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1998 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

LOGIC COLLOQUIUM '98

Prague, Czech Republic, August 9–15, 1998

The 1998 European Summer Meeting of the Association was held at the University of Economics in Prague, August 9–15, 1998. It was co-organized by the Academy of Sciences of the Czech Republic and the Charles University—on the occasion of the 650th anniversary of the founding of the university. The meeting was sponsored and supported by the Association for Symbolic Logic, the National Science Foundation, the United States Air Force European Office of Aerospace Research and Development, the Division of Logic, Methodology and Philosophy of Science of the International Union of the History and Philosophy of Science, Hewlett-Packard s.r.o. (Czech Republic), Compulog Neg, the Charles University of Prague, the Academy of Sciences of the Czech Republic, and the University of Economics, Prague.

The last time a Logic Colloquium was scheduled to be held in Prague was August 1980. Unfortunately, the Czechoslovakian government cancelled that meeting about 40 days beforehand, due to fears that participants would organize protests against the imprisonment of the Czech philosopher and mathematician V. Benda. Benda is presently a senator in the Czech parliament.

The program of the Logic Colloquium included fifteen invited one-hour plenary addresses, three tutorials, and 39 invited half-hour lectures in parallel special sessions on computability theory, proof theory, set theory, model theory, philosophical logic, and fuzzy logic. In addition, there were 98 contributed talks plus 26 papers presented by title. During the conference, the Bolzano medal of the Academy of Sciences of the Czech Republic was presented to Gaisi Takeuti in recognition of his work in mathematical logic and other areas. The number of registered participants was 241, coming from 30 countries.

The tutorial short courses were given by P. Hájek, *Metamathematics of fuzzy logic*; J. Krajíček, *Propositional logic, arithmetic and complexity*; and Z. Sela, *The elementary theory of free groups*.

Invited plenary addresses were given by: S. B. Cooper, *Hartley Rogers' 1965 agenda*; T. Coquand, *Formal topology and proof theory*; A. Ekert, *On some ramifications of quantum computation* (joint work with R. Lupacchini); L. Fortnow, *Separating classes*; B. Hart, *The uncountable spectra of countable theories*; G. Hjorth, *Vaught conjecture on analytic sets*; T. Jech, *Stationary sets*; C. Jockusch, *Ramsey's theorem, computability, and second-order number theory*; L. Lipshitz, *The model theory of algebraically closed non-archimedean valued fields*; P. Komjáth, *Results and problems in combinatorial set theory*; E. Palyutin, *Commutative theories*; A. Razborov, *Complexity of resolution proofs*; A. Setzer, *The role of large cardinals in ordinal notation systems*; T. Strahm, *Metapredicativity*; and P. Welch, *The maximality of inner models: questions related to the Jónsson property*.

The Program Committee consisted of C. Ambos-Spies, L. Bukovsky, S. Buss (chair), P. Clote, S. Cook, L. van den Dries, P. Hájek, A. Macintyre, Y. Moschovakis, D. Mundici, R.

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Parikh, P. Pudlák, J.-P. Ressayre, S. Wainer, and B. Zil'ber. The local Organizing Committee consisted of B. Balcar, P. Hájek (chair), J. Ivánek, P. Jirků, J. Krajíček, A. Kučera, P. Pudlák, J. Sgall, P. Štěpánek, L. Váchová, and M. Zeithamlová. The honorary chairman was P. Vopěnka.

The abstracts of the tutorials, invited plenary talks, invited talks in special sessions, and contributed talks (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
S. BUSS

Abstracts of tutorials

► PETR HÁJEK, *Metamathematics of fuzzy logic*.
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Fuzzy sets were introduced by Zadeh in 1965. The term "fuzzy logic" has been for long used almost exclusively by non-logicians and papers and books on fuzzy logic deep from the point of view of mathematical logic have been rather rare. But the time seems ripe for serious logical investigation of fuzzy logic as a logic of vague propositions, propositions that may be more or less true—a logic with a comparative notion of truth (hence a kind of many-valued logic). The tutorial will survey the present state of knowledge in what could be called "mathematical fuzzy logic", indicate (briefly) use of formal systems in analyzing methods of fuzzy engineering (as fuzzy control) and present many open problems.

Part I: Propositional logic. Continuous t-norms as truth functions of conjunction. Basic fuzzy propositional logic, BL-algebras, completeness. Particular stronger logics: Łukasiewicz, Gödel and product logics. Standard completeness theorems (with respect to standard semantics over the real unit interval). Adding truth constants: Pavelka logic. Results on computational complexity.

Part II: Predicate logic. Basic predicate logic, completeness theorem. Recursive axiomatizability of Gödel predicate logic (with standard semantics). Recursive non-axiomatizability of Łukasiewicz and product predicate logic. Pavelka predicate logic.

Part III: Using fuzzy logic. Fuzzy functions. Zadeh's approximate inference as deduction. Generalized quantifiers; "many". The liar paradox and fuzzy logic.

[1] P. HÁJEK, ***Metamathematics of fuzzy logic***, Kluwer, 1998.

[2] P. HÁJEK and J. PARIS, *A dialogue on fuzzy logic*, ***Soft Computing***, vol. 1 (1997), pp. 1–3.

[3] P. HÁJEK, J. PARIS, and J. SHEPHERDSON, *The liar paradox and fuzzy logic*, ***Journal of Symbolic Logic***, to appear.

► JAN KRAJÍČEK, *Propositional logic, arithmetic, and complexity*.
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My ambition for the tutorial is to explain how the three areas and their main open problems (the length-of-proofs question for Frege systems, the finite axiomatizability of bounded arithmetic, and the P vs. NP problem, respectively) are related, and why it is interesting.

I shall put emphasis on independence results for bounded arithmetic and lower bounds for proof complexity.

An extensive bibliography can be found in [1]

[1] J. Krajíček, ***Bounded arithmetic, propositional logic, and complexity theory***, Encyclopedia of Mathematics and Its Applications, vol. 60, Cambridge University Press, 1995.

► ZLIL SELA, *The elementary theory of free groups*.
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We borrow techniques and concepts from low dimensional topology, algebraic geometry, and geometric group theory to analyze sets of solutions to systems of equations over a free group. The obtained structure theory enables the study of elementary sets and elementary predicates defined over a free group.

Abstracts of invited plenary talks

► S. BARRY COOPER, *Hartley Rogers' 1965 agenda*.
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In his lecture at the 1965 Logic Colloquium in Leicester, Hartley Rogers raised a number of very general questions which have since become basic to computability theory.

Starting out from a reassessment of Rogers' concerns of that time, as described in his seminal paper [1], we review the current situation in regard to the related research program.

[1] H. Rogers, Jr., *Some problems of definability in recursive function theory*, in ***Sets, models and recursion theory*** (J. N. Crossley, editor), North-Holland, Amsterdam, 1967, pp. 183–201, Proceedings of the Summer School in Mathematical Logic and Tenth Logic Colloquium, Leicester, August–September, 1965.

► THIERRY COQUAND, *Formal topology, unwinding of proofs and proof theory*.
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The notion of "topology without points", where the space is described in terms of "observable elements" and not of ideal points, provides a concrete and natural approach to topological notions. (This was stressed for instance by K. Menger.) We illustrate in this talk its potential for unwinding proofs on the example of a topological proof of van der Waerden's theorem on arithmetical progressions. This approach, if carried out in a weak metalanguage, can also be applied to proof theory. We give as an example of such an application a direct reduction of the classical theory of Σ_1^0 induction to its intuitionistic version.

► ARTUR EKERT, *On some ramifications of quantum computation*.
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The development of the theory of quantum computation has shown quite clearly that computation is a physical process and can only be properly understood within a context of physical theory—certain conjectures have to be resolved by experimentation rather than by pure reasoning. This leads to very interesting problems regarding, for example, the nature and status of mathematical truth, proof, inference.

I will introduce basic concepts and methods of quantum computations and show how the classical understanding of a proof as 'a sequence of propositions each of which is either an axiom or follows from earlier propositions in the sequence by the standard rules of inference' must be modified to account for quantum proofs. We must accept that in the future, quantum computers will prove theorems that neither a human brain nor any other arbiter will ever be able to check step-by-step, since if the 'sequence of propositions' corresponding to such a proof were printed out, the paper would fill the observable universe many times.

► LANCE FORTNOW, *Separating classes*.
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One of the greatest challenges in theoretical computer science and computational complexity is to separate complexity classes. We have very few interesting cases where we can show that one class of languages strictly contains another other than by straightforward simulation and diagonalization.

In this talk we look at the possibility of separating the complexity classes L (problems solvable in logarithmic space) and NP (problems solvable in nondeterministic polynomial time). While answering this question will not settle the famous P versus NP question, it would still separate two important complexity classes.

We will discuss two recent approaches to this problem. The first approach tries to settle this question by trading off time by alternation. We show that if Boolean formula satisfiability, the seminal NP-complete problem, requires a small amount of time we can simulate a large number of alternations with a small amount of time. We can then contrast this with an extension of a result of Nepomnjaščiĭ showing that large number of alternations require a large amount of space. Combining these ideas yields new time-space tradeoffs for satisfiability and may lead to a separation of nondeterministic time (NP) and space (L). In particular we show that no computer can solve satisfiability using $n \log^k n$ time and $\sqrt{n}$ space where n is the size of the formula and k is any constant. This is the first nontrivial lower bounds for satisfiability on general random-access Turing machines.

The second approach follows along the lines of Post's program. We will try to separate classes by looking at properties of classes, in particular autoreducibility. A set A is autoreducible if we can determine whether an element x is in A by querying the set $A - \{x\}$.

We show that all of the complete sets for the class of languages computable in exponential-time are autoreducible. We also show that there is some complete set for the class of languages computable in doubly-exponential space that is not autoreducible. This gives a new qualitative difference between these two classes.

Using translation arguments we show that settling the autoreducibility of complete sets for sets computable in exponential space would yield separations of important lower complexity classes. If all such languages are autoreducible then we have achieved our goal of separating NP from L. Otherwise we can exhibit some language computable in polynomial-space but not in NP.

The research on autoreducibility is joint work with Harry Buhrman, Leen Torenvliet and Dieter van Melkebeek.

Both of these papers are available from http://www.cs.uchicago.edu/~fortnow.

► BRADD HART, *The uncountable spectra of countable theories*.
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Consider the relation that, for a fixed complete countable theory T with infinite models, assigns to every uncountable cardinal κ the number of non-isomorphic models of T of cardinality κ. This is called the uncountable spectrum of T. After Shelah completed the Main Gap for countable theories, the question of the possible uncountable spectra remained open. Several years ago, Hrushovski outlined a program to answer this question. E. Hrushovski, M. C. Laskowski and I have carried out this program and in this talk I would like to discuss some of the history of the problem and at the same time indicate what new elements beyond

Shelah's Main Gap play a role in the complete computation.

► GREG HJORTH, *The Vaught conjecture on analytic sets*.
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Definition. A *Polish group* is a topological group that is Polish as a space—that is to say, separable and allows a complete metric. If a Polish space is acted on continuously by a Polish group G then we say it is a *Polish G-space*.

From the point of logic perhaps the most important Polish group is $\mathrm{Sym}(\mathbb{N})$—the group of all permutations of the natural numbers with the topology of pointwise convergence. This group stands out for its role in inducing the isomorphism relation on countable models. For $\mathcal{L}$ a countable language, we may give the space of all $\mathcal{L}$-structures on $\mathbb{N}$ a Polish topology, either that generated by first order or that generated by quantifier free formulas. There is a natural action of $\mathrm{Sym}(\mathbb{N})$ on this space of all countable models such that the isomorphism relation is exactly the resulting orbit equivalence relation.

Definition. A subset of a Polish space is *perfect* if it is closed, non-empty, and contains no isolated points. A perfect set necessarily has size continuum.

Let us say that a Polish group G *satisfies the Vaught conjecture on analytic sets* if whenever it acts continuously on a Polish space X, and $A \subset X$ is the continuous image of a closed set in a Polish space, and A/G (the orbits of G in A) is uncountable, then there is a perfect $P \subset A$ such that any two points in P have distinct orbits.

It is known that there are PC counterexamples to the original Vaught conjecture for countable models, and hence the infinite symmetric group $\mathrm{Sym}(\mathbb{N})$ does not satisfy Vaught's conjecture on analytic sets. Here I will present a result to the effect that the only groups not satisfying the Vaught conjecture are the ones that are in some sense as complicated as $\mathrm{Sym}(\mathbb{N})$.

Theorem. *A Polish group G satisfies Vaught's conjecture on analytic sets if and only if there is no closed subgroup H and continuous homomorphism from H onto* $\mathrm{Sym}(\mathbb{N})$.

► THOMAS JECH, *Stationary sets*.
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We examine the role of stationary sets in modern set theory. We give an overview of the current research on stationary sets and present some open problems.

► CARL G. JOCKUSCH, JR., *Ramsey's theorem, computability, and second-order number theory*.
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(Joint work with P. Cholak and T. Slaman and, separately, with T. Hummel.)

I will survey some results on the complexity of infinite homogeneous sets for the various forms of Ramsey's theorem and the formal strength of these forms as statements of second-order arithmetic. These two topics are closely related.

Let $\mathrm{RT}(n, k)$ be the assertion that for any k-coloring of the n-element sets of natural numbers there is an infinite set H which is homogeneous (i.e., all n-element subsets of H have the same color).

The following result refutes a conjecture of mine in [3].

THEOREM 1 (with P. Cholak and T. Slaman, [1]). *For any $n > 1$ and any computable k-coloring of the n-element sets of natural numbers, there is an infinite homogeneous set H such that H'' (the second jump of H) is computable from $0^{(n)}$ (the nth jump of the empty set).*

The proof of the next result is closely related to the proof of the $n = 2$ case of Theorem 1. Let RCA_0 be the system of second-order arithmetic based on algebraic axioms, Δ^1_1-comprehension and $\mathrm{I}\Sigma_1$ (Σ_1 induction) (see [5]). This theorem improves Seetapun's theorem [4] that $\mathrm{RT}(2,2)$ does not imply ACA_0 (arithmetic comprehension) over RCA_0. In contrast, Simpson [5] has shown that $\mathrm{RT}(n,k)$ is equivalent to ACA_0 for each $n > 2,\ k > 1$.

