Box$-free.) Thus, $\mathcal{A}^1_2$

is defined without any reference to bounding terms, and admitting induction over formulas having arbitrarily many alternations of unbounded quantifiers, albeit for so-called "tier 0" values only. The system also uses only a very small set of initial functions.

To obtain the characterization, one extends the Curry-Howard isomorphism to include modal operations. This leads to a realizability translation based on recent results in higher-type ramified recursion. The fact that induction formulas are not restricted in their logical complexity, allows one to use the Friedman A translation in a way that would be impossible for conventional bounded arithmetics.

We claim that $\mathcal{A}_2^1$ provides a "foundational" definition of polytime provability. The development also leads us to propose a new Frege rule, the "Modal Extension" rule: if $\vdash \alpha$, then $\vdash A \leftrightarrow \alpha$ for new symbol A.

The assistance of the Fields Institute for Research in Mathematical Sciences is gratefully acknowledged.

[1] S. Bellantoni, *Ranking arithmetic proofs by implicity ramification*, in ***Proof complexity and feasible arithmetics*** (P. Beame and S. Buss, editors), DIMACS Series in Discrete Mathematics, no. 39, 1998.

[2] Martin Hofmann, *Type systems for polynomial-time computation*, ***Habilitationsschrift***, Darmstadt University of Technology, 1998, available via `www.dcs.ed.ac.uk/home/mxh/`.

[3] Edward Nelson, *Understanding intuitionism*, presented at the *Recontre du Reseau Georges Reeb*, March 24–28, 1997.

► JAN JOHANNSEN AND CHRIS POLLETT, *Bounded arithmetic, threshold circuits, and counterexample computations*.
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The rule Δ_1^b-*COMP* is defined as follows: If a predicate $A(x)$ has been proven to be Δ_1^b, i.e., equivalent to both a Σ_1^b- and a Π_1^b-predicate, then we conclude the comprehension axiom for A

$$\exists y < 2^{|a|}\, \forall x < |a|\, (\mathrm{Bit}(y, x) = 1 \leftrightarrow A(x)).$$

We consider the theory of Bounded Arithmetic Δ_1^b-*CR* axiomatized by *BASIC*, *open-LIND* and the Δ_1^b-*COMP* rule, and show that this theory is closely related to the complexity class (uniform) TC^0.

First, the Σ_1^b-definable functions of Δ_1^b-*CR* are TC^0. In fact, we show that the theory C_2^0 of [1], whose Σ_1^b-definable functions are TC^0, is $\forall\Sigma_1^b$-conservative over Δ_1^b-*CR*.

Moreover, a closer relationship between Δ_1^b-*CR* and TC^0 is established by the following witnessing theorem using counterexample computations:

Theorem 1. *Assume Δ_1^b-CR $\vdash \exists x\, \forall y\, A(a, x, y)$, where A is Δ_1^b with respect to Δ_1^b-CR. Then there are $k \in \mathbb{N}$ and functions $f_1, \dots, f_k \in TC^0$, that are Σ_1^b-definable in Δ_1^b-CR, such that Δ_1^b-CR proves*

$$A(a, f_1(a), b_1) \vee A(a, f_2(a, b_1), b_2) \vee \cdots \vee A(a, f_k(a, b_1, \dots, b_{k-1}), b_k).$$

As in [2], this is used to show that a collapse of arithmetic theories implies a collapse of complexity classes.

Theorem 2. *If $S_2^1 = \Delta_1^b$-CR, then NP is contained in non-uniform TC^0.*

[1] J. Johannsen and C. Pollett, *On proofs about threshold circuits and counting hierarchies* (extended abstract), in ***Proceedings of the 13th IEEE symposium on logic in computer science***, 1998, pp. 444–452.

[2] J. Krajíček, P. Pudlák, and G. Takeuti, *Bounded arithmetic and the polynomial hierarchy*, ***Annals of Pure and Applied Logic***, vol. 52 (1991), pp. 143–153.

► JULIETTE KENNEDY, *On an unprovability result in bounded arithmetic*.
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We give a proof of the following result: if D_2' proves double length induction for $\Sigma_0^b(\Sigma_1^b)$-formulas then $P = NC$. We discuss consequences of the above for incompleteness theorems in bounded arithmetic theories. This is joint work with A. Ignjatovíc.

► CHRIS POLLETT, *Using translation to separate bounded arithmetic theories*.
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In this talk, we consider the use of translations to separate bounded arithmetic theories. Many of the known non-relativised separations of bounded arithmetic theories were achieved using an upward RSUV-like translation or an abstraction thereof. We will survey some of these results, and then consider the use of downward translations to separate theories. In particular, we will sketch a proof that IOpen(exp) and $\mathrm{I}\Delta_0(\exp)$ are different.

► RICHARD SOMMER, *The computational complexity of uniform reflection*.
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We present a mathematical (complexity-theoretic) characterization of the metamathematical principle of uniform reflection. In particular, we will state results asserting that, over elementary recursive arithmetic (ERA), the uniform Σ_1^0-reflection principle for T (i.e., the assertion that every T-provable Σ_1^0-formula is true) is equivalent to the assertion of the totality of a specific function in the fast-growing hierarchy, for various commonly studied subsystems T of first- and second-order arithmetic. Also, statements in the hierarchy generated by iterating uniform Σ_1^0-reflection, starting with ERA, correspond exactly to statements of totality of functions in the fast-growing hierarchy.

► ALASDAIR URQUHART, *Complexity of some substructural logics*.
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We prove two complexity results for propositional linear logic with the unrestricted weakening rule, and an extension of it.

Theorem 1. (1) *The decision problem for* **LLW** *is exponential space hard with respect to* log-lin *reducibility.*
(2) *There is a constant $c > 1$ so that any Turing machine solving the decision problem for* **LLW** *must use space c^n for infinitely many inputs of length n.*
(3) *The decision problem for* **LLW** *can be solved in doubly exponential space.*

The upper bound in part (3) improves on the upper bound derivable from Kopylov's original decision procedure [2], which is non-primitive-recursive.

