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1998–1999 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

San Antonio, Texas, January 15–16, 1999

The Winter Meeting of the Association for Symbolic Logic was held in San Antonio, Texas, on January 15–16, 1999, in conjunction with the annual meeting of the American Mathematical Society. The program committee consisted of Peter Cholak (chair), Lou van den Dries, and Steve Jackson. The program included eight invited talks, thirteen contributed talks, and two papers presented by title only.

The following invited talks were presented:

Michael Benedikt (Bell Laboratories), *What you can and can't define in a model.*

Geoff LaForte (University of West Florida), *A barely* Σ^0_2 *degree.*

Chris Laskowski (University of Maryland), *Beyond simplicity: What's next?*

Bill Mitchell (University of Florida), *Club sets and large cardinals.*

Anand Pillay (University of Illinois at Urbana-Champaign), *Simple theories.*

Robert Soare (University of Chicago), *Understanding automorphisms using templates.*

Sławomir Solecki (University of Indiana), *Aspects of non-local compactness.*

Reed Solomon (University of Wisconsin), *Computable aspects of ordered groups.*

Abstracts of invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
PETER CHOLAK

Abstracts of invited talks

► MICHAEL BENEDIKT, *What you can and can't define in a model.*
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We overview some work in the definability of second order properties with first-order sentences. It works like this: You fix a model M (e.g., the real field), and a class of subsets C of the model (e.g., the definable subsets), and some property $P(S)$ of elements S from C, e.g., "S is of even cardinality", and you ask:

Is there a first-order formula in the language of M plus a predicate for S that defines within C exactly the sets that have $P(S)$?

In the case where C is the collection of finite subsets of M, you get a generalization of finite model theory parameterized by the model M. In the case where M has a natural topology—as with models based on the real line—you get a kind of topological model theory parameterized by M. In both cases there are connections between the resulting theory and questions in stability theory and real algebraic geometry.

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► GEOFFREY LAFORTE, *A barely Σ^0_2 degree* (joint work with Rod Downey and Steffen Lempp).
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While the lion's share of effort in classical computability theory over the last fifty years has naturally been directed toward the study of relative *computability*, the more neglected relation of relative *enumerability* is almost as interesting. This relation holds between sets A and B when A is computably enumerable using B as an oracle: in other words, when A is definable with one arithmetic existential quantifier over B.

By the (strong) Sacks Jump Theorem [1], given any noncomputable, computably enumerable C, any Σ^0_2 set with degree at least as great as $\mathbf{0}'$ has the same degree as the jump of some c. e. B such that $C \not\leq_T B$. Clearly, any such B must itself be incomplete. Thus we have the weaker fact that any Σ^0_2 set with degree at least as great as $\mathbf{0}'$ has Σ^B_1-degree for some incomplete c. e. B. Of course, any Σ^0_2 set is itself relatively computably enumerable in the complete set K, and so has Σ^K_1-degree. The natural question in this context is whether given any Σ^0_2 degree we can find some incomplete c. e. set over which it is Σ_1-definable. We present a recent result by Downey, LaForte, and Lempp [2] answering this question negatively. We also mention some other recent results and some open questions.

[1] G. SACKS, *Recursive enumerability and the jump operator*, ***Transactions of the American Mathematical Society***, vol. 108 (1963), pp. 223–239.

[2] R. DOWNEY, G. LAFORTE, and S. LEMPP, *A Δ^0_2 set of barely Σ^0_2 degree*, ***Journal of Symbolic Logic***, in press.

► MICHAEL C. LASKOWSKI, *Beyond simplicity: What's next?*.
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We offer a number of new characterizations of simple theories. In particular, a theory is simple if and only if every (partial) type that divides over a set A, weakly divides over A. As well, we give characterizations of simplicity in terms of extendibility of types that naturally generalize characterizations of stable theories and are suggestive of what the 'next' dividing line might be.

Along this vein, we define Shelah's universality spectrum [2]. By modifying the machinery in [2], we offer a sufficient condition on a theory for it to be consistent that its universality spectrum differs from that of any theory with the strict order property.

This is joint work with Alfred Dolich.