THEOREM 2 (with P. Cholak and T. Slaman, [1]). *Any Π^1_1 sentence of the language of second-order number theory provable from $\mathrm{RCA}_0+\mathrm{I}\Sigma_2+\mathrm{RT}(2,2)$ is provable from just $\mathrm{RCA}_0+\mathrm{I}\Sigma_2$. In particular,* PA *(Peano arithmetic), and even $\mathrm{B}\Sigma_3$ (Σ_3 bounding), are not provable from $\mathrm{RCA}_0 + \mathrm{I}\Sigma_2 + \mathrm{RT}(2,2)$.*

Cholak, Slaman, and I also analyze the strength of the statement $(\forall k)\ \mathrm{RT}(2,k)$ and show that it implies $\mathrm{B}\Sigma_3$ but not $\mathrm{B}\Sigma_4$ over RCA_0.

The work with T. Hummel concerns n-cohesive sets. An infinite set A of natural numbers is called n-cohesive if A is almost homogeneous for each computably enumerable 2-coloring of n-element sets.

THEOREM 3 (with T. Hummel, [2]). *For each n there is a Δ^0_{n+1} set which is n-cohesive.*

For $n = 2$, Hummel and I show in [2] that Theorem 3 can be improved to show the existence of a Π^0_2 set which is 2-cohesive. However, we do not know whether there exists a Π^0_n set which is n-cohesive for any $n > 2$.

[1] P. CHOLAK, C. JOCKUSCH, and T. SLAMAN, *The strength of Ramsey's theorem for pairs*, in preparation.

[2] T. HUMMEL and C. JOCKUSCH, *Generalized cohesiveness*, ***Journal of Symbolic Logic***, to appear.

[3] C. JOCKUSCH, *Ramsey's theorem and recursion theory*, ***Journal of Symbolic Logic***, vol. 37 (1972), pp. 268–280.

[4] D. SEETAPUN and T. SLAMAN, *On the strength of Ramsey's theorem*, ***Notre Dame Journal of Formal Logic***, vol. 36 (1995), pp. 570–582.

[5] S. SIMPSON, *Subsystems of second order arithmetic*, to appear.

► PÉTER KOMJÁTH, *Results and problems in combinatorial set theory*.
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We survey some recent results and mention some corresponding problems in combinatorial set theory.

THEOREM 1 (Komjáth-Shelah). *There are natural numbers t_n such that for any $n < \omega$ it is consistent that there is a set mapping $f : [\omega_n]^4 \to [\omega_n]^{<\omega}$ with no free sets of cardinal t_n.*

THEOREM 2 (Komjáth-Shelah). *For every $n < \omega$ it is consistent that there is a set mapping $f : [\omega_n]^2 \to [\omega_n]^{<\omega}$ with no infinite free sets.*

THEOREM 3. *If A is a set of reals, $|A| \leq \aleph_2$ and $f : A \to P(A)$ is a set mapping with $f(x)$ meager for every $x \in A$ then A has an ordering in which every initial segment is meager.*

THEOREM 4. *It is consistent that the previous theorem fails for some $A \subseteq \mathbb{R}$ with $|A| = \aleph_3$.*

THEOREM 5. *It is consistent that $\omega_1 \to (\omega_1, (\omega : 2))^2$ yet $\omega_1 \not\to (\omega_1, \omega + 2)^2$.*

THEOREM 6. *It is consistent that $\omega_2 \not\to (\omega_1, \omega + 2)^2$.*

THEOREM 7. *$\omega_3 \to (\omega_1, \omega + n)^2$ holds for every $n < \omega$.*

► LEONARD LIPSHITZ, *The model theory of algebraically closed non-archimedean valued fields*.
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We consider algebraically closed fields K, with a non-archimedean norm $|\cdot|\colon K \to \mathbb{R}_+$ satisfying the ultrametric triangle inequality

$$|a+b| \leq \max\{|a|, |b|\}$$

in the language with symbols for the field operations, the norm and multiplication and order on $\mathbb{R}_+$.

In 1956 A. Robinson proved the model-completeness and decidability of the theories of such fields. In the 1970s V. Weispfening (and others) gave quantifier eliminations for these theories. In the last 10 years (in the case that K is complete in $|\cdot|$), stimulated by work of Denef and van den Dries on the reals and p-adics, there has been renewed interest in the model theory of these fields in various analytic languages—i.e., the above mentioned language enriched by function symbols for various classes of analytic functions, and a symbol for bounded division

$$D(x,y) = \begin{cases} x/y & \text{if } |x| \leq |y| \neq 0, \\ 0 & \text{otherwise.} \end{cases}$$

Results include quantifier elimination in various languages (Lipshitz (1993), Schoutens (1994), Gardener and Schoutens (1997)) and model-completeness (Z. Robinson and Lipshitz (1997)). These results lead to the development of theories of semi-analytic and subanalytic sets over these fields with the results (but not the proofs) being in close analogy to the real and p-adic cases.

The talk will outline this development from 1956 to the present.

► E. A. PALYUTIN, *Commutative theories*.
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Commutative model theory generalizes Model theory of modules. The formation of the notion “Commutative theories” is connected with papers of author [1, 2, 3]. These papers had no connections with modules, but the paper [4] had binded these two directions. In this paper was proved a combinatorial fact, which generalizes the well known Neumann’s lemma about coverings groups by cosets of their subgroups [5].

Definition. An elementary theory T (not complete in general case) is commutative if for each model A of T the theory $\mathrm{Th}(A^\omega)$ of its direct power A^ω is stable and has no Dimention Order Property (DOP).

The property DOP was introduced by S. Shelah to describe elementary theories without structural model theory [6]. Stable theories without DOP are called weakly classicable theories. There is an “elementary” definition of commutative theories, but we have not enough place here to give it. The basic result for Commutative model theory is the following quantifier elimination theorem. It generalizes the well known Baur-Garavaglia-Monk theorem from Model theory of modules [7].

Theorem. *Let T be a commutative theory. Then each formula in T is equivalent to a Boolean combination of positive primitive formulas and one-placed formulas.*

In particular, such quantifier elimination is true for weakly classificable Horn theories.

[1] E. A. Palyuntin, ***Algebra i Logika***, vol. 19 (1980), no. 5.
[2] ———, ***Algebra i Logika***, vol. 24 (1985), no. 5.
[3] ———, ***Algebra i Logika***, vol. 30 (1991), no. 5.
[4] ———, ***Algebra i Logika***, vol. 34 (1995), no. 3.

[5] B. H. NEUMANN, ***Journal of the London Mathematical Society***, vol. 29 (1954).
[6] S. SHELAH, ***Classification theory***, North-Holland, 1990.
[7] M. ZIEGLER, ***Annals of Pure and Applied Logic***, vol. 26 (1984).

► ALEXANDER A. RAZBOROV, *Complexity of resolution proofs*.
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Complexity of propositional proofs plays as important a role in the theory of feasible proofs as the role played by the complexity of Boolean circuits in the theory of efficient computation. In many cases it provides a very elegant and combinatorially clean framework for studying provability of Σ_0^b-formulae in first-order theories of feasible arithmetic that bears essentially the same message as the original framework of first-order ("uniform") provability.

During several last years, many talks were delivered in various places on general aspects of either the whole discipline or its rather broad parts like Algebraic Proof Systems. It seems, by now the field has matured enough, and the time for more specialized survey talks has come. This talk is designed in such a way, and we choose for it one of the lowest levels in the hierarchy of propositional proof systems, Resolution. (Historically this was also the first p.p.s. considered by Tseitin in his seminal paper of 1968!)

Despite its simplicity, establishing lower bounds is a hard task not only for general resolution, but even for its more restricted versions like Regular Resolution. We survey known results in this direction, proved both by purely combinatorial arguments (like Haken's bottleneck technique) and by more general reductions to related problems in Complexity Theory (interpolation-like theorems).

Much of the research in the area is concentrated on the proof complexity of so-called *Pigeon-Hole-Principle*, and we will see how drastically it changes when we vary the number of pigeons (comparatively to the number of holes). We focus our attention on the central open problem in the area: understanding the proof complexity of this principle in case when the number of pigeons is very big, potentially infinite. We sketch some partial results and approaches toward this goal aimed at *regular* resolution. On our way, we recall a beautiful characterization of the latter system in purely computational terms of read-once branching programs for some specific search problems. The existence of such a characterization is something absolutely unique in the whole area of complexity of propositional proofs. We will see some related models of Rectangular Calculus and Transversal Calculus, and we will see some related (computational) bounds for read-once branching programs.

The last part of the talk consists of results obtained jointly with A. Wigderson and A. Yao.

► ANTON SETZER, *The role of large cardinals in ordinal notation systems*.
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Since Gentzen's analysis of Peano Arithmetic, one goal in proof theory has been the reduction of the consistency of mathematical theories to the well-ordering of ordinal notation systems. In the case of Gentzen's system and slight extensions the well-ordering of the systems is quite intuitive. Stronger ordinal notation systems are usually developed by using cardinals, large cardinals or their recursive analogues and the main intution is developed from set theory. Therefore they are no longer as intuitively well-ordered as the weaker systems, an obstacle for the understanding of such systems for non-specialists.

Ordinal systems is an alternative presentation of ordinal notation systems in such a way, that we have intuitive well-ordering arguments. They are defined in such a way that we can transfinitely enumerate all ordinal notations by repetitively selecting out of the set of ordinals, which are denoted by using ordinals previously chosen, the least element not chosen before with respect to some (well-ordered) termination ordering. In order to guarantee the correctness of this process and that it enumerates all ordinals, the following conditions are required:

- An ordinal notation is finite and refers only to smaller ordinals.
- If $\alpha < \beta$ can be denoted, then α is below some of the ordinals, β is denoted from, or the denotation of α is with respect to the termination ordering less than the denotation of β.
- If A is a set of ordinal notations which is well-ordered, the set of ordinals which can be denoted from ordinals in A is well-ordered with respect to the termination ordering.

An ordinal system is elementary, if the above condition can be verified in primitive recursive arithmetic, and elementary ordinal systems reach all ordinals below the Bachmann-Howard ordinal. In order to get beyond this bound, the analogue of cardinals in this approach is needed.

It turns out that what is needed are subprocesses: Instead of choosing in the main process the next ordinal directly, at every stage we need to start a subprocess in an ordinal system, which is relativized with respect to the ordinals already denoted in the main process. Once this subprocess is complete and has enumerated all ordinals in the relativized system, we can verify that the set of ordinals denotable from the ones previously selected in the main system is well-ordered with respect to the termination ordering and we can therefore select the next ordinal. For stronger ordinal notation systems, a more complicated arrangement of such subprocesses is necessary and therefore the role of cardinals in ordinal notation systems becomes clear: they are a way of organizing these processes.

We will analyze the relationship between these processes and cardinals in the case of regular cardinals, inaccessible cardinals and Mahlo cardinals.

► THOMAS STRAHM, *Metapredicativity*.
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The foundational program to study the principles and ordinals which are implicit in a predicative conception of the universe of sets of natural numbers led to the progression of systems of ramified analysis up to the famous Feferman-Schütte ordinal Γ_0 in the early sixties. Since then numerous theories have been found which are not prima facie predicatively justifiable, but nevertheless have predicative strength in the sense that Γ_0 is an upper bound to their proof-theoretic ordinal. It is common to all these predicative theories that their analysis requires methods from predicative proof theory only, in contrast to the present proof-theoretic treatment of stronger impredicative systems. On the other hand, it has been well-known for a long time that there are natural systems which have proof-theoretic ordinal greater than Γ_0 and whose analysis makes use just as well of methods which every proof-theorist would consider to be predicative. Nevertheless, not many theories of the latter kind have been known until recently.

Metapredicativity is a new general term in proof theory which describes the analysis and study of formal systems whose proof-theoretic strength is beyond the Feferman-Schütte ordinal Γ_0, but which are nevertheless amenable to *predicative methods*. It has turned out only recently that the world of metapredicativity is extremely rich and that it includes many natural and foundationally interesting formal systems.

In this talk we give a general survey and introduction to metapredicativity. In particular, we discuss various examples of metapredicative systems, including (i) subsystems of second order arithmetic, (ii) first and second order fixed point theories, (iii) extensions of Kripke-Platek set theory without foundation by reflection principles, and (iv) systems of explicit mathematics with universes.

Relevant keywords for our talk are: arithmetical transfinite recursion and dependent choice; restricted bar induction; transfinite hierarchies of fixed points; transfinite fixed point recursion; hyper inaccessibility, Mahloness and higher reflection without foundation; universe operators.

► P. D. WELCH, *The maximality of inner models: Questions related to the Jónsson property*.
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We shall look at various problems arising from embeddability properties to the universe V (or initial segments thereof), and their interactions with inner models.

It is well known that embeddings of certain inner models L, L^μ, ... give rise to larger cardinal like properties, than the models themselves contain. The rigidity of such canonical inner models allows certain properties of V to reflect down to the inner model concerned.

We shall look at questions related to the "maximality" properties of such models, and in particular, how the Jónsson property interacts with them. (A cardinal κ has the *Jónsson property* if every algebra $\mathcal{A} = \langle A, (f_n)_{n<\omega}\rangle$, with $\kappa \subseteq A$ (where f_n is a sequence of finitary functions) has a proper elementary subalgebra $\mathcal{A}'$ of the same cardinality κ.)

There has been a history of interactions between this property and inner models, as such cardinals provide elementary embeddings of part of a submodel to the model itself.

We shall consider the maximality of canonical inner models with respect to this and other properties, and the effect of the assumption of the existence of such a Jónsson cardinal on, say, its power set, and on embeddings of submodels of V to V.

Abstracts of invited talks in the Special Session in Computability

► MARAT M. ARSLANOV, *The enumeration degree structures*.
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A set A is enumeration reducible (e-reducible) to a set B if there exists a recursively enumerable (r.e.) set Φ (called in this context an e-operator) such that

$$A = \{\, x : (\exists u)\, [\langle x, u\rangle \in \Phi \,\&\, D_u \subseteq B]\,\}.$$

The structure $\mathcal{D}_e$ of the e-degrees is the structure of the equivalence classes (called e-degrees) of sets of numbers under the equivalence relation $\equiv_e$ generated by the preordering $\leq_e$; the e-degree of A is denoted by the symbol $\deg_e(A)$. $\mathcal{D}_e$ is in fact an upper semilattice with least element $\mathbf{0}_e$, where $\mathbf{0}_e = \deg_e(W)$, any r.e. set W.