Let **ELLW** be the sequent calculus of Extended Linear Logic of Kanovich [1], with the rule of weakening added. This calculus admits cut elimination, but does not satisfy the subformula property.

THEOREM 2. *There is no primitive recursive decision procedure for* **ELLW**.

[1] MAX I. KANOVICH, *Linear logic as a logic of computations*, ***Annals of Pure and Applied Logic***, vol. 67 (1994), pp. 183–212.

[2] A. P. KOPYLOV, *Decidability of linear affine logic*, in ***Proceedings of the 10th annual IEEE symposium on logic in computer science***, IEEE Computer Society Press, 1995.

Abstracts of invited talks in the Special Session on Set Theory

► HOWARD BECKER, *Ideals without ccc and without property* (M).
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We prove a strong version of a theorem of Balcerzak- Roslanowski-Shelah by showing, in ZFC, that there exists a simply definable Borel sigma-ideal for which both the ccc and property (M) fail. The proof involves Polish group actions.

► STEVE JACKSON, *On the theory of* $\mathbf{\Pi}^1_5$ *sets*.
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Kechris and Martin proved, assuming the axiom of determinacy, that the collection of $\mathbf{\Pi}^1_3$ sets is closed under $< \boldsymbol{\delta}^1_3$ length unions and intersections. In fact, they proved a stronger lightface version from which the boldface version follows. We extend these results to $\mathbf{\Pi}^1_5$ sets, and indicate how to generalize to all $\mathbf{\Pi}^1_{2n+1}$.

Abstracts of invited talks in the Special Session on Nominalism and Realism

► JOHN BURGESS, *Mathematics and Bleak House*.
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In the wake of the questionable outcome of various projects for the nominalist reconstrual of mathematics, there has been a recrudescence of fictionalism in philosophy of mathematics. As compared with reconstructive nominaelism, fictionalism has "the advantages of theft over honest toil". But it also has certain disadvantages, some of which it will be my aim to bring out.

► PENELOPE MADDY, *Methodology for the naturalist*.
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Among the outstanding methodological questions of contemporary set theory are these: do statements independent of the standard set theoretic axioms remain genuine mathematical problems? and if so, what means are appropriate for answering them? In this talk, I explore a naturalistic approach to these questions.

Abstracts of contributed talks

► TAREK SAYED AHMED, *The class of neat reducts is not elementary*.
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We solve a problem of Tarski, and his co-authors Andréka, Henkin, Monk, and Németi

[1]. We will show that for any pair of ordinals $1 < \alpha < \beta$, the class of α-dimensional neat reducts of β-dimensional cylindric algebras is not elementary.

This work has been partially supported by the Hungarian National Foundation for Scientific Research Grant No. T16448.

[1] L. Henkin, J. D. Monk, A. Tarski, H. Andréka, and I. Németi, ***Cylindric set algebras***, Lecture Notes in Mathematics, no. 881, Springer-Verlag, Berlin, 1981.

► ANDREW ARANA, *Degrees of n-diagrams of nonstandard models of completions of arithmetic*.

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For a nonstandard model A of first order Peano arithmetic (PA), let $SS(A)$ denote the Scott set (or standard system). Let $D_n(A)$ denote the n-diagram. First, considering models of true arithmetic (TA), we show that for any Scott set S which includes the arithmetical sets, the following are the same for all $n \geq 0$:

$$\{ \deg(D_n(A)) : A \text{ a nonstandard model of } TA \text{ with } SS(A) = S \}.$$

We then consider models of other completions of PA.

[1] J. F. Knight, *True approximations and models of arithmetic*, to appear in ***Colloquium '97*** (Cooper and Truss, editors).

► MATTHIAS BAAZ AND RICHARD ZACH, *Infinite-valued Gödel logics*.

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Takeuti and Titani [4] investigated the first-order version of infinite-valued Gödel logic with the real interval [0, 1] as its set of truth values. This logic was shown to be axiomatizable, but no cut-free system is known. We present such a cut-free axiomatization based on Avron's [1] hypersequent calculus for infinite-valued propositional Gödel logic. The proof of completeness of the system uses only the fact that the set of truth values contains a densely ordered subset (we call such sets *saturated*). It is thus shown that all Gödel logics based on such sets of truth values are the same. We show furthermore that any other infinite-valued first-order Gödel logic is not axiomatizable. (This was shown for a special case in [2].) In contrast to the propositional case, the order structure and not just the cardinality of the set of truth values determines the set of valid first-order formulas. The situation is analogous to the propositional case insofar as all Gödel logics based on saturated sets of truth values determine the same consequence relation, which is compact. All other infinite-valued Gödel logics are not compact [3].

[1] A. Avron, *Natural 3-valued logics—characterization and proof theory*, ***Journal of Symbolic Logic***, vol. 56 (1991), no. 1, pp. 276–294.

[2] M. Baaz, A. Leitsch, and R. Zach, *Incompleteness of an infinite-valued first-order Gödel logic and of some temporal logics of programs*, in ***Computer science logic, selected papers from CSL '95*** (E. Börger, editor), Springer-Verlag, Berlin, 1996.

[3] M. Baaz and R. Zach, *Compact propositional Gödel logics*, in ***Proceedings of the 28th international symposium on multiple-valued logic, May 1998, Fukuoka, Japan***, IEEE Press, Los Alamitos, 1998, pp. 108–113.

[4] G. Takeuti and T. Titani, *Intuitionistic fuzzy logic and intuitionistic fuzzy set theory*, ***Journal of Symbolic Logic***, vol. 49 (1984), pp. 851–866.

► PAUL BANKSTON, *The model theory of continua*.
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Model theory provides an excellent conceptual platform for examining (classes of) algebraic objects. Somewhat less straightforwardly, but just as elegantly, it can be used as a conceptual platform in topology. Our long-term study applies model-theoretic (mainly ultraproduct) methods to create conjectures and prove theorems about compacta (notably continua) and the maps between them. One payoff of this study has been the identification of the class of *co-existentially closed continua*, a subclass of the class of unidimensional indecomposable continua. Every continuous surjection from a continuum onto a co-existentially closed continuum is weakly confluent, so our new class is contained in Lelek's class W. What is more, from the standpoint of category duality, this class bears the same relationship to the class of continua as the class of algebraically closed fields does to the class of fields; and one of many open questions is whether every unidimensional indecomposable continuum is co-existentially closed.