[1] S. SHELAH, *Simple unstable theories*, ***Annals of Mathematical Logic***, vol. 19 (1980), pp. 177–203.

[2] ———, *The universality spectrum: Consistency for more classes*, in ***Combinatorics, Paul Erdős is eighty***, proceedings of the meeting in honor of Paul Erdős, Keszthely, Hungary, 1993.

► BILL MITCHELL, *Club sets and large cardinals*.
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I will present a survey of recent results involving methods of forcing to add closed unbounded subsets of large cardinals, and applications of these methods.

► ANAND PILLAY, *Simple theories*.
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I will survey the progress made in the past few years in our understanding of simple theories. I will discuss both the similarities with stability theory as well as the new phenomena, such as the hyperdefinability of compact Lie groups, which appear to arise.

► ROBERT I. SOARE, *Understanding automorphisms using templates*.
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For about twenty years the Extension Theorem [Soare, 1974] was the main tool for building *effective* automorphisms of the computably enumerable (c. e.) sets. In the mid 1990s Harrington and Soare, and independently Cholak, developed a tree method for constructing Δ^0_3 automorphisms and used it to obtain results which the effective method would not produce. The proof for a Δ^0_3 automorphism $\Phi\colon U_\alpha \mapsto \widehat{U}_\alpha$ used, for each node α in the priority tree T, certain finite lists (called *templates*), $\mathcal{M}_\alpha$, $\mathcal{R}_\alpha$, and $\mathcal{B}_\alpha$, etc. of α-states (partitions in the Venn diagram) which during the construction are well (i.e., infinitely often) visited, emptied by the opponent (RED), or by the automorphism player (BLUE), respectively, or have certain other properties. However, the extension phenomenon was not addressed in the Δ^0_3 method, and certain extensions had to be proved over and over in building specific automorphisms.

We now formulate a New Extension Theorem (N. E. T.) appropriate for Δ^0_3 automorphisms on trees, with a very short, simple statement with *hypotheses* (as opposed to proofs) stated in terms of templates for the first time. The N. E. T. is needed implicitly or explicitly in almost every present Δ^0_3 automorphism proof (including all the effective automorphism results, but there it produces only a Δ^0_3 not necessarily effective automorphism). Another advantage is that the templates and N. E. T. give an easy way to survey the main automorphism results (effective and noneffective) of the last twenty-five years and demonstrate exactly how such standard computability properties, such as low, promptly simple, simple, prompt, high, etc. are translated into template properties which meet the hypotheses of the N. E. T. and therefore yield an automorphism. A systematic examination using templates has unearthed new theorems and questions. This survey will include old and new results.

This will be a very simple and understandable lecture for nonexperts with lots of diagrams and intuition on automorphisms (effective and Δ^0_3), trees, templates, the N. E. T. and its applications. You are guaranteed to understand the main ideas.

► SŁAWOMIR SOLECKI, *Aspects of non-local compactness*.
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Compact and locally compact groups have been studied extensively by analysts and topologists. However, many topological groups occurring in mathematics (for example, S_∞ the group of all permutations of ω inducing the "logic action" in model theory) are not compact or locally compact. These groups, as S_∞ above, usually turn out to be Polish, that is, metric, separable, complete. This situation made it important to investigate properties of general Polish groups and their continuous actions. Using methods from Descriptive Set Theory, quite interesting differences between this general situation and the compact and locally compact cases have been discovered. I will survey some recent results that explore these differences which have to do with continuous actions of groups (or, more precisely, with orbit equivalence relations induced by such actions), a notion of smallness called Haar null, some specific topological properties of groups, and others.

► REED SOLOMON, *Computable aspects of ordered groups*.
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An ordered group is a group together with a linear order such that the order is preserved under multiplication on both the right and the left. From the point of view of computability theory, one aspect of orderable groups is the relationship between Π^0_1 classes and the space of orders on an orderable computable group. We will discuss several methods of representing Π^0_1 classes (such as by Turing degree preserving bijections) and show that the space of orders on an orderable computable abelian (or nilpotent) group is too general to allow it to represent an arbitrary Π^0_1 class in any reasonable way. A second aspect is the presentation of an orderable computable group. We will show that every orderable computable abelian group has a presentation under which it admits some computable order. A third aspect is the notion of computable categoricity. When is it the case that any two computable presentations of an ordered group are computably isomorphic? We will examine this question for abelian and nilpotent groups (joint work with Sergey Goncharov and Steffen Lempp). Many of these issues were first raised by Rod Downey and Stuart Kurtz.