We study the structures $\mathcal{D}_{\alpha,e}$ of α-r.e. e-degrees for $\alpha \leq \omega$. In [1] it was shown that for any Δ^0_2-e-degree $\mathbf{a} >_e \mathbf{0}_e$ there exists a Δ^0_2-e-degree $\mathbf{b} <_e \mathbf{0}'_e$ such that $\mathbf{0}'_e = \mathbf{a} \cup \mathbf{b}$. In [2] we showed that here in general we cannot choose $\mathbf{b}$ as an ω-r.e. e-degree: there is a Δ^0_2-e-degree $\mathbf{a} >_e \mathbf{0}_e$ such that $\mathbf{a} \cup \mathbf{b} <_e \mathbf{0}'_e$ for any ω-r.e. e-degree $\mathbf{b} <_e \mathbf{0}'_e$.

In this paper we obtain further results of this kind. In particular, we prove that structures of n-r.e., ω-r.e. and *total* e-degrees for $n < \omega$ are not pairwise elementarily equivalent, and discuss the analog of Downey's Conjecture for the structures of n-r.e. e-degrees, $1 < n < \omega$.

[1] S. B. Cooper, A. Sorbi, and X. Yi, *Cupping and noncupping in the enumeration degrees of Σ^0_2-sets*, to appear.

[2] M. M. Arslanov and A. Sorbi, *A note on the Δ^0_2 enumeration degrees*, to appear.

► C. T. CHONG, *The Friedberg jump inversion theorem revisited: The role of a definable cut*.
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We report on a joint work with Yue Yang and Lei Qian.

The Friedburg Jump Inversion Theorem states that every degree above $\mathbf{0}'$ is the Turing jump of a degree. We provide a level-by-level analysis of this Theorem. The main result states that Σ_n induction is necessary and sufficient for every degree between $\mathbf{0}'$ and $\mathbf{0}^{(n)}$ (inclusive) to be the Turing jump of a degree.

The proof hinges on an anatomy of Σ_n definable cuts in models of fragments of Peano arithmetic. Despite its seemingly simple looking set-theoretic definition, there is a wealth of information hidden in a definable cut, and its existence affects the structure of definable degrees of unsolvability in an essential way.

As a by-product, we also show that the existence of a low recursively enumerable degree is equivalent to Σ_1 induction, implying that its proof-theoretic complexity is greater than that of the Friedbert-Muchnik Theorem, known to hold without Σ_1 induction (earlier result of Chong and Mourad).

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There has been increasing interest over the last few decades in the study of effective model-theoretic and algebraic structures. Several different notions of effectiveness in structures have been investigated. We focus on structures whose functions and relations can be decided by Turing machines.

Definition. A structure $\mathcal{A}$ is *computable* if its domain is computable and its functions and relations are uniformly computable. An isomorphism from a structure $\mathcal{M}$ to a computable structure is called a *computable presentation* of $\mathcal{M}$. If $\mathcal{M}$ has a computable presentation then it is *computably presentable*.

In Model Theory we identify isomorphic structures. From the point of view of Computable Model Theory, however, two isomorphic structures might be very different. Thus, for our purposes, studying structures up to isomorphism is not enough. Instead, we study structures up to *computable* isomorphism. This is reflected in the following definition.

Definition. The *computable dimension* of a computably presentable structure $\mathcal{M}$ is the number of computable presentations of $\mathcal{M}$ up to computable isomorphism. A structure of computable dimension one is said to be *computably categorical*.

One way in which we may attempt to understand the differences between noncomputably isomorphic computable presentations of a structure $\mathcal{M}$ is to compare (from a computability-theoretical point of view) the images in these presentations of a particular relation on the universe of $\mathcal{M}$. One way to do this is to look at the (Turing) degrees of the images of a relation in different computable presentations of a structure.

Definition. Let U be a relation on the domain of a computable structure $\mathcal{A}$. The *degree spectrum* of U on $\mathcal{A}$ is the set of degrees of the images of U in all computable presentations of $\mathcal{A}$.

We study issues of computable dimension and possible degree spectra in the settings of particular algebraic theories, such as those of groups, rings, and integral domains. For instance, we show that there exists a computable group (respectively integral domain) of computable dimension two which contains a computable, intrinsically c.e. subgroup (respectively subring) with a two element degree spectrum, where we think of a subgroup as a unary relation on the domain of a group and of a subring as a unary relation on the domain of a ring.

► O. V. KUDINOV, *Intrinsically arithmetical relations and autostability*.
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After some nontrivial results by Ash, Nerode, Chisholm and others dealing with intrinsically recursively enumerable relations on recursive models, a natural question arises related to arithmetical relations considered on recursive or arithmetical models. Obtained by using the technique of recursive labelling systems results in this direction possess rather long lists of conditions and the corresponding problem of verification is too hard for some of them. However, under some natural conditions considered relations are definable in some recursive fragment of a standard language $L_{\omega_1\omega}$ with a finite number of parameters. We need the following definitions.

We denote the carrier set of a model $\mathbf{M}$ of language L as M, the set of all finite tuples as $M^{<\omega}$ and for a, b from $M^{<\omega}$ we put $a \leq_1 b$ if and only if for any existentional formula $\varphi(x)$ the condition $\varphi(a)$ implies $\varphi(b)$ and $\operatorname{length}(a) = \operatorname{length}(b)$. In addition, we put $a \leq_2 b$ if and only if $a \leq_1 b$ and for any $y \in M^{<\omega}$ there exists $v \in M^{<\omega}$ such that $b\hat{\ }v \equiv_1 a\hat{\ }u$. Given Turing degree a, a relation R on a model $\mathbf{M}$ is said to be a-intrinsically Σ^0_n-relation if and only if for any a-recursive representation $\mathbf{N}$ of the model $\mathbf{M}$ the set $f^{-1}(R)$ lies in Σ^0_n for any isomorphism f between $\mathbf{N}$ and $\mathbf{M}$. The first result is a simple compilation of known constructions.

Proposition. *Let* $\mathbf{M}$ *be a* 1*-decidable model, the set* $\leq_1$ *is decidable and a relation* R *on this model is* $0'$*-intrinsically* Δ^0_2*-relation. Then* R *is formally* Δ^0_2*-relation.*

The next theorem is the most interesting.

Theorem. *Let* $\mathbf{M}$ *be a* 1*-decidable model, the sets* $\leq_1$ *and*

$$\{\, \langle a, d\rangle \in (M^{<\omega})^2 \mid \exists x \in M^{<\omega}\ d\hat{\ }x \equiv_1 a \,\}$$

are decidable and a relation R *on this model is intrinsically* Σ^0_2*-relation. Then for some* $a \in M^{<\omega}$ *for any* $b, c \in M^{<\omega}$ *the conditions* $b\hat{\ }a \leq_2 c\hat{\ }a$ *and* $b \in R$ *imply the condition* $c \in R$*. In particular, the relation* R *is* Π_4*-definable in recursive fragment* $L^r_{\omega_1\omega}$ *of a standard language* $L_{\omega_1\omega}$ *with finite number of parameters.*

Proposition. *Let* $\mathbf{M}$ *be a* 1*-decidable model. If some invariant under action of the group* $\operatorname{Aut}(\mathbf{M})$ *relation* R *is* Σ^0_2*-set and it is not* Σ_2*-definable in* $L^r_{\omega_1\omega}$ *under any finite constant enrichment then* $\mathbf{M}$ *is not autostable.*

[1] C. J. Ash and A. Nerode, *Intrinsically recursive relations*, ***Aspects of Effective Algebra***, pp. 26–41, Proceedings of a Conference at Monash University, Australia.

► A. S. MOROZOV, *On recovering recursion-theoretic objects from groups of computable symmetries*.
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The first part of the talk contains the results on recovering Turing degrees from the groups of all permutations of natural numbers recursive in these degrees as well as from all possible nontrivial quotients of these groups, i.e., from quotients by subgroups of all finitary and even permutations. We study how these degrees are defined by isomorphism types of these groups as well as by their elementary theories.

In the second part, some similar problems for groups of Σ-definable permutations of recursively listed locally countable admissible sets are studied.

► BENJAMIN SCHAEFFER, *Abstract complexity theory and the degrees of unsolvability*.
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Abstract complexity theory was developed by M. Blum in order to exploit the underlying similarities between different complexity measures for computable sets to prove general theorems holding for all such measures. Simply by relativizing Blum's definition, one can apply abstract complexity theory to the degrees of unsolvability.

In these investigations we focus on Δ^0_2 functions, and the main complexity measure we use is the mind-change measure, a well-known notions of complexity for Δ^0_2 functions. This measure has been investigated by Ershov and Epstein, Hass, and Kramer.

Definition. A function f is *g-c.e.* if there exists a computable function $h(x, s)$ such that $\lim_s h(x, s) = f(x)$ and

$$|\{ s : h(x, s+1) \neq h(x, s) \}| \leq g(x)$$

for all x.

Abstract complexity theory focuses our investigations in two ways. First of all, it points out the importance of considering functions that are g-c.e. for noncomputable g. Also, since an analogue of the Gap Theorem holds in our context, we develop a notion of a Δ^0_2-honest function that is conceptually related to the honest functions of computational complexity theory.

An interesting notion of complexity for degrees is f-approximability.

Definition. A degree **a** is f-approximable if every function it computes is f-c.e.

We introduce a hierarchy of Δ^0_2 functions, the composition hierarchy, that is suitable for investigating the complexity theoretic properties of degrees. It turns out that the L_2 degrees are especially convenient to analyze in this way. Using this framework, we produce hierarchy results for generic degrees, c.e. degrees, and array noncomputable degrees in L_2. Transfinite levels of the composition hierarchy such as c^{ω} or $c^{\omega+1}$ can be easily defined by extending this definition.

Definition. Let c^0 denote the computable functions. By induction, a function $f \in c^{n+1}$ if f is g-c.e. for some $g \in c^n$.

We examine the array noncomputable (**anc**) degrees introduced by Downey, Jockusch, and Stob. To indicate the type of result proved and the methods needed, one can show, using a simple finite injury argument, that there exist F-approximable **anc** degrees, where $F \in c^{\omega}$. Using a $\mathbf{0}''$ tree argument, one can show there exist F-approximable **anc** degrees, where $F \in c^1$.

It turns out that a degree's structural properties can be linked to its complexity theoretic properties. For instance, we can examine complexity-based generalizations of the **anc** property. In our framework, the **anc** degrees are the c^1-nc degrees. Downey and Shore prove that every $c^{\omega+1}$-nc c.e. degree bounds a copy of M_5 in the c.e. degrees. On the other hand, Walk shows that there exists a c^1-nc degree that fails to bound a copy of M_5 in the c.e. degrees.

► FRANK STEPHAN, *On the structures inside truth-table degrees*.
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The following theorems on the structure inside nonrecursive truth-table degrees are established: Dëgtev's result that the number of bounded truth-table degrees inside a truth-table degree is at least two is improved by showing that this number is infinite. There are even infinite chains and antichains of bounded truth-table degrees inside every truth-table degree which implies an affirmative answer to a question of Jockusch whether every truth-table degree contains an infinite antichain of many-one degrees. Some but not all truth-table degrees have a least bounded truth-table degree. The technique to construct such a degree is used to solve an open problem of Beigel, Gasarch and Owings: there are Turing degrees (constructed as hyperimmune-free truth-table degrees) which consist only of 2-subjective sets and do therefore not contain any objective set. Furthermore a truth-table degree consisting of three positive degrees is constructed where one positive degree consists of enumerable semirecursive sets, one of co-enumerable semirecursive sets and one of sets, which are neither enumerable nor co-enumerable nor semirecursive. So Jockusch's result that there are at least three positive degrees inside a truth-table degree is optimal. The number of positive degrees inside a truth-table degree can also be some other odd integers as for example nineteen, but it is never an even finite number.

► SEBASTIAAN A. TERWIJN, *Randomness and lowness*.
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We will discuss recent joint work with Antonín Kučera and Domenico Zambella on the class $\mathcal{R}$ of Martin-Löf random reals and the class $\mathcal{S}$ of Schnorr random reals.

Martin-Löf randomness was introduced to give a consistent definition of the notion of "random sequence" or, in other words, recursive notion of measure-zero set of sequences. Later Schnorr introduced a more effective version of this notion. (Roughly, Schnorr's notion corresponds to recursiveness whereas Martin-Löf's notion corresponds to recursive enumerability.) We show that there are nonrecursive r.e. sets that are low for $\mathcal{R}$. (Following standard terminology, we say that a set A is *low* for a class $\mathcal{C}$ if for its relativized version $\mathcal{C}^{\mathcal{A}}$ it holds that $\mathcal{C} = \mathcal{C}^{\mathcal{A}}$.) This answers a question of van Lambalgen and Zambella. The question could be viewed as a "probabilistic version" of Post's problem. For the class of Schnorr random reals $\mathcal{S}$ we show that lowness has a purely recursion theoretic characterization in terms of approximability of functions. This gives us the existence of uncountably many sets that are low for $\mathcal{S}$.

[1] A. KUČERA and S. A. TERWIJN, *Lowness for the class of random sets*, ***Journal of Symbolic Logic***, to appear.

[2] S. A. TERWIJN and D. ZAMBELLA, *Algorithmic randomness and lowness*, manuscript, 1997.

Abstracts of invited talks in the Special Session in Fuzzy and Many-Valued Logic

► MATTHIAS BAAZ, *On the proof-theory of fuzzy logic*.
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We use the proof-theory of first-order Gödel logics as starting point of a general discussion on the proof-theory of fuzzy logics. We show that first-order Gödel logics are provably best represented by Hypersequent calculi, where we obtain cut-elimination and analogues of mid-sequent theorem and Maehara's lemma for cut-free proofs. The construction of Schütte-valuation trees[1] for cut-free completeness leads to a broader view of the proof-theoretic properties of basic logics like Urquhart's logic and Hajek's logic. We conclude with a discussion of the principal obstacles involved in a construction of proof-theoretic calculi for Łukasiewicz and Product logics.