► M. BURGIN, *Algorithmic problems for super-recursive algorithms*.
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The traditional formalism for a substantive concept of algorithm presupposes two essential conditions: the finiteness and constructibility of the input, processed, and output information over a finite interval of time, as well as the finiteness and constructibility of the list of rules for realization of the processing. In contrast to an informal notion, other conditions of algorithm specification appear only in mathematical models. One of these conditions is the demand that to obtain a result of a computation, it is necessary to finish the computational process. However, it is superfluous in many situations because quantities of processes that are naturally considered as algorithmic do not satisfy this restriction. That is why a more general model, called a super-recursive algorithm, has been developed, making it possible to increase to a great extent performance abilities of systems of algorithms [1].

Inductive Turing machines (iTm) are the closest super-recursive systems to the ordinary algorithms having at the same time much more computing power [2].

Let K, H be two classes of algorithms. We consider the following algorithmic problem.

(K, H)-PROBLEM. Is there some A from H that determines for any input value x whether an arbitrary algorithm B from K gives the result being applied to x?

Let RITM and TM be the classes of all iTm with a recursive memory and ordinary Turing machines, correspondingly.

THEOREM.
(a) (TM, RITM)-*problem has the positive solution*;
(b) (RITM, RITM)-*problem has the negative solution*.

[1] M. S. BURGIN, *The notion of algorithm and the Turing-Church's thesis*, ***VIII International Congress on logic, methodology and philosophy of science***, vol. 5, part 1, Moscow, 1987, pp. 138–140.

[2] ———, *Inductive Turing machines*, ***Soviet Mathematics Doklady***, vol. 270 (1983), pp. 1289–1293.

► DOUG BURKE AND GARVIN MELLES, *Generalized Levy-Shoenfield absoluteness*.
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Let P be a partial order and κ a cardinal. Let $\mathrm{MA}_\kappa(P)$ be the statement that for each $p \in P$ and each collection $\mathcal{D}$ of cardinality $\leq \kappa$ of subsets of P which are dense below p, there is a $\mathcal{D}$ generic $G \subseteq P$ such that $p \in G$. Let $A \subseteq \kappa$ and let $\phi(x)$ be a Σ_1 formula. Suppose $V \models \mathrm{MA}_\kappa(P)$ and $V[G]$ is a forcing extension of V via the partial order P. Then

$$V \models \phi(A) \iff V[G] \models \phi(A).$$

This theorem is originally due to Bagaria.

► RICCARDO CAMERLO AND ALEXANDER S. KECHRIS, *Countable structures with a fixed group of automorphisms.*

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Let $\mathcal{L}$ be a language which contains a countable infinity of unary function symbols or a relation or function symbol of arity at least 2. Let $X_\mathcal{L}$ be the Polish space of structures for $\mathcal{L}$ with universe $\mathbb{N}$, as in [2, 16.C]. We have the following descriptive set theoretic result:

THEOREM 1. *Let G and H be countable groups. Then*

$$\{\, x \in X_\mathcal{L} \mid \mathrm{Aut}(\mathcal{A}_x) \simeq G \,\} \qquad \textit{and} \qquad \{\, x \in X_\mathcal{L} \mid \mathrm{Aut}(\mathcal{A}_x) \simeq H \,\}$$

are $\mathbf{\Pi}^1_1$-complete sets which are Borel inseparable.

From the correspondence between $\mathcal{L}_{\omega_1\omega}$-sentences and invariant Borel subsets of $X_\mathcal{L}$ we get the following model-theoretic interpretation of the preceding theorem:

THEOREM 2. *Let G and H be countable groups and let σ be a sentence of $\mathcal{L}_{\omega_1\omega}$ satisfied by all countable structures for $\mathcal{L}$ whose automorphism group is isomorphic to G. Then there is a countable structure whose automorphism group is isomorphic to H satisfying σ.*

The result concerning complexity of $\{\, x \in X_\mathcal{L} \mid \mathrm{Aut}(\mathcal{A}_x) \simeq G \,\}$ given in Theorem 1 cannot be generalised to uncountable groups. In fact we have:

THEOREM 3. *Let p be a prime number. Then $\{\, x \in X_\mathcal{L} \mid \mathrm{Aut}(\mathcal{A}_x) \simeq \mathbb{Z}_p^\mathbb{N} \,\}$ is $\mathbf{\Pi}^1_1$ & $\mathbf{\Sigma}^1_1$-complete.*

Every countable structure can be coded in a countable graph. From that construction (see [1]) we obtain:

THEOREM 4. *Let $L = \{R\}$ be the language consisting of one binary relation symbol and let $\mathcal{G} \subseteq X_L$ be the closed set of graphs. Then the results of Theorems* 1, 2, *and* 3 *hold replacing everywhere $X_\mathcal{L}$ with $\mathcal{G}$.*

[1] W. HODGES, ***Model theory***, Cambridge University Press, 1993.
[2] A. S. KECHRIS, ***Classical descriptive set theory***, Springer-Verlag, 1995.

► SAMIR CHOPRA AND ROHIT PARIKH, *An inconsistency tolerant model for belief representation and belief revision.*

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In [1], the authors gave axioms for revising consistent, logically closed belief sets. The AGM axioms, however, allow the trivial update where all previous beliefs are destroyed by a new, contradictory piece of information. They also require a survey of our entire belief set before revisions can take place.

The *Language Splitting* (LS) model [2] provided an axiomatization for update operators that blocked the trivial update, and by splitting our total theory via a language partition, ensured that the only part of a theory that would be revised would be the part relevant to the new belief.