Abstracts of contributed talks

► NORBERT BRUNNER, *Social welfare theory and permutation models* (joint work with Reiju Mihara).
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The talk reports about joint work [4] with H. Reiju Mihara of Kagawa University about an application of independence proofs to economy.

Arrow's axioms of social welfare theory are known to be contradictory for finite societies (Arrow's theorem [2]) but consistent for infinite sets I of individuals (Fishburn's theorem [5]). Armstrong [1] has investigated the consequences of restricting the coalitions to an algebra $\mathbb{B}$ (a subalgebra of $\mathcal{P}(I)$) and Mihara [7] those of anonymity (an axiom due to May [6]). We prove Arrow's theorem for infinite societies in the context of restricted coalition algebras $\mathbb{B}$ and the axiom of Γ-anonymity (where Γ is a notion of "equal treatment" of the individuals). Our argument applies Weglorz' models of set theory without the axiom of choice (c.f. [3]).

THEOREM. *In ZFC, assume that $I = \omega$ and let either $\mathbb{B} = \mathcal{P}(I)$ or $\mathbb{B}$ be countable and structured such that $[I]^1 \subset \mathbb{B} \subseteq \mathcal{P}(I)$. Then a reasonable $\mathbb{B}$-social welfare function F on I which satisfies* $\mathrm{Aut}(\mathbb{B})$*-anonymity is dictatorial.*

Here a *reasonable* $\mathbb{B}$*-social welfare function* F on I is defined on the $\mathbb{B}$-measurable profiles $\mathbf{p}$ of the (transitive and complete) individual preferences on a finite set X of at least three alternatives. It defines a transitive and complete social preference $F(\mathbf{p})$, it is independent of irrelevant alternatives and it satisfies a condition of unanimity. F is *dictatorial*, if there is a dictator $i \in I$ whose strict preferences are obeyed. $\mathbb{B}$ is *structured*, if each infinite $b \in \mathbb{B}$ splits into disjoint infinite b_1 and $b_2 \in \mathbb{B}$. $\mathrm{Aut}(\mathbb{B})$ is the automorphism group of $\mathbb{B}$. Then $\pi \in \mathrm{Aut}(\mathbb{B})$ corresponds to a permutation $\{\pi i\} = \pi(\{i\})$ of the individuals and this in turn to the permutation $\mathbf{p}^\pi(i) = \mathbf{p}(\pi i)$ of the individual preferences. F is Γ*-anonymous*, where Γ is a subgroup of $\mathrm{Aut}(\mathbb{B})$, if for all permutations $\pi \in \Gamma$ we have $F(\mathbf{p}) = F(\mathbf{p}^\pi)$.

[1] T. E. ARMSTRONG, *Arrow's theorem with restricted coalition algebras*, ***Journal of Mathematical Economics***, vol. 7 (1980), pp. 55–75; erratum: vol. 14 (1985), pp. 57–59.

[2] K. J. ARROW, ***Social choice and individual values***, Yale University Press, New Haven, 1963.

[3] N. BRUNNER, P. HOWARD, and J. E. RUBIN, *Choice principles in Weglorz' models*, ***Fundamenta Mathematicae***, vol. 154 (1997), pp. 97–121.

[4] N. BRUNNER and H. R. MIHARA, *Social welfare theory and the axiom of choice*, preprint.

[5] P. C. FISHBURN, *Arrow's impossibility theorem*, ***Journal of Economic Theory***, vol. 2 (1970), pp. 103–106.

[6] K. O. MAY, *A set of independent necessary and sufficient conditions for simple majority decision*, ***Econometrica***, vol. 20 (1952), pp. 680–684.

[7] H. R. MIHARA, *Anonymity and neutrality in Arrow's theorem with restricted coalition algebras*, ***Social Choice Welfare***, vol. 14 (1997), pp. 503–512.

► WILLIAM C. CALHOUN, *Embedding lattices into the lattice of computably enumerable Turing degrees*.