► ULRICH HÖHLE, *Classification of separated subpresheaves over GL-monoids*.
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Let $M = (L, \leq, *)$ be a GL-monoid (cf. [1]); i.e., M is a strictly two-sided commutative quantale satisfying the following divisibility law

$$\forall(\alpha, \beta) \in L \times L \text{ with } \alpha \leq \beta \ \exists \gamma \in L \text{ such that } \alpha = \beta * \gamma.$$

A quadruple $(X, E, \mathbb{E}, \upharpoonright)$ is called a *presheaf* over M if and only if the pair (X, E) is a M-valued set (cf. Section 3 in [2]), the triple $(X, \mathbb{E}, \upharpoonright)$ is an ordinary presheaf over L and the following compatibility axioms are valid:

$$(\Pi 1) \qquad \mathbb{E}(x) = E(x, x)$$

$$(\Pi 2) \qquad E(x, y) * ((E(x, x) \to \alpha) \wedge (E(y, y) \to \beta)) \leq E(x \upharpoonright \alpha), y \upharpoonright \beta)).$$

A presheaf $(X, E, \mathbb{E}, \upharpoonright)$ is called *separated* if and only if the underlying M-valued set (X, E) is separated (cf. Section 3 in [2]). The category $\mathbf{SPSH}(M)$ of separated presheaves over M is complete and cocomplete (cf. Section 5 in [2]). Moreover there exists a separated presheaf Ω over M and a $\mathbf{SPSH}(M)$-morphism $t : 1 \mapsto \Omega$ provided with the following properties (cf. Theorem 6.4 in [2]):

- Every strict and extensional, L-valued map admits an internalization as a $\mathbf{SPSH}(M)$-morphism with codomain Ω.
- Every (Ω, t)-classifiable subobject is *unique* (Ω, t)-classifiable.

In general, a separated presheaf over M has more subobjects than characteristic morphisms.

[1] U. HÖHLE, *Commutative, residuated l-monoids*, in ***Non-classical logics and their applications to fuzzy subsets*** (U. Höhle and E. P. Klement, editors), Kluwer, Boston, Dordrecht, 1995, pp. 53–106.

[2] ———, *Presheaves over GL-monoids*, in ***Non-classical logics and their applications to fuzzy subsets*** (U. Höhle and E. P. Klement, editors), Kluwer, Boston, Dordrecht, 1995, pp. 127–157.

► JEFF PARIS, *Rational Pavelka predicate logic is a conservative extension of Łukasiewicz predicate logic*.
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Rational Pavelka logic extends Łukasiewicz infinitely valued logic by adding truth constants $\bar{r}$ for rationals in $[0, 1]$. This enables one to prove *partially* true conclusions from *partially* true premisses since φ has truth value $\geq r$ if and only if $\bar{r} \to \varphi$ has truth value 1. It is natural to ask whether this is a conservative extension. This is easily seen to be the case for the propositional logic, but for the predicate logic it was posed as an open problem by Hájek, [2]. In my talk I shall sketch a proof of this result and show how this allows Pavelka's notion of *provability degree* to be defined already in Łukasiewicz infinitely valued logic.

The crux of this conservation result is proving the existence of the suprema and infima giving the truth values of quantified formulae involving the new constants $\bar{r}$. This would be

[1] The Schütte valuation trees incorporate fuzzy properties themselves.

unnecessary if one used a claim of Belluce and Chang ([1], Theorem 1) that every theorem of Łukasiewicz predicate logic is true in all evaluations for which its truth value is defined (rather than the more demanding requirement of the existence of the possibly infinite many suprema and infima used in the proof). Unfortunately, as I shall indicate, their claim is false, not only for Łukasiewicz Logic but also for Product Logic.

The new results in this talk are all joint with Petr Hájek (Czech Academy of Sciences) and John Shepherdson (Bristol University), see [3].

[1] L. P. BELLUCE and C. C. CHANG, *A weak completeness theorem for infinite valued first-order logic*, ***Journal of Symbolic Logic***, vol. 28 (1963), pp. 43–50.

[2] P. HÁJEK, ***Metamathematics of fuzzy logic***, Kluwer, in preparation.

[3] P. HÁJEK, J. PARIS, and J. C. SHEPHERDSON, *Rational Pavelka predicate logic is a conservative extension of Łukasiewicz predicate logic*, 1998, submitted to the ***Journal of Symbolic Logic***.

Abstracts of invited talks in the Special Session in Model Theory

► JAN DENEF, *An application of quantifier elimination to singularity theory*.
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(Joint work with F. Loeser.)

We study the scheme of formal arcs on a singular algebraic variety and its images under truncations. These were first considered by J. F. Nash. We prove a rationality result for the Poincaré series of these images which is an analogue of the rationality of the Poincare series associated to the p-adic points on a p-adic variety. The main tools which are used are quantifier elimination for the field of power series over $\mathbb{C}$ and motivic integration (a notion first introduced by M. Kontsevich). Motivic integration on the scheme of formal arcs is used to obtain new geometric invariants of singularities.

► BERNHARD HERWIG, *On weakly o-minimal ω-categorical structures*.
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This is on joint work with H.D.Macpherson, G.Martin, A.Nurtazin, J.K.Truss. We try to classify in a certain sense all weakly o-minimal ω-categorical structures.

The first case: We assume there is only one 1-type. Otherwise the structure can be divided into finitely many pieces where this analysis works. We classify the binary structure up to isomorphism. It turns out, that the structure will consist of m many densely refining equivalence relations with convex classes, where m is the number of 2-types for ordered 2-tuples. This can be reformulated in terms of an $(m+1)$-valued ultrametric space. We cannot say anything about the structure of the first-order-definable higher arity relations. It will turn out later that this structure can be quite complicated.

The second case: We assume 2-indiscernibility (there is only one 2-type). We classify the ternary structure up to isomorphism. It turns out that again the number of 3-types already completely terminates the structure (only up to ternary relation). The structures are certain refining classes of C-relations. One can think of a C-relation as a one-parameter family of equivalence relations. Or one can think of a refining sequence of m C-relations as a one-parameter family of $(2m+2)$- or $(2m+3)$-valued ultrametric spaces.

The third case: If we assume 3-indiscernibility we get k-indiscernibility for every k. There remains only one structure: The dense linear ordering.

Some examples: We give a number of examples to illustrate what can happen in the higher arities. We construct continuum many examples. They require an infinite language.

► JORIS VAN DER HOEVEN, *Undecidability versus undecidability*.
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To several mathematicians, the undecidability theorm of Gödel came as a shock. Under the influence of this shock, a general opinion has arisen that as soon as one proves a problem to be undecidable, this problem is too hard and in a certain sense, not worth studying it any longer. In our talk, we want to put into perspective the importance of the concept of undecidability in the sense of Gödel on the hand of our main research topic: the automatic resolution of nonlinear differential equations.

First of all, many theoretically undecidable problems may be almost decidable in practice in the sense that the undecidable part of the problem is "degenerate". For instance, the problem of solving algebraic differential equations locally (when stated in a sufficiently general way) is undecidable. Nevertheless, when we write down such an equation at random, we have a large probability that we may solve it by the implicit function theorem.

On the other hand, certain questions which are theoretically decidable may be very hard to decide in practice. For instance, there exists an "algorithm" to compute the sign of $\sin 10^{10^{10^{10}}}$, but we would be very happy to own a computer which would actually be able to compute this sign.

The above discussion shows that the classical concept of undecidability is somehow not adapted to practice. Now the important question is how to build a theory of practical decidability? Ideally speaking, we would like to have algorithms for solving differential equations, which are able to detect whether the resolution of a given equation essentially involves a practically undecidable problem and, if not, to actually solve the equation.

In order to make this work, it is important to make a "catalogue" of practically undecidable problems, which may be very different from the undecidable problems we are used to. We will see that in absence of oscillation, we do have practical decidability. In case of oscillation, we shall discuss the problems of levels of exponentiality, Diophantine approximation, small divisors and a link between "chaos" and resummation theory.

A final interesting aspect of our approach is that it may be necessary to found the theory on a new system of "plausible axioms". The logical problem we are faced with here is the following: it is highly probable that many mathematical truths cannot be proved, because the classical set of mathematical axioms is too weak. For instance, consider the zero-test problem for exp-log constants (i.e., constants built up from the rationals by $+$, $-$, $\times$, $/$, exp and log). Such a zero-test was given by Richardson if Schanuel's conjecture holds. This algorithm has the particularity that it fails on a particular input if and only if this input leads to an explicit counterexample. For this reason, Schanuel's conjecture would be a good candidate for a "plausible axiom", but others will be discussed in our talk.

► LUDOMIR NEWELSKI, $*$-*algebraic groups*.
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In 1985, Hrushovski and Pillay proved that any 1-based group interpretable in a stable theory is abelian-by-finite.

I introduced the notion of m-independence (being the strongest natural notion of independence on finite tuples, which refines the forking independence of Shelah, in a small stable theory) and developed a theory of m-independence parallel in many ways to the theory of forking independence.

$*$-finite $*$-algebraic tuples are a special kind of countable tuples of elements of a model of

T. They play for m-independence a role similar to imaginaries in forking.

Restricting just to $*$-algebraic tuples, I extended m-independence to an arbitrary small theory. I defined the notion of an m-normal theory, corresponding in the theory of m-independence to that of a 1-based theory in forking.

A $*$-algebraic group is a type-definable group of uniformly $*$-finite $*$-algebraic tuples. For instance, if G is a group definable in a model of T, then the group G/G^0 is $*$-algebraic, where G^0 is the connected component of G.

I obtained some results on $*$-algebraic groups in small m-normal theories similar to the results of Hrushovski and Pillay on groups interpretable in 1-based stable theories. For example I proved that any $*$-algebraic group interpretable in a small m-normal theory is abelian-by-finite. I proved that any superstable theory T with $<$ continuum countable models is m-normal (and small). So in this case, for any group G interpretable in T, the group G/G^0 is abelian-by-finite.

It is open, whether any $*$-algebraic group interpretable in a small theory is abelian-by-finite.

[1] E. Hrushovski and A. Pillay, *Weakly normal groups* in ***Logic colloquium '85*** (Paris Logic Group, editors), North-Holland, 1987, pp. 233–244.

[2] L. Newelski, *m-normal theories*, preprint.

► HANS SCHOUTENS, *Model theoretic aspects of Artin approximation*.
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The following theorem is due to Artin. Let K be a field and $X = (X_1, \dots, X_n)$ be a set of variables. Let $R = K[\![X]\!]^{\text{alg}}$ denote the ring of algebraic power series. Then this ring is existentially closed inside $K[\![X]\!]$ and we just say that R has the Artin Approximation Property. This theorem has many important applications both in algebraic geometry and in commutative algebra. For instance, using model theoretic considerations, van den Dries showed how it can be put to use to prove the existence of Big Cohen-Macaulay modules in zero characteristic.

One observes that one of the key properties of the (Noetherian) local ring R is that it is Henselian (i.e., satisfies Hensel's Lemma) and it is dense inside the complete local ring $K[\![X]\!]$. This lead Artin to the following general conjecture: any Henselian excellent local ring has the Artin Approximation Property, i.e., is extentially closed inside its completion (with respect to the maximal ideal topology). Spivakovsky has now proven this theorem, using a technique called *smoothening along a section*. However, his proof is still inaccessible, so that alternative approaches to this conjecture would be welcome. We indicate how a model theoretic analysis might yield a different proof, at least in some special cases. The idea is to find an appropriate elementary class of local rings which is closed under taking the completion of a model and then to study its existentially closed models.

The class $\mathcal{C}_{d,e}$ of local rings we have in mind is the collection of all equicharacteristic local Cohen-Macaulay rings of fixed Krull dimension d and fixed multiplicity e. Unfortunately, no elementary class of positive dimensional rings containing only Noetherian models can exist and hence in particular the above class is not elementary. But it turns out that there is a first order theory $\mathcal{T}_{d,e}$ of rings of which the Noetherian models are precisely the models in $\mathcal{C}_{d,e}$. Moreover, any complete model is Noetherian and the class is closed under taking completion, as required.

Using the results of an earlier work on existentially closed Artinian local rings, we show that any Gorenstein ring with algebraically closed residue field and for which the Artin Approximation Property holds, is an existentially closed model of the class $\mathcal{C}_{d,e}$. Hence a new proof to Artin's Conjecture would be provided by showing that a Henselian local Gorenstein ring R with algebraically closed residue field, is existentially closed in $\mathcal{C}_{d,e}$. Of course, as part of the latter one needs to show that R is existentially closed inside its completion, but perhaps there exist other means to prove the result without going through this particular exemplification of it. For instance one could try to prove amalgamation for models which together with a non-first order version of Robinson's test would yield a form of model-completeness.

► PATRICK SPEISSEGGER, *The Pfaffian closure of an o-minimal structure*.
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Every o-minimal expansion $\tilde{\mathbb{R}}$ of the real field has an o-minimal expansion $\mathcal{P}(\tilde{\mathbb{R}})$ in which the solutions to Pfaffian equations with definable C^1 coefficients are definable.

► SERGEI STARCHENKO, *On groups definable in o-minimal structures*.
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Let M be an o-minimal expansion of a real closed field R, and G be a subgroup of $\mathrm{GL}(n, R)$ definable in M.

THEOREM (A. Pillay, Y. Peterzil, and S. Starchenko). *Either*

G is semialgebraic;

or

definable in the structure $\langle R, r_1(x), \ldots r_k(x)\rangle$, where each $r_i(x)$ is an M-definable automorphism of the multiplicative group of positive elements;

or

definable in the structure $\langle R, \exp\rangle$, where exp *is an M-definable isomorphism between the additive group of R and the multiplicative group of positive elements.*

Abstracts of invited talks in the Special Session in Philosophical Logic

► HORACIO ARLÓ-COSTA, *Acceptance conditions for knowledge and qualitative probability*.
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The first part of the paper provides acceptability conditions for knowledge (or full belief). Let L be a modal language containing the knowledge operator K. An information model is a set E of L-theories such that for every consistent T in E: (1) $A \in T$ if and only if $K(A) \in T$; (2) $A \notin T$ if and only if $\neg K(A) \in T$.