We generalize the *LS* model and allow subtheories with overlap, via a notion of *partial language splitting*. This allows for the tolerance of inconsistency in our explicit beliefs while retaining (locally) classical logic, but blocking outright contradictions. Our model enables an agent to answer queries using Belnap's four-valued logic. Axioms analogous to the AGM axioms hold and the relevance considerations of the original *LS* model are preserved. The distinction between implicit and explicit beliefs is adequately represented and psychologically plausible, computationally tractable procedures for query answering and belief base revision are obtained.

[1] C. ALCHOURRON, P. GÄRDENFORS, and D. MAKINSON, *On the logic of theory change: Partial meet functions for contraction and revision*, ***Journal of Symbolic Logic***, vol. 580 (1985), pp. 510–530.

[2] ROHIT PARIKH, *Beliefs, belief revision and splitting languages*, ***Preliminary proceedings of information theoretic approaches to logic, language and computation***, 1996, pp. 183–187, final version to appear, CSLI, 1999.

► NORMAN DANNER AND DANIEL LEIVANT, *Stratified polymorphism and primitive recursion*.

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In [2], one of the authors introduced the system of finitely stratified polymorphism and characterized the numeric functions representable in it as $\mathcal{E}_4$, the fourth level of the Grzegorczyk hierarchy (the super-elementary functions). Finitely stratified polymorphism is a restriction of the polymorphic λ-calculus in which types are assigned finite levels, and type quantification is restricted by the levels in a way that breaks the circularity inherent in full polymorphism. The relationship between stratified polymorphism and full polymorphism bears some similarity to that between ramified or predicative second-order logic (as in, e.g., [1] and [3]) and classical second-order logic.

Here we extend finitely stratified polymorphism to transfinite levels through ω^ω. Introduction of limit ordinal levels is handled by a highly uniform and finitely presentable inference similar to the Limit Generalization rule of [1]. We then generalize the result of [2] and show that the functions representable in the calculus using levels $< \omega^k$ consist of precisely $\mathcal{E}_{k+3}$ in the Grzegorczyk hierarchy; in particular, the full calculus captures exactly the primitive recursive functions.

[1] SOLOMON FEFERMAN, *Systems of predicative analysis*, ***Journal of Symbolic Logic***, vol. 29 (1964), no. 1, pp. 1–30.

[2] DANIEL LEIVANT, *Finitely stratified polymorphism*, ***Information and Computation***, vol. 93 (1991), no. 1, pp. 93–113.

[3] KURT SCHÜTTE, ***Proof theory***, Springer-Verlag, Berlin, 1977, translated from the revised German edition by J. N. Crossley, Grundlehren der Mathematischen Wissenschaften, Band 225.

► GEOFFREY HELLMAN, *Maximality vs. extendability: A Zermelian-structuralist view of some recent results of McGee on set-theoretic determinateness.*
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In an interesting recent paper [1], Vann McGee has proved a "Categoricity Theorem" to the effect that any two models, having the same universe of discourse, of second-order ZFC set theory with urelements satisfying the additional axiom that the urelements form a set are such that their pure sets are isomorphic. A particular case of this asserts that any two models of this theory "in which the first-order variables range over everything have isomorphic pure sets". An apparent virture of this is that we can appeal to this theory to support the determinateness in truth-value of arbitrary, unbounded sentences of pure set theory, much as we can appeal to second-order Dedekind-Peano arithmetic to support the determinateness of arbitrary sentences of the language of that theory.

Two interrelated problems with this are raised and discussed. The first is that it conflicts with the open-endedness and extendability of universes of discourse for set theory, as articulated forcefully by Zermelo in his famous paper of 1930 [2]. Indeed, Zermelo's vision is shown here to be perfectly compatible formally with McGee's results. It is, however, useful toward this end to introduce modal concepts to avoid conflicts with second-order logical comprehension principles [3]. The second problem concerns the "non-mathematicality" and even circularity in invoking unrestricted quantification ("absolutely everything") for the purpose of establishing determinateness. Here it is granted that we do use unrestricted quantification in our ordinary discourse, but it is also urged that it be treated as open-ended and context-dependent, in line with the Zermelian view rather than the argument for full determinateness. Finally, we emphasize that the determinateness of the Continuum Hypothesis and indeed of any statement concerning accessible sets is not in question, and is sustained on both views.

[1] VANN MCGEE, *How we learn mathematical language*, ***The Philosophical Review***, vol. 106 (1997), no. 1, pp. 35–68.

[2] ERNST ZERMELO, *Über Grenzzahlen und Mengenbereiche: neue Untersuchungen über die Grundlagen der Mengenlehre*, ***Fundamenta Mathematicae***, vol. 16 (1930), pp. 29–47.

[3] GEOFFREY HELLMAN, ***Mathematics without numbers: Towards a modal-structural interpretation***, Chapter 2, Oxford University Press, 1989.

► KEJIA HO AND FRANK STEPHAN, *On bounding simple sets inside m-degrees.*
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We study nontrivial upper bounds with respect to m-reducibility for simple sets both as natural classes and as individual sets. The results along the former line are:

(1) No m-incomplete computably enumerable set is an upper bound for all simple sets.
(2) No effectively simple set is an upper bound for all strongly effectively simple sets; in addition, for every effectively simple set A there is a strongly effectively simple and c-complete (hypersimple) set B such that $B \not\leq_m A$.
(3) If $\mathbb{C}$ is a Σ_3 class of computably enumerable sets containing only simple sets and cofinite sets, then the ideal $\mathbb{I}$ generated by $\mathbb{C}$ has an exact pair of simple sets and every set in $\mathbb{C}$ is 1-reducible to any one half of this exact pair; in particular, the ideal generated by the (strongly) effectively simple sets and cofinite sets has an exact pair of simple sets B and C, and every (strongly) effectively simple set is 1-reducible to B and to C.

The results along the latter line are:

(1) Below some noncomputable computably enumerable set there exists no simple set;

in addition, this noncomputable computably enumerable set could be chosen to be p-complete.
(2) Below every c-complete computably enumerable set there exists a simple set.