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This talk will be a survey of results concerning lattice embeddings into the lattice of ideals of computably enumerable Turing degrees, **I**. The problem of embedding lattices into **I** is related to the well-known problem of embedding lattices into the computably enumerable Turing degrees, **R**.

Calhoun [2] showed that any *cleanable* lattice is embeddable into **I**. The cleanable lattices include the S_8 lattice, and infinitely many others that are not embeddable into **R**. Weber [4] showed that a certain set of lattices (including S_8) is embeddable into every initial segment of **I**.

Ambos-Spies, Lempp and Lerman [1] characterized the distributive lattices that can be embedded into **R** preserving both 0 and 1. It is easy to show that the same characterization applies to embeddings into **I**. Recently, Lempp and Lerman [3] found the first example of a lattice without critical triple that is not embeddable into **R**. There is work in progress by Calhoun and Lerman to adapt the proof to **I**.

[1] K. AMBOS-SPIES, S. LEMPP, and M. LERMAN, *Lattice embedding into the r. e. degrees preserving* 0 *and* 1, **Journal of the London Mathematical Society**, Second Series, vol. 49 (1994), no. 1, pp. 1–15.

[2] W. C. CALHOUN, *The lattice of ideals of recursively enumerable degrees*, ***Ph. D. dissertation***, University of California, Berkeley, 1990.

[3] S. LEMPP and M. LERMAN, *A finite lattice without critical triple that cannot be embedded into the enumerable Turing degrees*, ***Annals of Pure and Applied Logic***, vol. 87 (1997), pp. 167–185.

[4] F. O. WEBER, *Initial segments of the lattice of ideals of r. e. degrees*, ***Journal of Symbolic Logic***, vol. 59 (1994), no. 4, pp. 1326–1350.

► NATHANIEL S. K. HELLERSTEIN, *Diamond, a paradox logic*.

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This talk is about a four-valued logic of paradox and self-reference. Diamond has four truth values; true, false, 'true but false', and 'false but true'. The mixed values are wave states, corresponding to oscillations. This logic follows the De Morgan laws; in addition it has a 'phase order' structure that is preserved by all operations. Diamond's wave-state truth values allow us to resolve the classic foundational paradoxes; and from them one can generate fixed points for all self-referential diamond-logic systems. In addition to resolving general self-reference, diamond logic also embeds the continuum. That is, the continuum embeds in the space of all infinite diamond-logic sequences; and any function from the reals to the reals extends to a function from the diamond sequences to the diamond sequences. Embedding the continuum in diamond space, along with diamond's general self-reference properties, implies "Zeno's theorem"; any continuous real function has a fixed point in diamond space.

[1] Nathaniel S. K. Hellerstein, ***Diamond, a paradox logic***, World Scientific Publishing Co., Singapore, 1997.

[2] Francisco Varela and Louis Kauffman, *Form dynamics*, ***Journal of Social and Biological Structure***, vol. 3 (1980), pp. 171–206.

[3] Francisco Varela, *A calculus for self-reference*, ***International Journal of General Systems***, vol. 2 (1975), pp. 5–24.

[4] Louis Kauffman, *De Morgan algebras; Completeness and recursion*, ***Proceedings of the 8th international symposium for multiple-valued logic***, 1978, pp. 209–213.

► JEFFRY L. HIRST, *Reverse mathematics and derived sortings*.
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A derived sorting for a directed graph G assigns elements of a well ordering to the vertices of G in a fashion that preserves the ordering induced by the edges. The formal definition follows.

Definition. Suppose G is a directed acyclic graph with source node v_0, vertex set V, and edge set E. The pair $\langle f, \alpha\rangle$ is a *derived sorting* for G if α is a well ordering, f maps V onto α, and the following three properties hold:

(1) $f(v_0)$ is the least element of α,
(2) $(u, v) \in E$ implies $f(u) < f(v)$, and
(3) for any $v \in V$ and $x \in \alpha$, if $x < f(v)$, then there is a $u \in V$ such that $(u, v) \in E$ and $x \leq f(u)$.