Acceptance in T is represented by set theoretical membership in T. Such theories are usually called stable in autoepistemic logic, and we will use here the same terminology. A sentence is valid in an information model if it is accepted in all the stable theories of the model. A sentence is valid if it is valid in all information models. Let Th(S5) be the set of theorems of the modal system S5. We show that A is in Th(S5) if and only if A is valid. We compare this result with a similar theorem proved by Robert Moore in [6].

The intuitionistic notion of assertability differs from the notion of acceptability used in the above characterization. Although the tertium non datur is valid there might be a piece of information T such that $(A \vee B)$ is accepted in T, although neither A nor B are accepted in T. In fact, there is an interesting resemblance between the behavior of the *forcing* relation used in quantum logics and our notion of acceptance. Recent work done in this area in philosophy of science ([1], [2] and [3]) is reviewed and contrasted with our approach.

In the second part of the paper we introduce a notion of qualitative probability Prob. The notion in question circumvents qualitative versions of the so-called lottery paradox.

Although we abandon the rule of Adjunction for Prob (see [4]), we maintain the deductive closure of the stable theories used in the model. Full belief is not defined here in terms of a more basic notion of probability. Infinitary versions of the lottery paradox considered in [5] are presented as the main obstacle to accomplish such reduction.

[1] M. L. DALLA CHIARA and R. GIUNTINI, *Quantum logical semantics*.

[2] ISAAC LEVI, *The logic of full belief*, in ***The covenant of reason*: *Rationality and the commitments of thought***, Cambridge University Press, 1997, pp. 40–69.

[3] B. C. VAN FRAASSEN, *Fine-grained opinion, probability and the logic of full belief*, ***Journal of Philosophical Logic***, vol. 24 (1995), pp. 349–377.

[4] HENRY E. KYBURG, JR., *The rule of adjunction and reasonable inference*, ***Journal of Philosophy***, March 1997.

[5] PATRICK MAHER, ***Betting on theories***, Cambridge University Press, 1993.

[6] ROBERT MOORE, ***Logic and representation***, CSLI Lecture Notes, no. 39, 1995.

► SHAUGHAN LAVINE, *Skolem was wrong*.
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The following apparent dilemma began with Skolem: A theory must be either first order or else higher order or infinitary. But neither alternative is adequate to categorize a subject of mathematical research, say, for example, the natural numbers or the sets. First-order logic is too weak—any theory of the natural numbers or the sets that is couched in it will have unintended models, in addition to the ones desired, and so will fail to characterize its subject. On the other hand, second-order and infinitary logics are themselves inadequately characterized, and they require axiomatization—either in higher-order or infinitary logic, which leads to a vicious regress, or else in first-order logic, which leaves theories just as inadequate as those stated directly in first-order logic. The dilemma has led to skepticism about whether mathematical theories can coherently be taken to refer to mathematical objects at all and to attempts to claim that the subject matter of mathematics needs no independent characterization. No such drastic reactions are necessary, since the dilemma is only apparent. Skolem and his heirs have missed a third possibility, one that solves the problem posed.

I show, using an idea developed by Feferman to a different end, that there are suitably categorical axiomatizations of the usual mathematical structures in an extension of first-order logic that is, for present purposes, adequately characterized and therefore not subject to the objections made to higher-order and infinitary logics.

Feferman's idea is that of a full schema: a structure for the language $\mathcal{L}$ is a model of an ordinary *schema* presented in the language $\mathcal{L}$ if it is a model of each instance of the schema in the language $\mathcal{L}$; but a structure in any language $\mathcal{L}^+$ is a model of a *full schema* presented in a language $\mathcal{L}$ if $\mathcal{L}^+$ includes $\mathcal{L}$ and the structure is a model of each instance of the schema in the language $\mathcal{L}^+$ (not just $\mathcal{L}$). To compare two models of, for example, Peano arithmetic with full schematic induction, consider a structure that includes both models. The full induction schemas will have instances in the combined language that are not in the language of either model taken separately, and those instances suffice to show that the two models are isomorphic. Analogous results hold for standard axiomatizations of set theory, the real numbers, and so forth.

I argue first, that the full induction schema is a codification of our intentions in defining the natural numbers, second, that the possibility of combining two models of Peano arithmetic with full schematic induction into one structure cannot coherently be denied so long as one is willing to raise the question whether our conception of the natural numbers suitably characterizes them, and finally, that the notion of a full schema is not subject to the problems posed for higher-order and infinitary logics. Related proposals by Parsons, McGee, and

Shapiro will be discussed, as well as criticisms by Field of the present proposal.

► HANS ROTT, *Making belief dynamics more dynamic*.
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The paper addresses conceptual problems in the change of beliefs, or more precisely: change of information states, that have shown up as a result of formal work on the logic of belief change in the last two decades. Special emphasis is given to the problem of iterated revisions which is hardly treated in the classical theories of the paradigm founded by Alchourron, Gärdenfors and Makinson (AGM). Some important problems about the representation and processing of possibly inconsistent information, however, surface only in this truly dynamic context.

Three different dimensions of coherence may be distinguished in the dynamics of belief systems. A synchronic concept (roughly, consistency and/or closure), a diachronic one (roughly, minimal change) and a dispositional one (usually codified in special 'rationality postulates'). Contrary to how the folklore has it I argue that the idea of minimal change has not been an important factor driving the investigations in the field; much more research has been devoted to dispositional coherence.

An important idea I discuss in the paper is that dispositions may stay as a relatively stable feature of a cognitive system that develops in time, thereby serving as a basis for conservative strategies of iterated belief change. I follow up that line of argument using orderings of sentences that are commonly referred to as relations of 'epistemic entrenchment.' A logical characterization of entrenchment is given that frees the notion of many of its (annoyingly) strong requirements. I also present a semantic approach in terms of choice functions. When the usual assumptions for belief change made by AGM are added, the conservative method for iterated theory change is equivalent to a method due to Boutilier who suggested a model-theoretic construction a few years ago.

Unfortunately, even in my more flexible setting the conservative method leads to undesirable results. My diagnosis is that conservatism in belief dynamics leads to a violation of a fourth dimension of coherence, which may be called temporal coherence. Conservative belief change is then compared with other methods describable in the AGM framework: external belief change (which is ultraconservative), radical belief change and moderate belief change. Only the latter approach, due to Nayak, complies with constraints regarding temporal coherence.

[1] C. Boutilier, *Iterated revision and minimal change of conditional beliefs*, ***Journal of Philosophical Logic***, vol. 25 (1996), pp. 263–305.

[2] P. Gärdenfors, ***Knowledge in flux: Modeling the dynamics of epistemic states***, Bradford Books, MIT Press, Cambridge, Massachusetts, 1988.

[3] A. C. Nayak, *Iterated belief change based on epistemic entrenchment*, ***Erkenntnis***, vol. 41 (1994), pp. 353–390.

[4] H. Rott, *Logic and choice*, in ***Theoretical aspects of rationality and knowledge, Proceedings of the seventh conference (TARK '98)*** (I. Gilboa, editor), Morgan Kaufmann, San Francisco, 1998, pp. 235–248.

Abstracts of invited talks in the Special Session in Proof Theory

► JEREMY AVIGAD, *A realizability interpretation for classical arithmetic*.
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A realizability interpretation for classical arithmetic in a Tait-style sequent calculus is presented. On the one hand, this interpretation is shown to coincide with the familiar form of intuitionistic realizability, under a suitable translation of the sequent calculus to natural deduction followed by an application of the Friedman-Dragalin translation. On the other hand, normalization of the realizing term and the corresponding intuitionistic proof are shown to be compatible with typical cut-elimination procedures. As a result, witnessing functions extracted from proofs of Π_2 sentences by each of the two methods compute the same result.

► ARNOLD BECKMANN, *Dynamic ordinal analysis of weak fragments of bounded arithmetic*.
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We define the *dynamic ordinal* of a theory which is a suitable analogue of the usual proof-theoretic ordinal for weak theories of arithmetic. We compute the dynamic ordinal of certain fragments of bounded arithmetic by adapting methods from the Schütte-style ordinal analysis of the fragments $\mathrm{I}\Sigma^0_n$ of Peano-arithmetic. Then different dynamic ordinals immediately imply the separation of the associated fragments of bounded arithmetic.

This way we also investigate very weak fragments of bounded arithmetic based on induction for $s\Sigma^{bb}_n$-formulas. The $s\Sigma^{bb}_n$-formulas are built up from double sharply bounded formulas by counting alternations of bounded quantifiers.

► L. BEKLEMISHEV, *Proof-theoretic analysis by iterated reflection*.
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Transfinite recursive progressions of iterated reflection principles can be used for a meaningful ordinal classification of arithmetical theories. This approach provides an alternative proof-theoretic analysis of Peano Arithmetic PA and, among other things, yields familiar characterizations of its provably recursive functions. From a technical point of view, this kind of analysis only uses the simplest form of cut-elimination for pure predicate calculus and is easily formalizable in the theories as weak as $\mathrm{I}\Delta_0 + \mathrm{Supexp}$.

This method is also interesting for the reason that it allows to classify Π^0_1-consequences of theories via iterated consistency assertions, whereas the more usual characterizations of provably recursive functions via fast growing hierarchies only capture their Π^0_2-consequences. As an example, we calculate the Π^0_1-ordinal of $\mathrm{PA} + \mathrm{Consis}(\mathrm{PA})$, which happens to be equal to $\varepsilon_0 + \varepsilon_0$.

This approach is especially advantageous in the study of fragments of PA. We prove that the class of provably recursive functions of the fragment of PA axiomatized by the induction schemas for Σ_1-formulas (with parameters) and Π_2-formulas without parameters coincides with the class of doubly-recursive functions of R. Peter, that is, with the ω^2th class in the Grzegorczyk hierarchy. Each of these theories taken separately corresponds to the class of primitive recursive functions (for the case of pure Σ_1-induction this is a well-known theorem of Parsons, Mints and Takeuti [3], the case of pure Π_2-induction without parameters is analyzed in [1]). A generalization of this to Σ_n-induction together with Π_{n+1}-induction without parameters yields the ordinal $\omega_n(2)$.

These results are based on a generalization to an essentially wider class of theories of the so-called Fine structure theorem of U. Schmerl on iterated uniform reflection principles over Primitive Recursion Arithmetic. We also obtain and apply a number of new results on local reflection principles. Over $\mathrm{I}\Delta_0 + \mathrm{Exp}$ the uniform reflection schema for Σ_1-formulas (for any reasonable theory) is Σ_2-conservative over the corresponding local reflection schema. A generalization of this fact to higher levels of arithmetical hierarchy is related (and gives new proofs of) some results in [2].

[1] L. D. BEKLEMISHEV, *Parameter free induction and reflection*, in ***Computational logic and proof theory*** (G. Gottlob, A. Leitsch, and D. Mundici, editors), Lecture Notes in Computer Science, no. 1289, Springer-Verlag, Berlin, 1997, pp. 103–113.

[2] R. KAYE, J. PARIS, and C. DIMITRACOPOULOS, *On parameter free induction schemas*, ***Journal of Symbolic Logic***, vol. 53 (1988), no. 4, pp. 1082–1097.

[3] C. PARSONS, *On a number-theoretic choice schema and its relation to induction*, in ***Intuitionism and proof theory*** (A. Kino, J. Myhill, and R. E. Vessley, editors), North-Holland, Amsterdam, 1970, pp. 459–473.

► HERMAN GEUVERS, *The meta-theory of typed lambda calculi and its relation to logical systems*.

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Due to the formulas-as-types embedding theorem, various logical systems in natural deduction style can be embedded into systems of typed lambda calculus. In this embedding the derivations become terms, which are 'first class citizems' of the type theory. This allows systems of typed lambda calculus to be used as interactive proof assistants. There is a strong relation between various meta-theoretic results of type theory and proof-theoretic results of the logic, e.g., normalization and cut-elimination.

In the talk we give an overview of the formulas-as-types embedding for various logical systems and we discuss the issue of completeness of the embedding. Furthermore we present several different presentations of typed lambda calculi and discuss the issue of their equivalence. The work presented here will partly be of an introductory nature, covering results in [3] (partially also reported in [1]). We will also discuss relations with some new results, notably in [2].

[1] HENK BARENDREGT, *Lambda calculi with types*, in ***Handbook on logica in computer science*** (Abramsky, Gabbay, and Maibaum, editors), Oxford University Press, pp. 117–310.

[2] GILLES BARTHE and MORTEN HEINE SORENSEN, *Domain free pure type systems*, in ***Proceedings of LFCS '97*** (S. Adian and A. Nerode, editors), Lecture Notes in Computer Science, no. 1234, Springer-Verlag, Berlin.

[3] HERMAN GEUVERS, *Logics and type systems*, ***Ph.D. Thesis***, Nijmegen University, 1993.

► JAN JOHANNSEN, *Bounded arithmetic for threshold circuits and counting hierarchies*.

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(Joint work with Chris Pollett.)

For notation and background see [2]. The theory C_k^0 is the theory in the language of first-order Bounded Arithmetic with the function symbols $\sharp_2, \ldots, \sharp_k$, axiomatized by the *BASIC* axioms, *LIND* for quantifier-free formulas and the replacement scheme $BB\Sigma_0^b$ for sharply bounded formulas.

Since C_2^0 is equivalent to the theory $\bar{R}_2^0$ of [1], the Σ_1^b-definable functions of C_2^0 are exactly those in the complexity class TC^0. We generalize this result to $k > 2$, so that, e.g., the Σ_1^b-definable functions of C_3^0 are exactly those in qTC^0, the class of functions computable by uniform constant-depth threshold circuits of quasi-polynomial size.