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[3] ———, ***Classical recursion theory***, North-Holland, Amsterdam, New York, Oxford, Tokyo, 1989.
[4] E. L. Post, *Recursively enumerable sets of positive integers and their decision problems*, ***Bulletin of the American Mathematical Society***, vol. 50 (1944), pp. 284–316.
[5] R. I. Soare, *Recursively enumerable sets and degrees*, Perspectives in Mathematical Logic, Omega Series, Springer-Verlag, Berlin, Heidelberg, New York, London, Paris, Tokyo, 1987.
[6] ———, *Computability and recursion*, ***Bulletin of Symbolic Logic***, vol. 2 (1996), pp. 284–321.

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This paper compares two strategies for dealing with problems of intensionality in the framework of Epistemic Arithmetic.

The first strategy requires that the objects in the range of the first-order quantifiers (which are, in Epistemic Arithmetic, the natural numbers) can only be referred to in the epistemic language by means of *transparent* terms, whereas objects in the range of higher-order quantifiers can be referred to by means of *opaque* expressions. The second strategy also allows the formal language to contain *opaque* terms referring to objects in the range of the first-order quantifiers.

The first strategy was pursued in [1] to construct a natural system of Epistemic Type Theory. The second strategy was sketched in [2, pp. 31–38].

It will be indicated how the intended interpretations of formal systems based on the two strategies differ. Moreover, it will be shown that the two strategies lead to different formal systems of higher-order Epistemic Arithmetic. In fact, the two strategies can already lead to different first-order systems of Epistemic Arithmetic. For if a description operator is available, then *opaque* terms referring to objects in a range of the first-order quantifiers can be constructed.

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[2] S. Shapiro, *Epistemic and intuitionistic arithmetic*, in ***Intensional mathematics*** (S. Shapiro, editor), North-Holland, Amsterdam, 1985, pp. 11–46.

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Recent decades have seen the emergence of a new strand of philosophy of mathematics known as *structuralism*, Michael Resnik and Stewart Shapiro among its most vocal advocates. An important part of the overall project of structuralism is to provide an account of mathematical ontology that respects the idea familiar from Benacerraf (and others) that mathematical objects are to be individuated solely by the relations they have to each other.

The basic *realist* elaboration of this idea is that mathematical objects, such as numbers, are *places in relational structures*. However, there is a vast class of mathematical structures that contain places that are *indiscernible* if intra-structural relations are all we have to go by. Given that the most straightforward structuralist account of place-individuation entails the "Leibniz principle" of the identity of indiscernibles, and given that there are no other such accounts the structuralist could adopt without compromising the very motivation of her position, she must treat structurally indiscernible places as *identical* (for example, $1 = -1$). Thus, at least in the case of systems with structurally indiscernible elements, the idea of 'extracting' the structure of such a system and treating it as an entity on its own right is incoherent. The structuralist will then either have to show why this predicament does not vitiate her overall project or else try to find other means of individuating places that respect the basic motivation of structuralism. We will argue that neither response can be sustained and conclude that opting for a nominalist account of the ontology of structures is probably the safest route for the structuralist at the present time.

► AMIR LESHEM, $0^{\#}$ *and elementary end extensions of* V_κ.
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In this paper we consider the question of existence of elementary end extensions of models of the form (V_κ, ε). The interest in the structure of elementary end extensions of models of set theory goes back to the results of Keisler, Morley, and Silver. Villaveces [3] studied the structure of the partial order of elementary end extensions. A good overview of the subject is given in [3].

In this paper we prove the following:

THEOREM 1. *If $0^{\#}$ exists then for every cardinal κ there is an inner model M such that*

$$M \models \text{“}(V_\kappa, \in) \text{ has no elementary end extension”}.$$

In particular weak compactness is never downwards absolute, once we have $0^{\#}$ in the universe. On the other hand we will prove that any cardinal above $\aleph_1$ is Mahlo in any strict inner model of $L[0^{\#}]$. The proof of Theorem 1 uses ideas of Kunen [1] for constructing a Suslin tree and of M. Stanley [2] for constructing generic objects for reverse Easton iterations inside $L[0^{\#}]$, together with a simple coding of the generic Suslin tree.

[1] K. KUNEN, *Saturated ideals*, ***Journal of Symbolic Logic***, vol. 102 (1978), pp. 173–181.
[2] M. C. STANLEY, *Backwards Easton forcing and* $0^{\#}$, ***Journal of Symbolic Logic***, vol. 53, (1988).
[3] A. VILLAVECES, *Height of models of ZFC and the existence of end elementary extensions*, 1996, preprint.

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Here we explore the use of diagrammatic and topological methods within the model theory of lattices (i.e., R-torsion-free finitely generated modules) over group rings RG where G is a finite group and R is a Dedekind domain. In particular, first we obtain some partial decidability results when a Sylow 2-subgroup of G is noncyclic of order 4 (and G is not a 2-group), then we discuss how to define a Ziegler topology in this setting and we study the outcoming topological space for certain choices of G.

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Extensions of the technique used to prove the nondensity of the D.R.E. degrees are used to obtain results about the non-dense embedding of certain lattices into that structure.

[1] S. B. COOPER, L. HARRINGTON, A. H. LACHLAN, S. LEMPP, and R. I. SOARE, *The d-r.e. degrees are not dense*, ***Annals of Pure and Applied Logic***, vol. 55 (1991), pp. 125–151.

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First-order model theory on infinite structures is a well-developed field of abstract mathematics with a wealth of powerful methods and sweeping results. In contrast, on finite structures we have very few general methods and equally few general results. In this paper we try to bridge this gap. We enrich finite model theory by adding certain infinite limits of finite structures. In this way we can develop *quasi-finite* model theory using some of the powerful methods of model theory on infinite structures without losing the finite model property (every sentence with a model has a finite model).

► AMY K. C. S. VANDERBILT AND DOUGLAS CENZER, *Intuitionist default theories and their common derivations*.
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Various questions concerning extensions of default theories in Nonmonotonic Logic, specifically Default Logic, have been explored in detail by Marek, Truszczyński, and others. We explore the problem of finding a common derivation for a formula ϕ occurring in each extension of a default theory (D, W). Further, we take an intuitionist view, and show how this enhances as well as simplifies the result. This work is part of the first author's dissertation, which is in preparation.