For a finite acyclic directed graph with a source node, the derived sorting assigns to each vertex v the length of the longest path from the source node to v. We will use the framework of reverse mathematics to analyze the strength of some theorems about derived sortings of infinite graphs. In the following example, $\mathbf{ATR}_0$ denotes a subsystem of second order arithmetic that includes an arithmetical transfinite recursion scheme.

Theorem ($\mathbf{RCA}_0$). *The following are equivalent*:

(1) $\mathbf{ATR}_0$.
(2) *Every countable well founded directed graph with a source node has a derived sorting.*

► COLLEEN M. HOOVER, *A transfer theorem for Σ-minimal types in a superstable theory with few countable models*.
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Through the use of a "pseudo-algebraic" closure operator, almost atomic closure over finite rank, together with the meager forking machinery developed by L. Newelski, we may in some sense treat a type of U-rank omega in a superstable theory with few countable models much like a weakly minimal type. The focus of this talk will be to briefly outline a result for certain types of rank omega which parallels the transfer theorem in the weakly minimal case.

► ALBIN L. JONES, *Partitioning triples*.
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We will summarize the state of the art in the partition calculus of triples. As an example, consider a partition of the triples of all countable ordinals into two classes. We have recently proven that either for each integer m the first class contains all triples from a set of order type

$\omega + m$, or for each integer n the second class contains all triples of an n element set. That is, we have shown that $\omega_1 \to (\omega + m, n)^3$ for all integers m and n.

► HIROTAKA KIKYO, *Model companions of theories with an automorphism*.
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For a theory T in L, T_σ is the theory of the models of T with an automorphism σ. If T is the theory of the algebraically closed fields, it is known that T_σ has a model companion called *ACFA*. But usually it is hard to find a model companion of T_σ. The following results are obtained: If T is an unstable model complete theory without the independence property, then T_σ has no model companion. If T is an unstable model complete theory and T_σ has the amalgamation property, then T_σ has no model companion. Combining with a result of Kudaibergenov, if T is model complete and some model of T has the fcp, then T_σ has no model completion.

► BORIS A. KUSHNER, *Almost every computable real number is hard to compute*.
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A (normal) algorithm α computes a (say, classical) real number x if for every n the value $\alpha(n)$ is a rational number and $|\alpha(n) - x| < 2^{-n}$. A real number x is computable if there exists an algorithm that computes x. Let g be a total recursive function that we choose to be a measure of the complexity of calculations. A computable real number x is hard-computable with respect of g if for every algorithm α that calculates x one can find an m such that for every $n > m$ α takes more than $g(n)$ steps on the input n. A set S of computable numbers is of constructive measure 0 if for every n there is an algorithmic sequence of rational open intervals that covers S and such that all finite sums of lengths of its intervals are less than 2^{-n}. The following theorem holds (cf. Kushner [2]).

THEOREM. *For every total recursive function g almost every computable real number is hard-computable with respect of g.*

This result holds in Blum's axiomatic setting, as well.

[1] M. BLUM, *A machine-independent theory of complexity of recursive functions*, ***Journal of the Association for Computing Machinery***, vol. 14 (1967), no. 2, pp. 322–336.
[2] B. A. KUSHNER, *Slozhno-vychislimye deystvitel'nye chisla* (Russian), ***Zeitschrift für mathematische Logik und Grundlagen der Mathematik***, vol. 19 (1973), pp. 447–452.
[3] ———, ***Lectures on constructive mathematical analysis*** (translation from the Russian; Russian original, 1973), American Mathematical Society, Providence, Rhode Island, 1984.

► ALEX M. MCALLISTER, *Bounded Scott set saturation*.
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If $\mathcal{A} \models \mathcal{PA}$, then the family of the standard parts of parametrically definable subsets of $\mathcal{A}$ (commonly denoted $\mathcal{SS}(\mathcal{A})$) is a Scott set and we say that $\mathcal{A}$ is Scott set saturated. There are general notions of a structure being Scott set saturated. In particular, consider the following:

DEFINITION. If $\mathcal{A}$ is a structure and $\mathcal{S} \subseteq \mathcal{P}(\omega)$ is a Scott set, then $\mathcal{A}$ is *$\mathcal{S}$-saturated* (*$\mathcal{S}$-B_n-saturated*) if and only if for every $\bar{a} \subset A$ and every complete type (complete B_n-type) $\Gamma(\bar{a}, x)$, the following are equivalent:

(1) $\Gamma(\bar{a}, x) \in \mathcal{S}$ and $\Gamma(\bar{a}, x) \cup \mathcal{D}^c(\mathcal{A})$ is consistent,
(2) $\Gamma(\bar{a}, x)$ is realized in $\mathcal{A}$.