The second-order theory $\mathbf{D}_k^0$ is formulated in a second-order language that includes $\sharp_2, \ldots, \sharp_k$. It is axiomatized by *BASIC* axioms, *IND* and *BCA* for quantifier-free formulas, the choice scheme $\Sigma_0^{1,b}$-AC for first-order bounded formulas and a counting axiom that allows to count the number of elements in a set. The theory $\mathbf{D}_2^0$ characterizes the Counting Hierarchy *CH*: the $\Sigma_1^{1,b}$-definable functions in $\mathbf{D}_2^0$ are exactly the functions in *CH*. This result can also be generalized for $k > 2$.

We then show that for every k, $\mathbf{D}_k^0$ is isomorphic to C_{k+1}^0 under the so-called *RSUV*-isomorphism. In particular, the theory characterizing *CH* is isomorphic to the theory for qTC^0. As an application, we give some partial conservativity results for subtheories of $\mathbf{D}_k^0$, which can then be translated to the first-order world via the *RSUV*-isomorphism. In particular, we get weaker first-order theories that suffice to characterize TC^0 and qTC^0.

[1] J. JOHANNSEN, *A bounded arithmetic theory for constant depth threshold circuits*, in ***Gödel '96*** (P. Hájek, editor), Lecture Notes in Logic, no. 6, Springer-Verlag, 1996, pp. 224–234.

[2] J. KRAJÍČEK, ***Bounded arithmetic, propositional logic and complexity theory***, Cambridge University Press, 1995.

► RALPH MATTHES, *Functoriality of monotonicity witnesses in the system of positive (interleaved) inductive types*.
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As part of the definition of system M2LJ in [1] (p. 311), for every type ρ and every type variable α which occurs only (not necessarily strictly) positively in ρ, a closed term of type $(\alpha \to \alpha') \to \rho \to \rho[\alpha := \alpha']$ is defined. This term $\mathsf{map}_{\lambda\alpha\rho}$ witnesses monotonicity of ρ in the argument α and is used to define β-reduction for the type $\mu\alpha\rho$, which intuitively represents the least pre-fixed-point of $\sigma \mapsto \rho[\alpha := \sigma]$. For the work to be reported it is essential that already ρ may have been built up by the help of some $\mu\alpha'\rho'$. This may happen by nesting as, e.g., the countably-branching trees are defined by the help of the inductive type of the naturals. The more difficult case is given by interleaving μ-types as, e.g., $\mu\alpha.1+(\mathsf{tree}(\alpha) \to \alpha)$ with $\mathsf{tree}(\rho) := \mu\alpha.1 + (\rho \to \alpha)$. (This example is due to Ulrich Berger.) In [2] for the non-interleaved μ-types (more precisely: only for positive type schemes) it is shown that the terms $\mathsf{map}_{\lambda\alpha\rho}$ are functorial, i.e., they preserve identity and composition with respect to β- and η-equality. We also find in [2] the notion of initial $\lambda\alpha\rho$-algebra as opposed to weakly initial algebra which is modelled by β-equality.

We now extend functoriality to any $\mathsf{map}_{\lambda\alpha\rho}$ in the system M2LJ. For this to be true we have to add the canonical η-rule for $\mu\alpha\rho$ but also the conditional equality expressing (full) intiality of $\mu\alpha\rho$, i.e., every term of type $\mu\alpha\rho \to \sigma$ which behaves on constructor terms of type $\mu\alpha\rho$ as some μ-elimination is equal to the function defined by this elimination. This makes the resulting equality a priori undecidable because it gives a formal induction principle. The proof of functoriality obviously also needs a similar statement for the terms $\mathsf{comap}_{\lambda\alpha\rho}$ with α only negatively in ρ which had to be defined simultaneously with the terms $\mathsf{map}_{\lambda\alpha\rho}$. Moreover, we simultaneously have to prove a permutation rule for map and comap (for different indicies) which in some sense expresses naturality. The essential case is the interleaved μ. We need all the induction hypotheses due to a lack of permutative conversions for $\mu\alpha\rho$. The idea is to introduce permutative conversions following the well-known example of permutative conversions for sum types which are confluent and strongly normalizing together with β- and η-reduction.

I define equalities in order to permute μ-eliminations with any elimination which follows—including another μ-elimination. Unlike the initiality rule these equalities are not conditional. They do not apply to any μ-elimination followed by another elimination but to sufficiently many so as to ensure functoriality: Functoriality may be proved from these equalities (plus β- and η-equality) without the rule of (full) initiality. On the other hand, they are provably correct in the former theory, and the proof uses initiality, functoriality and the permutation rule which had to be proved simultaneously with functoriality.

I believe that the proposed equations may be turned into rewrite rules which preserve strong normalization and confluence. The resulting decidable equality theory would be a great help in the study of the relation between iteration and (full) primitive recursion on positive inductive types and positive fixed-point types.

[1] DANIEL LEIVANT, *Contracting proofs to programs*, in ***Logic and computer science*** (P. Odifreddi, editor), Academic Press, pp. 279–327.

[2] HERMAN GEUVERS, *Inductive and coinductive types with iteration and recursion*, in ***Preliminary proceedings workshop on types for proofs and programs*** (Nordström et al., editors), pp. 193–217 (`ftp://ftp.cs.chalmers.se/pub/cs-reports/baastad.92/proc.dvi.Z`).

► MICHAEL RATHJEN, *Universes in type theory*.
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The introduction of universes in Martin-Löf type theory resembles in many aspects the process of extending classical set theory via large cardinal axioms, and can thus be considered a constructivist's large cardinal programme.

The idea of forming universes in type theory is to introduce a universe as a set closed under a certain specified ensemble of set constructors, say C. The universe then "reflects" C. It is well in keeping with the spirit of Martin-Löf type theory as an open ended system to add new set constructors to existing formalizations. As a result, the formation of universes reflecting C can be employed as a new set constructor, which then gives rise to new universes (dubbed "superuniverses"), etc.

During the last three years, the exact proof-theoretic strength of many formalizations of Martin-Löf type theory with universes has been charted out. The talk will survey some of these results, including very recent ones about the strength of superuniverses.

► SERGEI TUPAILO, *Finitary reductions for local predicativity*.
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There is an extensive literature connecting infinitary "Schütte-style" and finitary "Gentzen-Takeuti-style" sides of proof theory. For example, in papers [4, 5, 2] this was done for systems not exceeding in strength Peano Arithmetic. But most recently, there has been an interest to what one can get on the side of finitary proof theory from the methods which are used for proof-theoretical analysis of impredicative theories. Especially we want to mention paper [3], where it was shown that Takeuti's reduction steps for $\Pi^1_1 - \mathrm{CA} + \mathrm{BI}$ can be derived from Buchholz' *method of* $\Omega_{\mu+1}$*-rule*.

Here we continue this line. As far as we know, the method of $\Omega_{\mu+1}$-rule has certain limitations as far as its power is concerned (no generalizations of it for theories stronger than ID_μ are known). On the contrary, modern ordinal analysis employs another very powerful tool, bearing a name of *local predicativity*, originally introduced by W. Pohlers and developed further by W. Buchholz, G. Jäger, M. Rathjen and others. On the side of finitary proof theory, Gentzen-Takeuti's finitary methods have been pushed very far by T. Arai [1]. So, in order to bring these two directions of modern proof-theoretic analysis closer together, a need arises to adapt methodology used in [3], to the context of local predicativity. This is what we are concerned with in the present research.

We have worked out our method for two theories: a theory T_Σ of recursively regular ordinals and a much stronger theory T_M of recursively Mahlo ordinals. Since our translation is pretty much the same in both cases, we will describe briefly only the latter case.

First we consider an infinitary version of T_M, T_M^∞, and define standard collection of operators on infinitary derivations, needed for proof-theoretical analysis of that theory: *First Cut-elimination* $\mathcal{R}$, *Second Cut-elemination* $\mathcal{E}$, *Predicative Cut-elimination* $\mathcal{E}_\gamma$, *Regular Bounding* $\mathcal{B}_{\beta,\kappa}$, *Regular Collapsing* $\mathcal{D}_\kappa^\mu$, *Mahlo Bounding* $\mathcal{B}_{\beta,\mathrm{M}}$ *and Mahlo Collapsing* $\mathcal{D}_\mathrm{M}$ (cf. [6]). Then, based on definitions of the operators above, for a finitary theory T_M^*, which extends T_M, we derive finitary cut-elimination steps together with corresponding ordinal assignments.

Cut-elimination for the finitary system formally makes no reference to its infinitary version, reduction step being defined by primitive recursion on the derivation. However, a convenient way to prove that ordinals decrease during reduction procedure comes from considering infinitary translations.

It should be noted that our translation method seems to be completely universal, and we do not expect any conceptual difficulties applying it to stronger theories. This would then provide a one-to-one correspondence between finitary and infinitary approaches to ordinal analysis.

[1] T. Arai, *Proof theory for theories of ordinals*, vol. I—III, 1996–1997, preprints.

[2] W. Buchholz, *Notation system for infinitary derivations*, ***Archive for Mathematical Logic***, vol. 30 (1991), pp. 277–296.

[3] ———, *Explaining the Gentzen-Takeuti reduction steps*, 1997, preprint.

[4] G. Mints, *Finite investigations of transfinite derivations*, in ***G. Mints, Selected papers in proof theory***, Bibliopolis, 1992, pp. 17–71. (Russian original: 1975.)

[5] ———, *A new reduction sequence for arithmetic*, in ***G. Mints, Selected papers in proof theory***, Bibliopolis, 1992, pp. 77–96. (Russian original: 1979.)

[6] M. Rathjen, *Proof theoretic analysis of KPM*, ***Archive for Mathematical Logic***, vol. 30 (1991), pp. 377–403.

[7] S. Tupailo, *Finitary reductions for local predicativity, I: Recursively regular ordinals*, 1998, preprint.

Abstracts of invited talks in the Special Session in Set Theory

► JÖRG BRENDLE, *Cardinal invariants of the continuum and combinatorics on uncountable cardinals*.

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A classical result of Bartoszyński, Raisonnier and Stern says that additivity of measure implies additivity of category, that is, if the union of any family of less than $\mathfrak{c}$ many sets of Lebesgue measure zero is still of measure zero, then the same statement holds with *measure zero* replaced by *meager*. Here, $\mathfrak{c}$ denotes the cardinality of the continuum. The main step in Bartoszyński's argument for this result is to show that additivity of measure implies Martin's axiom (MA) for Cohen forcing $\mathbb{C}_\omega$. Here, $\mathbb{C}_\kappa$ denotes the algebra for adding κ Cohen reals. We generalize this to

Theorem 1. *Additivity of measure implies Martin's axiom for any Cohen algebra* $\mathbb{C}_\kappa$.

While additivity of category easily implies MA for $\mathbb{C}_\omega$ (because of the connection between Cohen forcing and meager sets), this is not true for larger Cohen algebras.

THEOREM 2. *It is consistent that additivity of category holds, yet Martin's axiom fails for* $\mathbb{C}_{\omega_1}$.

In fact, one can show something stronger. As usual, $\clubsuit$ is the combinatorial principle claiming the existence of a sequence $\langle A_\alpha; \alpha < \omega_1, \alpha \text{ limit} \rangle$ with A_α being a cofinal subset of α such that for each uncountable $A \subseteq \omega_1$ there is α with $A_\alpha \subseteq A$. Under CH, $\clubsuit$ is equivalent to the diamond principle $\Diamond$. MA for $\mathbb{C}_{\omega_1}$ easily entails that $\clubsuit$ fails. We have

THEOREM 3. *It is consistent that additivity of category and $\clubsuit$ both hold, while $\mathfrak{c}$ is of arbitrary regular size.*

For $\mathfrak{c} = \aleph_2$, this follows from a stronger result of Džamonja and Shelah [1]; however, their construction does not generalize to larger cardinals. Another result of Shelah [3] implies Theorem 3 for $\clubsuit_{\omega_2}$.

This suggests that one investigates to what extent cardinal invariants of the continuum influence combinatorial properties of uncountable cardinals. For example one can prove

THEOREM 4. *If $\kappa \leq \lambda$ are regular uncountable cardinals, and $\mathcal{F} \subseteq [\lambda]^\kappa$ is such that for all $A \in [\lambda]^\kappa$ there is $B \in \mathcal{F}$ with $B \subseteq A$, then the size of $\mathcal{F}$ is at least the unbounding number $\mathfrak{b}$.*

There are a few similar results, with $\mathfrak{b}$ replaced by some other cardinal invariant of the continuum. On the other hand, by results of Fuchino, Soukup and Shelah [2], the continuum may be large with MA for $\mathbb{C}_\omega$ holding, yet there is $\mathcal{F} \subseteq [\aleph_2]^{\aleph_1}$ as in Theorem 4 of size $\aleph_2$.

In the lecture, we shall give an outline of the proofs of Theorems 1 and 3. A by-product of the techniques proving Theorem 1 is

THEOREM 5. *Assume $\mathfrak{c} = \aleph_2$. Then there is a Gross space over every countable field.*

Here, an uncountable-dimensional quadratic space E over an at most countable field is called a Gross space if the orthogonal complement of every countable-dimensional subspace has dimension less than $\dim(E)$. Our work complements results of Shelah and Spinas [4, 3] showing that the non-existence of Gross spaces is consistent with $\mathfrak{c} \geq \aleph_3$, and that the non-existence of Gross spaces over finite fields is consistent with $\mathfrak{c} = \aleph_2$.

[1] M. DŽAMONJA and S. SHELAH, *$\clubsuit$ does not imply the existence of a Suslin tree*, ***Israel Journal of Mathematics***, to appear.

[2] S. FUCHINO, S. SHELAH, and L. SOUKUP, *Sticks and clubs*, ***Annals of Pure and Applied Logic***, vol. 90 (1997).

[3] S. SHELAH, *Historic iteration with $\aleph_\varepsilon$-support*, preprint.

[4] S. SHELAH and O. SPINAS, *Gross Spaces*, ***Transactions of the American Mathematical Society***, vol. 348 (1996).