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We show that given any finite partially ordered sets $P \subseteq Q_i$ with the least and greatest elements such that $\{ Q_i : i \in \omega \}$ is a recursive sequence, if each Q_i satisfies Slaman-Soare's second non-extension condition, then there is an embedding e of P into the structure of computably enumerable (c.e.) degrees $\mathcal{R} = (\mathbf{R}, \leq, \mathbf{0}, \mathbf{0}')$ which cannot be extended to any embedding of Q_i into $\mathcal{R}$ for all $i \in \omega$. Such embedding e is said with *saturation* properties (also called with *strong embedding* properties). This result strengthens Slaman-Soare's second non-embedding theorem. It is a partial result of the decidability of the Π_2-theory of c.e. degrees. In addition, we will give a full characterization of embeddability of finite distributive lattices into the c.e. degrees with saturation properties, namely, a finite distributive lattice can be embedded into c.e. degrees with saturation properties if and only if it is a d.d.c. (disjoint diamond chain) but not the diamond.

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[6] R. I. SOARE, *The history and concept of computability*, `http://www.cs.uchicago.edu/~soare/computability.ps`.

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A Dedekind algebra is an ordered pair (B, h) where B is a non-empty set and h is a similarity transformation on B. Each Dedekind algebra can be decomposed into a family of disjoint, countable subalgebras called the configurations of the Dedekind algebra. There are $\aleph_0$ isomorphism types of configurations. Each Dedekind algebra is associated with a cardinal-valued function on omega called its configuration signature. The configuration signature of a Dedekind algebra counts the number of configurations in the decomposition of the algebra in each isomorphism type.

The configuration signature of a Dedekind algebra encodes the structure of that algebra in the sense that two Dedekind algebras are isomorphic provided their configuration signatures are identical. Configuration signatures are used to establish various results in the first-order model theory of Dedekind algebras. These include categoricity results for the first-order theories of Dedekind algebras and existence and uniqueness results for homogeneous, universal and saturated Dedekind algebras. Fundamental to these results is a condition on configuration signatures which is necessary and sufficient for elementary equivalence.

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Classical logic founders on truth-value gaps when it comes to guide a consistent discourse involving unavoidable indeterminacies such as Heisenberg's uncertainty. Two chronic deficiencies disable it in this task. First, of the two Sheffer hypotheses underlying classical logic, the truth conditions falsifying joint-denial overlook the case where one propositional component is the denial of the other. Second, classical truth-values are limited to propositions determined to be true or determined to be false. Propositions not so determined are left without truth-values, hence truth-value gaps. Indeed, the indeterminacies of particle physics refute the law of bivalence. By exhausting the truth-values via the law of excluded middle, we gain the complementary truth-value 'neither true nor false,' which defines 'indeterminate.' The indicated accretions complete and preserve the core classical logic. The truth conditions for an indeterminate joint-denial are deduced. It remains to deduce the truth conditions conferring the three truth-values on the other propositional connectives defined by joint-denial.

Quantification truths accommodating indeterminacy are deduced from the hypotheses for quantification found in Quine's *Mathematical logic* with which familiarity is assumed. Thus, we deduce the correct form of existential instantiation. Quantification is shown to collapse in a true-false dichotomy. Undecidable propositions prove to bear two or three alternate truth-values.

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Elementary logic cannot do justice to the logic of arguments like "Ezra and Thomas are critics who admire only each other; so, some critics admire only one another", or "Russell and Whitehead cooperate, and they are the coauthors of *Principia Mathematica*; so, the coauthors of *Principia Mathematica* cooperate." This paper presents an extension of elementary logic, the logic of plurals, that can account for logical relations among plural constructions. The logic of plurals is a conservative extension of elementary logic that has wider jurisdiction than that of elementary logic, i.e., elementary languages. In this respect, the logic of plurals is akin to higher-order extensions of elementary logic. It differs from them primarily in jurisdiction. Higher-order logics govern vertical extensions of elementary languages; the logic of plurals governs its horizontal extension, which includes plural counterparts of the resources available in elementary languages.

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We say that two functions f, $g \in {}^{\omega}\omega$ are eventually different (e.d.) if and only if $|f \cap g| < \omega$. In [1], A. Miller stated a problem first asked by E. van Douwen:

QUESTION 1. CH implies there exists $\mathcal{A} \subseteq {}^{\omega}\omega$ which is maximal with respect to eventually different (e.d.) functions which is also maximal with respect to infinite partial functions. Is there always such an $\mathcal{A}$? What is $\mathfrak{a}_{\mathrm{m}}$, the cardinality of the smallest such $\mathcal{A}$?

We can partially answer Question 1 by proving the following theorems.

THEOREM 2 (MA). *There exists an $\mathcal{A} \subseteq {}^{\omega}\omega$, which is maximal with respect to e.d. functions which is also maximal with respect to infinite partial functions, and $\mathfrak{a}_{\mathrm{m}} = 2^{\omega}$.*

THEOREM 3. *Let $M \models (\mathrm{ZFC}+\neg\mathrm{CH})$. Let $\kappa \in M$ be a cardinal such that $\omega_1 \leq \kappa < 2^{\omega} = \lambda$. Then there exists a c.c.c. notion of forcing $\mathbb{P}$ such that the following statements holds in $M^{\mathbb{P}}$.*

(1) $2^{\omega} = \lambda$.
(2) *There exists a m.e.d. function set $A^{*} \subseteq {}^{\omega}\omega$ of cardinality κ with respect to both eventually different functions and infinite partial functions.*

THEOREM 4. *Let $M \models (\mathrm{ZFC} + \mathrm{CH})$. There is a m.e.d. family $\mathcal{A} \in M$ of size ω_1 in M such that for any Cohen generic G over M, $\mathcal{A}$ remains a m.e.d. family in $M[G]$. Here $\mathcal{A}$ is maximal with respect to eventually different functions and $\mathcal{A}$ is also maximal with respect to infinite partial functions.*

DEFINITION 5. Let $\mathfrak{a}$ be the least infinite λ such that there exists a maximal almost disjoint (m.a.d.) family $\mathcal{F} \subset \wp(\omega)$ of size λ.