Wilmers first isolated the notion of $\mathcal{S}$-saturation and identified many interesting properties, including close connections with recursive saturation. We have developed the theory of $\mathcal{S}$-B_n-saturation which closely parallels that of $\mathcal{S}$-saturation. We have shown that $\mathcal{S}$-B_n-saturation is strictly weaker than $\mathcal{S}$-saturation and examined connections with bounded recursive saturation.

Although nonstandard models of $\mathcal{PA}$ are uniquely $\mathcal{S}$-B_n-saturated, other structures are not so well-behaved; there are structures which are not $\mathcal{S}$-B_n-saturated for any Scott set, structures which are $\mathcal{S}$-B_n-saturated for every Scott set, and even structures which are $\mathcal{S}$-B_n-saturated if and only if $\emptyset' \in \mathcal{S}$. We have identified some natural sufficient conditions which guarantee that a model of a theory is uniquely $\mathcal{S}$-B_n-saturated. Finally, we have produced a generalization of Scott's characterization of countable Scott sets as the Scott sets of nonstandard models of $\mathcal{PA}$.

► TIMOTHY H. MCNICHOLL, *Intrinsic reducibilities*.
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Let R be an extra relation on the domain of a countable model $\mathcal{A}$. Let $k \in \omega$. If $\leq_Q$ is a reducibility, then we say that R is *intrinsically* $\leq_Q \mathcal{A}^{(k)}$ if for every isomorphism $g: \mathcal{B} \approx \mathcal{A}$, $g^{-1}(R) \leq_Q \mathcal{B}^{(k)}$. We continue the work in [1] and [2] by obtaining syntactic equivalents to the following.

1. Intrinsic weak truth table reducibility to $\mathcal{A}^{(k)}$.
2. Intrinsic truth table reducibility to $\mathcal{A}^{(k)}$.
3. Intrinsic bounded truth table reducibility to $\mathcal{A}^{(k)}$.
4. Intrinsic n-recursive enumerability to $\mathcal{A}^{(k)}$.
5. Intrinsic n-query reducibility to $\mathcal{A}^{(k)}$.

All of these results hold if k is replaced by a constructive ordinal provided that notational dependence is accommodated.

[1] C. Ash, J. Knight, M. Manasse, and T. Slaman, *Generic copies of countable structures*, ***Annals of Pure and Applied Logic***, vol. 42 (1989), pp. 195–205.

[2] C. Ash and J. Knight, *Recursive structures and Ershov's hierarchy*, ***Mathematical Logic Quarterly***, vol. 42 (1995), pp. 461–468.

► ROBERT MILNIKEL, JR., *An inductive characterization of nonmonotonic inference*.
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In nonmonotonic rule systems, inference is usually defined by the secured consequences of a collection of rules. (For terminology, see [1].) This inference is often referred to as the problem of skeptical reasoning. Until now, the secured consequences of a rule system have been characterized only as the intersection of its extensions. I will present an inductive, syntactic characterization of inference for nonmonotonic rule systems.

[1] W. Marek, A. Nerode, and J. Remmel, *A theory of nonmonotonic rule systems, I*, ***Annals of Mathematics and Artificial Intelligence***, vol. 1 (1990), pp. 241–273.

► ANDRZEJ ROSŁANOWSKI AND SAHARON SHELAH, *Norms on possibilities: ccc forcing notions*.
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The method of building forcing notions using *norms on possibilities* has a long history already. A systematic study of this technique and a number of its applications were presented in Rosłanowski and Shelah [1]. However, typical examples constructed in this way are not ccc.