► ILIJAS FARAH, *Liftings and finite combinatorics*.
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Quotients $\mathcal{P}(N)/\mathcal{I}$ over analytic ideals $\mathcal{I}$ on the integers have been extensively studied from various points of view. For example, they have recently played a prominent role in the study of Borel equivalence relations (for more see Kechris' lecture at this Colloquium). They have also been considered as quotient Boolean algebras, a route suggested by Erdős and Ulam a while ago. The key to understanding the relationship between analytic quotients is in understanding the connecting maps between them, like Borel-reductions of equivalence relations, or homomorphisms of quotient algebras. Clearly, the existence of lifting theorems saying that a given connecting map has a lifting of a particularly simple, or *canonical*, form greatly simplifies questions whether two quotients are isomorphic, or whether one embeds into another. It appears that statements of this form often reflect as finite combinatorial statements. In the case of Boolean algebras canonical liftings correspond to Rudin-Keisler reductions between the ideals, and in many interesting cases (but not always!) homomorphisms do have canonical liftings. In the case of Borel-reductions of equivalence relations induced by ideals the situation is by far more complex, in particular it is not so clear which maps should play the role of canonical liftings. It is worth remarking that the finite statements associated with the existence of liftings often strongly resemble ones encountered in rather distant fields of mathematics. We shall overview the current situation in the area and discuss some of the open problems, emphasizing possible directions for further research.

► MOTI GITIK, *A simple extender based forcing*.
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A simple forcing for blowing the power of a singular cardinal without adding new bounded subsets will be presented. It simplifies the previous techniques of M. Magidor and the author.

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The last problem from the list in [1] asks whether Projective Determinacy (PD) can be derived in ZFC from its regularity consequences for the projective sets. The projective sets form the smallest class of sets containing the Borel sets which is closed under complementation and the formation of continuous images. PD states that in certain two-person games of infinite length with projective pay-off sets, one of the players must have a winning strategy. It is a strong set theoretic assumption completely determining the behavior of the projective sets. Among its consequences are Lebesgue measurability of projective sets, its dual in terms of category (the property of Baire) and projective uniformization (a choice principle saying that for each projective set A in the real plane, there exists a function with projective graph which picks for each number x in the projection of A onto the real line a witness y putting the pair (x, y) into A). In 1981 Woodin [2] conjectured that (in ZFC) these regularity properties conversely imply PD. By a recent theorem of Steel this is not the case, however, *effective* versions of the question are still open and suggest an intimate connection with the theory of canonical models for large cardinal axioms.

In my talk I will present some partial results and discuss various conjectures which would settle effective versions of Woodin's conjecture.

[1] A. S. Kechris, D. A. Martin, and J. R. Steel, editors, *Appendix*: *Victoria Delfino problems II*, in ***Cabal seminar 81–85***, Lecture Notes in Mathematics, no. 1333, 1988, pp. 221–224.

[2] W. H. Woodin, *On the consistency strength of projective uniformization*, in ***Logic colloquium '81*** (J. Stern, editor), North-Holland, 1982, pp. 365–383.

► SHIZUO KAMO, *Cardinal invariants associated with certain games*.
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Following A. Blass [1], we call a function from $\omega^{<\omega}$ to ω a *predictor*. A function $f \in \omega^\omega$ is said to *be predicted constantly* by a predictor π, if there is $n < \omega$ such that, for any $k < \omega$, $\pi(f \restriction j) = f(j)$, for some $j \in [k, k+n)$. Let κ be the smallest size of a set of predictors Π such that every function $f \in \omega^\omega$ is predicted constantly by some predictor in Π. The motivation of κ is in some game-theoretical characterizations for cardinals in Cichoń's diagram. F. Galvin gave game-theoretical characterizations for **d** and $\mathbf{cov}(\mathcal{M})$, and M. Scheepers for **b**, $\mathbf{add}(\mathcal{M})$,

non($\mathcal{M}$) and **add**($\mathcal{M}$). (See [3, 4] for details.) After that, M. Kada [2] characterized **cof**($\mathcal{M}$) and **cof**($\mathcal{N}$). Also, M. Kada pointed out the relationship between game-theoretic properties and the notion of predictors. The games which Kada used in the characterization of **cof**($\mathcal{M}$) were associated with a certain function in ω^ω. He suggested another game which does not use such a function and has conjectured that the game also characterizes **cof**($\mathcal{M}$). The above κ is a translation of this game.

We will show, by using some well-known generic extensions, that κ may be distinct from the cardinals in Cichoń's diagram.

[1] A. Blass, *Cardinal characteristics and the product of countably many infinite cyclic groups*, ***Journal of Algebra***, vol. 169 (1994), pp. 512–540.

[2] M. Kada, *Cofinalities of null and meager ideals and infinite games*, manuscript.

[3] M. Scheepers, *Lebesgue measure zero subsets of the real line and an infinite game*, ***Journal of Symbolic Logic***, vol. 61 (1996), pp. 246–250.

[4] ———, *Meager sets and infinite games*, ***Contemporary Mathematics***, vol. 192 (1996), pp. 77–89.

► ALEXANDER S. KECHRIS, *Borel equivalence relations*.
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In this talk I will discuss recent results and open problems concerning the structure of Borel equivalence relations on Polish spaces.

► PAVOL ZLATOŠ, *Indiscernibles in the alternative set theory*.
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The alternative set theory (AST), developed by P. Vopěnka, aims to provide an alternative to the classical Cantor's set theory in the foundations of mathematics. AST is, first of all, an alternative theory of the *infinity*, which is treated as a phenomenon accompanying our views towards the *horizon*. As the various kinds of horizon, as, e.g., the *accessibility* or *discernibility* horizons, are always blurred and hazy phenomena, AST and the fuzzy set theory (FST) seem to overlap to a great deal. On the other hand, in AST, unlike in FST, the hazy subsets of unambiguous sets, called *semisets*, are treated within the classical two-valued logic. There are no intermediate membership degrees of elements into semisets—their haziness consists in violating different principles of the classical set theory, e.g., the principle of mathematical induction. From the formal point of view, AST is closely related to nonstandard analysis—it can be regarded as the theory of a saturated nonstandard universe of power $\aleph_1$ of hereditarily finite sets with two sorts of variables—for sets (elements of the universe) and classes (parts of the universe).

In the beginning of my talk I will sketch the main ideas and the "philosophy" of AST and give a brief introduction to its basic notions, axioms and some few results concerning the approach of AST to topology, based on the notion of an *indiscernibility equivalence*.

Then I will survey some papers on *indiscernibles* for extensions of the language of AST by countably many constants, due to K. Čuda, C. W. Henson, J. Mlček, B. Kussová-Vojtášková, A. Sochor, A. Vencovská and myself.

In the first part I will mention some general results of set-theoretical nature, concerning the existence of "large" classes of indiscernibles, the structure and location of monads of indiscernibles (which correspond to Ramsey ultrafilters in the classical theory) in the universe of sets, both with respect to a naturally definable order of the universe and with respect to a kind of Rudin-Keisler order of monads. In particular, each monad of indiscernibles is minimal in the Rudin-Keisler order, however, unlike for the classical Rudin-Keisler order of ultrafilters (where minimal and Ramsey ultrafilters coincide), the converse cannot be proved—its negation is consistent with AST. Some Ramsey type theorems, relating infinite sets of indiscernibles and equivalences of indiscernibility will be mentioned, as well.

The second part will be devoted to an application of indiscernibles to a problem from the theory of topological vector spaces in AST. In such a space, every infinite set contained in the *galaxy* of 0, such that no couple of its elements are infinitesimally close, contains an infinite subset of indiscernibles, such that none of its elements is infinitesimally close to the subspace spanned by the remaining ones.

Abstracts of contributed talks

► HAJNAL ANDREKA AND TAREK SAYED AHMED, *Omitting types in logics with finitely many variables*.
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Throughout n denotes a finite ordinal.

We show that, in contrast to first-order logic ($L_{\omega,\omega}$), the Henkin-Orey omitting types theorem fails (in a rather strong sense) for L_n, the first-order logic reduced to the first n variables, if and only if $2 < n$.

Also we characterize omissibility of a type, or a countable family of types, in a countable L_n theory, in terms of a certain algebraic notion formulated for CA_ns; (cylindric algebras of dimension n) the algebraic counterpart of L_n theories.

Generalizing to the infinite case, we obtain a new characterization of completely representable CA_αs, α a countable ordinal. In particular, we show that even the countable subdirectrly irreducible atomic representable CA_ωs may fail to have complete representations.

Finally, using results of Newelski [1] in investigating omitting $<$ cov K many types for $L_{\omega,\omega}$, we show how omitting uncountably many (noncomplete) types in countable L_n theories may lead to propositions that are independent of ZFC.

[1] Newelski, *Omitting types and the real line*, ***Journal of Symbolic Logic***, vol. 62 (1987), no. 4, pp. 1020–1026.

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We present an axiomatic framework for nonstandard analysis—the Nonstandard Class Theory (NCT) which extends von Neumann-Gödel-Bernays set theory (NBG) by adding a unary predicate symbol St to the language of NBG (St X means that the class X is standard) and axioms related to it—analogs of Nelson's idealization, standardization and transfer principles. Those principles are formulated as axioms rather than axiom schemes so that NCT is finitely axiomatizable. A version of NCT was considered in [1]. The main difference between that version and the theory presented here is that the last can be viewed as a theory of formula classes of Bounded Set Theory (BST) by V. Kanovei [2] rather than that of Internal Set Theory (IST) by E. Nelson, in particular every set is an element of a standard set. This modification makes it possible to strengthen the standardization axiom and to add the saturation axiom formulated below. An important place in NCT takes the notion of relative standardness. A class is called p-standard (or standard relative to the set p) if and only if it can be represented as a cut of a standard class by the set p. A class is an interval if and only if it is p-standard for a certain set p. Relativized transfer principle and restricted relativized idealization principle hold for the intersection of all p-standard classes containing the set x. A class is called p-saturated if and only if together with every one of its elements it contains the whole p-monad of that element. The saturation axiom states that every class is p-saturated for an appropriate set p. This axiom implies the usual saturation for arbitrary classes of standard size which makes it possible to formalize the Loeb measure theory and a certain analog of the theorem on completeness of a nonstandard hull of a normed space.

In many aspects NCT resembles Alternative Set Theory (AST) by P. Vopenka [3]. First of all there are semisets (proper subclasses of sets) here and a theorem can be proved that a class is standard if and only if its intersection with any standard set is a standard set. It is an open question whether a similar theorem for internal classes is true as well. Various other notions which stem from the concept of natural infinity can be formalized in NCT, but in contrast with AST it preserves all results of the classical mathematics. Thus, in a way NCT can be viewed as an alternative to the Alternatice Set Theory. Apparently, a more complete analog of AST can be obtained if we replace in NCT the axiom of infinity by its negation, but one should bear in mind that NCT does not obey the strong axiom of choice, which states that any semiset can be linearly ordered in such a way that every subsemiset has a least element—this can be done if and only if the semiset is of standard size. It should also be mentioned that NCT is a conservative extension of BST which, in turn, is a conservative extension of ZFC.

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[1] E. Gordon, ***Nonstandard methods in commutative harmonic analysis***, Providence, Rhode Island, 1997.

[2] V. Kanovei and M. Reeken, *Internal approach to external sets and universes, Part 1: Bounded set theory*, ***Studia Logica***, vol. 55 (1995), no. 2, pp. 229–257.

[3] P. Vopěnka, ***Mathematics in the alternative set theory***, Teubner, Leipzig, 1979.

► MOHAMMAD ARDESHIR, *Uniform interpolation in basic propositional logic*.
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The language of Basic Propositional Logic, BPC is $\mathcal{L} = \{\vee, \wedge, \rightarrow, \perp, \top\}$ and its axioms and rules are the same as Intuitionistic Proposition Logic, IPC, except *Modus Ponens*, which is weakened in BPC, [1].

Ordinary interpolation theorem in a logic says that if $A \vdash B$, then there is a formula C in the language containing only the shared propositional variables $p_1, \ldots, p_n$ in A and B such that $A \vdash C$ and $C \vdash B$.

The uniform interpolation is strengthening of ordinary interpolation in the sense that to find the interpolant C we need weaker data: it is enough to have A and (the shared) propositional variables $p_1, \ldots, p_n$ or B and $p_1, \ldots, p_n$. These forms are stated as follows:

Theorem (Uniform Post-Interpolation in BPC). *Let A and $p_1, \ldots, p_n \in \mathrm{PV}(A)$ be given. There is a formula $C(p_1, \ldots, p_n)$ such that $A \vdash C$, and for all formula B with*

$$\mathrm{PV}(A) \cap \mathrm{PV}(B) = \{p_1, \ldots, p_n\},$$

$A \vdash B$ if and only if $C \vdash B$.

Theorem (Uniform Pre-Interpolation in BPC). *Let B and $p_1, \ldots, p_n \in \mathrm{PV}(B)$ are given.*

There is a formula $C(p_1, \dots, p_n)$ *such that* $C \vdash B$, *and for all formula* A *with*

$$\mathrm{PV}(A) \cap \mathrm{PV}(B) = \{p_1, \dots, p_n\},$$

$A \vdash B$ *if and only if* $A \vdash C$.

[1] M. Ardeshir and W. Ruitenburg, *Basic propositional calculus, I*, ***Mathematical Logic Quarterly***, to appear.

[2] A. Visser, *Uniform interpolation and layered bisimulation*, in ***Gödel '96***, Springer-Verlag, 1996, pp. 139–164.

► MATTHIAS BAAZ AND ALEXANDER LEITSCH, *Fast elimination of monotone cuts*.
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Statman and Orevkov independently proved that cut-elimination is of non-elementary complexity even for Horn theories. By restricting the logical operators to $\{\wedge, \vee, \exists, \forall\}$ we obtain the type of *monotone* formulas. We show that the elimination of monotone cuts can be of non-elementary complexity (here generalized disjunctions in the antecedents of sequents play a central rôle). On the other hand we define a large class of problems (including all Horn theories) where elimination of monotone cuts is only exponential and show that this bound is tight. This implies that the elimination of monotone cuts in equational theories is easy. Particularly there are no short proofs of Statman's sequence with monotone cuts. The method of cut-elimination employed (*cut-projection*) differs strongly from Gentzen's method which can be shown to be of non-elementary complexity for equational theories with monotone cuts. Some applications to number-theories like **PRA** and Robinson's **Q** are provided.

[1] M. Baaz and A. Leitsch, *Cut normal forms and proof complexity*, ***Annals of Pure and Applied Logic*** (1998), to appear.