THEOREM 6. *It is consistent with* ZFC *that $\mathfrak{a}_{\mathrm{m}} = \omega_2$ and $\mathfrak{a} = \omega_1$.*

[1] A. MILLER, *Some interesting problems* (*updated on October 15, 1996*), in ***Set theory of the reals*** (Haim Judah, editor), Israel Mathematical Conference Proceedings, vol. 6, American Mathematical Society, 1993, pp. 645–654.

Abstracts presented by title

► TOSHIYASU ARAI, *Epsilon substitution method for theories of jump hierarchies*.
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Let $<$ denote a recursive well ordering of type Γ_0 and α_0 a fixed limit ordinal less than Γ_0. $(H)_{\alpha_0}$ denotes the theory of the absolute jump hierarchy generated by a fixed quantifier-free, arithmetical formula $\mathcal{A}(x, y, z, X)$ and up to $\alpha \leq \alpha_0$ having axioms $\alpha \leq \alpha_0 \rightarrow \{z \in H_\alpha \leftrightarrow \exists x\, \mathcal{A}(x, \alpha, z, H_{<\alpha})\}$, where $H_{<\alpha} = \sum_{\beta<\alpha} H_\beta$.

We formulate an epsilon substitution method for the ε-counterpart $(H)_{\alpha_0}\varepsilon$ of $(H)_{\alpha_0}$: Replace the existential quantifier in the RHS of the axiom for H by an epsilon term containing the predicate $H_<$. An *epsilon substitution* S is defined to be a function such that each $e \in \mathrm{dom}(S)$ is either a canonical ε-term with $S(e) \in \omega$ or e is one of formulas $n \in H_\alpha$ $(\alpha \leq \alpha_0)$ and $(\beta, n) \in H_{<\alpha}$ $(\beta < \alpha \leq \alpha_0)$, and in the latter case $S(e)$ is a Boolean value in $\{\bot, \top\}$. We give two termination proofs of the H-process: The first proof is an adaptation of [2], [3], i.e., based on a cut-elimination of a specially designed infinitary calculus. The second one is an adaptation of [1]. Each termination proof is based on transfinite induction up to an ordinal $\theta\alpha_0\varepsilon_0$, where the normal function $\theta\alpha\beta$ is the αth iterate of the function $\theta 1\beta = \omega^\beta$.

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[2] G. Mints, *Gentzen-type systems and Hilbert's epsilon substitution method, I*, in ***Proceedings of 9th International Congress of logic, methodology, and philosophy of science*** (D. Prawitz, B. Skyrms, and D. Westerstahl, editors), vol. IX, Elsevier Science B.V., 1994, pp. 91–122.

[3] G. Mints, S. Tupailo, and W. Buchholz, *Epsilon substitution method for elementary analysis*, ***Archive for Mathematical Logic***, vol. 35 (1996), pp. 103–130.

► JOHN CORCORAN, *Logical form of quantifier phrases*: *Quantifier-sortal-variable*.
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Sentences in mathematical English such as E below, expressing propositions about [natural] numbers, are routinely translated into sentences such as F below of an interpreted first-order language.

E: Every number x is either zero or the successor of some number y.
F: $\forall x\ (x = o \vee \exists y\ x = sy)$.

The English quantifier phrases *every number x* and *some number y* correspond to the symbolic quantifier phrases $\forall x$ and $\exists y$. However, there is disagreement among mathematical logicians concerning symbolic "location" of the meaning of the sortal [substantive] *number* in the symbolic formalism. Some logicians, the majority, take the meaning of *number* to be elliptically absorbed into the variable so that x means, in effect, *number x*. Others take the meaning of *number* to be absorbed into the quantifier so that $\forall$ means, in effect, *every number*. The former, the sortal variable approach, treats the quantifier as a logical symbol but, contrary to tradition, treats the variable as non-logical. The latter, the sortal quantifier approach, treats the variable as logical, but, contrary to tradition, treats the quantifier as non-logical. This paper traces the disagreement to Frege, who did not recognize universes of discourse, whose individual variables ranged over "the universe", and whose suppressed sortal was *object* (or *individual*), which is naturally taken to be logical. This paper favors a third alternative, the suppressed sortal approach, in effect, that $\forall x$ and $\exists y$ are elliptical for $\forall N\ x$ and $\exists N\ y$, where N is a sortal, or symbolic [common] noun, a category useful for understanding how number-theoretic propositions can be fully expressed in first-order.

According to this approach, the quantifier and the variable are both logical and the content of the quantifier phrase, $\forall x$ or $\exists y$, is carried by the suppressed non-logical sortal N.

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Let L be a classical language enriched with 'xB' a modal operator for 'x believes that', 'xE' a modal operator for 'x has adequate evidence that', and (EO read 'on epistemic grounds it is obligatory that') EOp wf when p is 'Aq', q wf and A a doxastic modality of degree > 0.

Axiom 1. $EPxBp \equiv\sim EO \sim xBp$.

Axiom 2. $EOxBp \supset EPxBp$.

Axiom 3. $EOxBp \supset EO \sim xB \sim p$.

Axiom 4. $EO \sim xBp \supset xE \sim p$.

Axiom 5. $xEp \supset EOxBp$.

From A1–A5, we get T1 and T2:

T1. $EPxBp \equiv\sim xE \sim p$.
T2. $EOxBp \equiv xEp$.

Recent work in epistemology includes the thesis: the analysis of knowledge in terms of true belief plus adequate evidence is inferior to an analysis in terms of true belief plus (epistemically) *justified belief* ('$xJBp$' hereafter). Some authors (e.g., Alston, Feldman, Plantinga) appear to use 'justified belief' so as to validate D1: $xJBp \equiv EPxBp$. Given D1, T1 shows that the justified belief analysis gives: $xKp \equiv (p \cdot xBp \cdot \sim xE \sim p)$, an absurdity. If we replace D1 by the stronger D2: $xJBp \equiv EOxBp$, T2 shows that the justified belief analysis gives us the traditional theory it was to replace: $xKp \equiv (p \cdot xBp \cdot xEp)$. Aside from its use in showing flaws in some recent epistemology, this logic of purely epistemic obligation is of interest as a theory of the sort that might have been endorsed by W. K. Clifford.