Here we develop the variant of the method of norms on possibilities presented in Rosłanowski and Shelah [2] which results in ccc forcing notions. We try to classify them using various dividing lines. We note that the method provides a number of examples of *ω-nw-nep forcing notions*. (The only examples of ω-nw-nep forcing notions known before were those coming from random forcing, the Cohen forcing and their FS iterations; see [3, §4] for a discussion of this topic.) Further we study the behaviour of our forcing notions when they are amalgamated (say, over a Cohen real). *The sweetness property* of Shelah [4, §7] is another cutting line here: some forcing notions are sweet (behaving like the Cohen forcing) while others are totally sour (which puts them, surprisingly, into the neighborhood of the random forcing).

[1] Andrzej Rosłanowski and Saharon Shelah, *Norms on possibilities I: forcing with trees and creatures*, ***Memoirs of the American Mathematical Society***, in press.

[2] ———, *Norms on possibilities II: more ccc ideals on 2^ω*, ***Journal of Applied Analysis***, vol. 3 (1997), pp. 103–127.

[3] Saharon Shelah, *On what I do not understand (and have something to say)*, preprint.

[4] ———, *Can you take Solovay's inaccessible away?*, ***Israel Journal of Mathematics***, vol. 48 (1984), pp. 1–47.

► AMY K. C. S. VANDERBILT AND DOUGLAS CENZER, *Common derivations in default theories*.

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The problem of determining the possible extensions of a given default theory has been studied by many—see [1]. Working within the field of Default Logic, an area of Nonmonotonic Logic, we consider a default theory (D, W) with a finite number of extensions. Restricting both D and W to being at most countable sets, we show that if ϕ is a formula of the language L, such that $\phi \in S$ for every extension S of (D, W), then ϕ has a common derivation $d^1{}_\phi$ that generates ϕ in every extension of (D, W). We further show that every extension of (D, W) is an extension of $(D \cup \{d^1{}_\phi\}, W)$. Although these two sets of extensions may be equal, we show that this is not always true. This work is part of the first author's dissertation, which is in preparation.

[1] Marek and Truszczyński, ***Nonmonotonic logics; Context-dependent reasoning***, Springer-Verlag, 1993.

Abstracts presented by title

► PAOLO LIPPARINI, *Decomposable ultrafilters and cardinal arithmetic*.

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Solovay showed that GCH holds at singular strong limit cardinals above a strongly compact cardinal. We show that some constraints on cardinal arithmetic are imposed by much weaker hypotheses.

Theorem 1. *Suppose that $(\mu_n)_{n<\omega}$ is a strictly increasing sequence of cardinals, and let*

$\mu = \sup_{n<\omega} \mu_n$, $\lambda = \mu^\omega$. *If D is an ω_1-complete ultrafilter which is μ_n-decomposable for all $n < \omega$ then D is λ-decomposable. Moreover, either*:

(i) $\lambda = \mu^+$, *or*

(ii) $\lambda = \nu^{+\alpha}$ *for some* α *with* $0 \leq \alpha \leq \omega + 1$ *and some* ν *such that either*

  (a) ν *is weakly inaccessible* (*and at least* ω*-weakly Mahlo*), *or*

  (b) ν *is singular and* D *is* cf ν*-decomposable.*

THEOREM 2. *Suppose that D is a μ-decomposable ω_1-complete ultrafilter. If $\mu^\omega > \mu$ then $\mu = \mu^{+n}$ for some $n < \omega$ and some μ satisfying one of clauses* (a) *and* (b) *in Theorem* 1.

A key argument in Solovay's proof is that if κ is strongly compact, μ is regular and $\kappa' < \kappa \leq \mu$ then $\mu^{\kappa'} = \mu$. Theorem 1 generalizes this result in the case $\kappa' = \omega$, since, by Theorem 2, only case (i) occurs above a strongly compact cardinal.

Natural generalizations of Theorems 1 and 2 hold when ω is replaced by any larger regular cardinal. They can be used to provide a simpler proof of Solovay's GCH theorem.

PROBLEM. Can case (ii) in Theorem 1 and $\mu^\omega > \mu$ in Theorem 2 actually occur?

► CYRUS NOURANI, *Positive forcing—the unrecorded specifics*.
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Start with a well-behaved countable fragment of an infinitary language $L_{\omega_1,\omega}$ defined by H. J. Keisler.

DEFINITION. A *generic diagram* for a model M is a diagram $D\langle A, G\rangle$, where A is a structure for M, definable by specific functions.

For example Σ_1 Skolem functions are a specific function set. We abbreviate generic diagrams by G-diagrams.

DEFINITION. A forcing property for a language L is a triple $\mathcal{F} = (S, <, f)$ such that

(i) $(S, <)$ is a partially ordered structure with a least element, for example, 0, thus $<$ is not always proper;

(ii) f is a function which associates with each p in S a set $f(p)$ of atomic sentences of $L[C]$;

(iii) whenever $p < q$, $f(p)$ is a subset of $f(q)$;

(iv) let l and t be terms of $L[C]$ without free variables and p in S, φ a formula of $L[C]$ with one free variable. Then if $(l\neg = t)$ is in $f(p)$ then $(t = l)$ is in $f(q)$ for some $q > p$. $\varphi(t)$ in $f(p)$ implies $\varphi(l)$ is in $f(q)$ for some $q > p$. For some c in C and $q > p$, $(c = l)$ is in $f(q)$.

The elements of S are called conditions for $\mathcal{F}$.

Let F be a family of functions which minimally defines the universe with the generic diagram for M. The inductive closure $T^*(P)$ is the set of formulas of the language provable from $T(P)$, in conjunction with a family of induction schemas $\langle I(f)\rangle$ for each f in F, through induction and modus ponens, in a language which includes implication $\rightarrow$ as a logical symbol and in which negation appears as a Boolean-valued operation only. Thus a theory $T^\#(P) = T(P) + \langle I[f] : f \in F\rangle$ is defined. For algebraic structures the universal reduct of $T^\#(P)$ is sufficient. We refer to the universal reduct by $T^*(P)$.

LEMMA. $\big(\mathcal{P}(T^*(P)), \subseteq, f\big)$, *where $\mathcal{P}$ is the powerset operation, and f an assignment of free variables in $T^*(P)$ into M, is a forcing property.*

DEFINITION. The relation $p \models +\varphi$ read "p positively forces φ" is defined for conditions p and q in S as follows: for an atomic sentence φ, $p \models +\varphi$ if and only if φ is in p; for an open formula φ of the form $f(X) = g(X)$, $p \models +\varphi$, if and only if for all c in $L[C]$, where c is an

n-tuple of constants form $L[C]$ and all q, $p < q$ implies there is an r such that $r > q$ and $r \models +f(c) = g(c)$. For arbitrary sentence φ, $p \models +\varphi$ if and only if p +forces a universal formula logically equivalent to φ.

DEFINITION. Let $\mathcal{F} = (S, <, f)$ be a positive forcing property, we say that a subset G of S is positively generic if and only if

(i) p and G and $q < p$ implies q in G;
(ii) p, q in G, implies there is an r in G such that $p < r$ and $q < r$;
(iii) for each sentence φ, there exists p in G, such that either $p \models +\varphi$ or there is no q in S, $q > p$ such that $q \models +\varphi$.

A special case of the above definition is when S consists of sets of formulas. For such S, we can make a substitution: subset relation for $<$. Clause (iii) can then be stated:

(iii$'$) for each sentence φ, there exists p in G such that either $p \models +\varphi$ or $p \cup \{\varphi\}$ is not a condition for $\mathcal{F}$.

THEOREM. *Let $T(P)$ be an equational theory, M its standard model encoded by a generic diagram. $\mathcal{P}(T^*(P))$ is positively generic for $T(P)$.*

Thus we have exhibited an explicit construction for a generic set. We have further shown that $T^*(P)$ has the genericity property.

By the theorem above $T^*(P)$ is the universal forcing companion for P.

An important special case for $T^*(P)$ is when P is Peano arithmetic. Of course, PA is not a universal theory, but by taking iterated Skolem expansion we can in principle get an extension of PA which is universal.

[1] H. J. KEISLER, *Forcing and the omitting types theorem*, in ***Studies in model theory***, Mathematical Association of America, New York, 1967.

[2] C. F. NOURANI, ***Forcing with universal sentences and initial models***, Association for Symbolic Logic, Boston, December 1983.