► ALBERT A. MULLIN, *Properties of recursive sequences*.
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This note deals with the role of consecutivity in the determination of new properties and relations for recursive sequences.

Lemma 1. *Put* $f_n = 2^n + 1$. *Then* $f_{43}, f_{44}, \ldots, f_{50}$ *are the only eight* consecutive *terms of* $\{f_n\}$ *with a prime number of* total *prime factors.*

Lemma 2. *Put* $g_p = 2^p + 1$, *where p is prime. Then* $g_3, g_5, \ldots, g_{47}, g_{53}$ *are the only* 15 consecutive *terms of* $\{g_p\}$ *with a prime number of* total *prime factors.*

Lemma 3. *Put* $h_p = 3^p + 2^p$, *where p is prime. Then* $h_3, h_5, \ldots, h_{17}, h_{19}$ *are the only seven* consecutive *terms of* $\{h_p\}$ *with a prime number of* total *prime factors.*

Problem 1. Give an example of a palindomic sonatina.

Problem 2. Characterize the finite groups with a prime number of *solvable* subgroups.

Problem 3. Put $k_p = (4^p + 3^p)$, where p is prime. Then $k_2, k_3, \ldots, k_{37}, k_{41}$ are the only 13 *consecutive* terms of $\{h_p\}$ with a prime number of *total* prime factors.

Problem 4. Give an example of a (validromic) Sestina which backwards is a *different* Sestina.

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A true computing basis to nonmonotonic logic characterizes a model theory for default logic and entailment. A positive forcing basis to nonmonotonic logic allows us to define a standard model theory for nonmonotonic logic. Specific theorems characterize a proof theory and model theory for nonmonotonic logic. A completeness theorem is proved on nonmonotonic logic by positive forcing [2]. Applications to model discovery and new computing areas are presented in brief proving a completeness theorem on predictive model diagrams.

1. Positive forcing contains default logic. A default theory consists of a set A of sentences of first-order logic together with a set of rules of inference called "defaults." Defaults have the form

$$p(x_1, \dots, x_n) \colon q(x_1, \dots, x_n) \to r(x_1, \dots, x_n),$$

where p, q, and r are first-order formulas.

Definition. We say that a formula $g(x_1, \dots, x_n)$ is provable by default if and only if it can be deduced by the above default rule schemata and a given set of axioms. Let a default extension of a theory $T(P)$ be $T(P) \cup \{r\}$, where r is a formula proved by default using the closure of the axioms of $T(P)$ under the rules of inference of $T(P)$ as the axiom set. Further, we say that a formula r is provable by default from a theory $T(P)$ if and only if r can be proved by default from any default extension of $T(P)$.

The notion of consistency in our formulation is that of consistency of a formula relative to an initial model. Thus if $\varphi(x) = t$ is consistent with the standard model, then φ can be applied as a default.

Theorem 1. *The formulas provable by default are only what is $+$forced.*

2. New applications. [4]'s theorist is a proof procedure for abductive reasoning based on [1]'s MESON system. Abduction involves constructing hypotheses, where each hypothesis is a set of atomic literals Δ_i; such that when some particular theory T is augmented with Δ_i, it entails the set of goal literals G, i.e., $T \cup \Delta_i \vDash G$. Minimally, the Δ_i must be a subset of a set of ground atomic abducibles A. In addition we must ensure $T \cup \Delta_i$ is consistent under $\vDash$. The set of all possible hypotheses is $\Delta = \{\Delta_i\}$. More general abductions are defined by applying more complex formulas as abducibles A. First-order existential formulas can form the abducibles. According to Poole an explanation problem can be stated formally as: given Facts $\mathcal{F}$: consistent formulas, known to be true, Defaults Δ: possible hypotheses, that we accept as part of an explanation and Observations $\mathcal{G}$: which are to be explained, an observation $g \in \mathcal{G}$ is explainable, if there exists a ground hypothesis $\mathcal{D} \subseteq \Delta$ such that: 1. $\mathcal{F} \cup \mathcal{D} \vDash g$; 2. $\mathcal{F} \cup \mathcal{D}$ is consistent. We have applied certain diagrams to abductive models since 1994 to model partial deduction and cumulative defaults. A *predictive diagram* for a theory T is a diagram $D[M]$, where M is a model for T, and for any formula q in M, either the function $f : q \to \{0, 1\}$ is defined, or there exists a formula p in $D[M]$, such that $T \cup \{p\}$ proves q; or that T proves q by minimal abduction. A *Gernalized predictive diagram* is a predictive diagram with $D[M]$ defined by a specific set of functions.

A specific to Poole's logic might be as follows.

Theorem 2. *A set of first-order observations $\mathcal{G}$ is explainable if and only if there exists a predictive diagram for the logical consequences to $\mathcal{G}$.*

[1] D. W. Loveland, ***Automated theorem proving: A logical basis***, North-Holland, 1978.

[2] C. F. Nourani, *Forcing with universal sentences and initial models*, ***Journal of Symbolic Logic***, 1981–1982, presented at Annual Meeting of the Association for Symbolic Logic, December 1983.

[3] ———, *Free proof trees and model-theoretic planning*, presented at Automated Reasoning AISB, England, April 1995.

[4] D. Poole, *A logical framework for default reasoning*, ***Artificial Intelligence***, vol. 36 (1988), pp. 27–47.

► RALF SCHINDLER, *Remarks on coding into $L(\mathbb{R})$*.
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We consider questions like: when is there an extension of V via some (set-sized) proper forcing in which there is a projective well-ordering of $\mathbb{R}$? Or in which $\mathcal{P}(\omega_1) \subset L(\mathbb{R})$?

Answers to such questions require "coding into K" techniques.