[2] ———, *Fast cut-elimination by projection*, in ***Proceedings of CSL '96***, Lecture Notes in Computer Science, no. 1258, Springer-Verlag, Berlin, 1997, pp. 18–33.

► STEFANO BARATELLA AND SIU-AH NG, *Neocompact quantifier elimination in structures based on Banach spaces*.
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We introduce a notion of quantifier elimination for Banach spaces, called Keisler's Quantifier Elimination (briefly *QE*) and we compare it with Henson's Quantifier Elimination (QE_H).

First of all, the "correct" logic for Banach spaces is not the first order one, but something such as the *positive bounded formulas* with *approximate satisfiability* developed by Henson in [2]. Nonstandard hulls of Banach spaces can be classified by positive bounded theories.

On the other hand, Fajardo and Keisler [1] formulated an abstract framework in which techniques from nonstandard analysis can be applied. As a related development, Keisler [4] defined and studied a class of infinitary expressions called *neocompact formulas*. In the same paper he proved general results on quantifier elimination. We consider here neocompact formulas in Banach spaces (notice that positive bounded formulas are neocompact) and we investigate the problem of reducing neocompact formulas to countable conjunctions of quantifier-free positive bounded formulas.

We deal with *Banach space structures* in the sense of Henson-Iovino [3] and consider sets definable using neocompact formulas from Keisler [4]. In particular, we consider also Keisler's *law structures*.

We make frequent use of the nonstandard hull construction due to Luxemburg (see [5]).

We first fix our setting by introducing basic definitions and we define a particular law structure, then we prove a technical result that, in the nonstandard hull of a standard Banach structure, neocompact formulas can be reduced to countable conjunctions of positive bounded formulas. This reduction will be applied later to give quantifier elimination.

We formulate some properties of our law structure taken from [4] and we prove equivalents of those properties in the setting of Banach space structures. We also prove the equivalence of the back-and-forth property and quantifier elimination in certain saturated spaces.

Sufficient conditions are studied under which *QE* transfers from nonstandard hull to the original space. Eventually, we show that *QE* is independent of the nonstandard hull construction. We also prove that *QE* and QE_H agree on nonstandard hulls of Banach spaces. One key ingredient leading to this result is a comparison of approximate satisfaction in the space, its nonstandard extension and its nonstandard hull.

[1] S. Fajardo and H. J. Keisler, *Neometric spaces*, ***Advances in Mathematics***, vol. 118 (1996), pp. 134–175.

[2] C. W. Henson, *Nonstandard hulls of Banach spaces*, ***Israel Journal of Mathematics***, vol. 25 (1976), pp. 108–144.

[3] J. Iovino, *A quick introduction to Banach space model theory*, preprint.

[4] H. J. Keisler, *Quantifier elimination for neocompact sets*, to appear.

[5] W. A. J. Luxemburg, *A general theory of monads*, in ***Applications of model theory to algebra, analysis and probability*** (W. A. J. Luxemburg, editor), Holt, Rinehart and Winston, New York, 1969.

► DAVID BASIN, SEÁN MATTHEWS, AND LUCA VIGANÒ, *Modal logics* K, T, K4, S4: *labelled proof systems and new complexity results*.

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`htp://www.mpi-sb.mpg.de/∼sean`.

In previous work [1, 5], we developed a framework for giving modular and uniform presentations of modal and other non-classical logics: we present logics as labelled (natural deduction or sequent) proof systems, in which we pair logical formulae with labels in order to formalize the consequence relations of the different logics, and which are sound and complete with respect to the appropriate Kripke semantics. We also showed how to use our framework to obtain complexity results for the modal logics K and T, by means of a proof-theoretical analysis of the structural rules of the corresponding labelled sequent systems. (This analysis also establishes the advantages of our approach with respect to related ones, in particular those similarly based on labelling, e.g. [2, 3].) We here improve and extend this analysis, and as a result we obtain $O(n \log n)$ space decision procedure for K, and $O(n^2 \log n)$ space procedures for T, K4 and S4. In the case of K, these space bounds are equal to the best currently known [4]. Note also that our analysis is completely modular and different from that of [4], which relies on the introduction of new modalities in standard (unlabelled) sequent calculi.

Let W be a denumerable set of *labels* $x, y, \dots,$ and R a binary relation over W. We factor our presentations into two parts, a fixed *base* system and a varying *relational theory*, corresponding to the two sorts of syntactic entities we consider: labelled formulas $x: A$, which pair a label x and a modal formula A (expressing that A holds at world x), and relational formulas $x\ R\ y$ (expressing that (x, y) is in the accessibility relation R). Let Γ and Δ (possibly annotated) vary over finite multisets of labelled and relational formulas, respectively. A sequent then has the form either $\Gamma, \Delta \vdash \Gamma'$ or $\Delta \vdash x\ R\ y$. The fixed base sequent system K consists of, among others, the following axioms and structural and logical rules, where $\Box R$ has the side condition that y does not occur in $\Gamma, \Delta \vdash \Gamma', x : \Box A$.

$$\frac{}{x : A \vdash x : A}\ AXl \qquad \frac{}{x\ R\ y \vdash x\ R\ y}\ AXr \qquad \frac{}{y : \bot \vdash x : A}\ \bot L$$

$$\frac{\Gamma, \Delta \vdash \Gamma'}{\Gamma, \Delta, x\ R\ y \vdash \Gamma'}\ WrL \qquad \frac{\Delta, x\ R\ y, x\ R\ y \vdash u\ R\ v}{\Delta, x\ R\ y \vdash u\ R\ v}\ Crl$$

$$\frac{\Gamma, \Delta \vdash \Gamma'}{x : A, \Gamma, \Delta \vdash \Gamma'}\ WlL \qquad \frac{\Gamma, \Delta \vdash \Gamma'}{\Gamma, \Delta \vdash \Gamma', x : A}\ WlR$$

$$\frac{x : A, x : A, \Gamma, \Delta \vdash \Gamma'}{x : A, \Gamma, \Delta \vdash \Gamma'}\ ClL \qquad \frac{\Gamma, \Delta \vdash \Gamma', x : A, x : A}{\Gamma, \Delta \vdash \Gamma', x : A}\ ClR$$

$$\frac{\Gamma, \Delta \vdash \Gamma', x : A \quad x : B, \Gamma, \Delta \vdash \Gamma'}{x : A \to B, \Gamma, \Delta \vdash \Gamma'} \to L \qquad \frac{x : A, \Gamma, \Delta \vdash \Gamma', x : B}{\Gamma, \Delta \vdash \Gamma', x : A \to B} \to R$$

$$\frac{\Delta \vdash x\ R\ y \quad y : A, \Gamma, \Delta \vdash \Gamma'}{x : \Box A, \Gamma, \Delta \vdash \Gamma'}\ \Box L \qquad \frac{\Gamma, \Delta, x\ R\ y \vdash \Gamma', y : A}{\Gamma, \Delta \vdash \Gamma', x : \Box A}\ \Box R.$$

Note that other connectives and modal operators are defined in terms of $\bot$, $\to$ and $\Box$, e.g., $x : \Diamond A$ is $x : (\Box(A \to \bot)) \to \bot$, and that we do not need right structural rules for relational formulas, since the sequents of the form $\Delta \vdash x\ R\ y$ are single-conclusioned. Cut-free labelled sequent systems for other logics are uniformly obtained from K by modularly extending the relational theory with rules formalizing properties of R, e.g., reflexivity and transitivity:

$$\frac{}{\Delta \vdash x\ R\ x}\ \mathit{refl} \qquad \frac{\Delta \vdash x\ R\ y \quad \Delta \vdash y\ R\ z}{\Delta \vdash x\ R\ z}\ \mathit{trans}.$$

That is, T is K $\cup$ {*refl*}, K4 is K $\cup$ {*trans*}, and S4 is T $\cup$ {*trans*} or K4 $\cup$ {*refl*}.

To develop complexity results, we perform a fine-grained proof-theoretical analysis of the structural rules of these systems. The key problem to tackle is that although the subformula property (which follows from the absence of cut) bounds the number of different formulae that can appear in a proof, it does not bound the number of times a formula can appear in a sequent. We provide this latter bound by proving that we can bound, and in some cases eliminate altogether, applications of the contraction rules. This, combined with an analysis of the accessibility relation of the corresponding Kripke frames, yields decision procedures with bounded space requirements. In particular, we first prove that *CrL* and *ClR* can be eliminated in K, T, K4 and S4. Then we investigate the use of *ClL*, and show that we can always transform a proof of a sequent $S = \ \vdash x : A$ in K, T, K4 and S4 so that only contractions of labelled formulas of the form $y : \Box B$ are needed. (In fact, we give an even more precise syntactic characterization of the subformula B in $y : \Box B$.) By a further analysis, we finally show that, given a provable sequent $S = \ \vdash x : A$ with size $n = |S|$, we have:

- *ClL* can be eliminated in K;
- *ClL* can be bounded in T, K4 and S4: in each branch of a proof of S, we need at most $O(n)$ contractions of labelled formulas of the form $y : \Box B$;

These bounds on contractions allow us to bound the depth of proofs and the size of sequents arising in them. By storing proofs efficiently (rather than the entire proof, we store only a sequent and a stack that maintains enough information to reconstruct branching points), we obtain:

THEOREM 1. *The modal logic* K *is decidable in* $O(n \log n)$ *space*; T, K4 *and* S4 *are decidable in* $O(n^2 \log n)$ *space.*

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[5] L. VIGANÒ, ***A framework for non-classical logics***, ***Ph.D. thesis***, Universität des Saarlandes, Saarbrücken, Germany, 1997.

► S. BELLANTONI, *Towards a new "feasible" arithmetic*.

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A ramified arithmetic $\mathcal{I}_2^1$ is defined whose provable functions are exactly the polynomial-time computable functions. Remarkably, $\mathcal{I}_2^1$ is defined without any reference to polynomial bounds, and using only a small set of initial functions. The system also admits induction over formulas containing arbitrarily many alternations of unbounded quantifiers—these quantifiers range over "tier 0" values, being binary strings not accompanied by any comprehension of the strings.

The characterization is obtained by extending the Curry-Howard isomorphism to include modal operations, and using a translation from $\mathcal{I}_2^1$ to the functional class recently defined by Bellantoni, Niggl, and Schwichtenberg. Modal operations are used to define the ramification levels.

A deficiency of the current system is that it is both modal and intuitionistic.

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► PATRICK BELLOT, *Imperative program synthesis using a logic of actions*.

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Program synthesis using logic usually produces functional programs, that is λ-expressions which do only computations on data represented by *abstract data types*. This is due to the purely mathematical nature of these systems. But *computer programming* does not deal with mathematical objects but with real data implementation, real objects such as disk files and pre-existing programs. Programming is not only a matter of sentencial logic but also of *resources* management and of *actions*. Our logic, the Ω logic, claims to take into account these two notions.

Formulae-as-actions. The formulae can be *action formulae* such as "Put block A on block B", *state formulae* such as "Block A is on block B", *mathematical formulae* such as "2 is even" or *term formulae* stating that the term is computable. A state formula A is seen as the action "to do things such that A holds". Mathematical formulae are modalized state formulae as in Linear Logic.

Connectives. We have the following connectives. Some are inspired from Linear Logic. Meanings are given in term of actions and are adapted to states using the view of state formulae given above.

- (*Parallelism*) $A \otimes B$, actions: to do A and B in parallel, states: both A and B are true and available;
- (*Deterministic choice*) $A \,\&\, B$, actions: to do A or to do B, one can choose, states: both A and B are true but one can use only one of them;
- (*Non-deterministic choice*) $A \oplus B$, actions: to do A or to do B but one does not know which one (outer non-determinism), states: one of A and B is true but one does not know which one;
- (*Dependent sequentiality*) $A > B$, actions: to do B by beginning to do A, states: B is true but one must first prove A in order to prove B;
- (*Independent sequentiality*) $A \gg B$, actions: to do A and then to do B, states: A and B are true but A is proved before B;
- (*Logical implication*) $A \Rightarrow B$, actions: if you know how to do A, you can do B, states: B can be logically deduced from A;
- (*Causality*) $A \rightarrow B$, action: doing A causes doing B, states: nothing;
- (*Universality*) $\forall x.A$, action: to do A for any value of x, states: universal quantification;
- (*Existence*) $\exists x.A$, actions: to do A for one value of x, states: existential quantification;
- (*Many*) $!A$, actions: to be able to do A as many times as we want (determinism), states: A is indefinitely reusable and discardable;
- (*Some*) $?A$, actions: to do A a certain number of times one does not know (non-determinism), states: it is possible that A is true forbidding to prove the intuitionistic negation of A.

There are a few other connectives in order to deal with repetition of actions.

Realizations. The technique of realizability allows to associate objects to theorems. These objects are built with the proof. In our case, the realizations are objects in the sense of object oriented programming. Their method *prove*() does the theorem considered as an action formula: if R realizes A, then $R.$*prove*() does A.

Axiomatisation. Our logic is a sequent calculus. The axiomatisation is made difficult because of the need of linearity. Moreover, realizations are imperative programs which cannot be combined in any way. In the sequent, the $\otimes$ connective intuitively links formulae on both sides of the entailment sign $\vdash$. Therefore, our structural rules are unusual:

$$\Gamma \vdash \Gamma \quad [id] \qquad \frac{\Gamma \vdash \Delta \quad \Delta \vdash \Theta}{\Gamma \vdash \Theta} \quad [tr] \qquad \frac{\Gamma \vdash \Delta}{\Gamma, \Theta \vdash \Gamma, \Theta} \quad [mn].$$

These rules make the proofs much more natural than classical sequent calculus rules by allowing a linear structure of proofs. They also allow a more human-like automatic research of proofs.

Applications. The Ω logic has been applied to find programs which move blocks in the well-known *World of Bocks* problems. It has also been applied to the *Candy Machine*, a trivial case of input/output automatum problems where the notion of action is essential. It has to be applied to functional computations using the term formulae.

Content of the talk. If accepted, the talk will be about the application of the Ω logic to the

World of Blocks, how it works, how it is used to synthesize non-trivial problems.

► ANATOLY P. BELTIUKOV, *Alternating time complexity bounds for protothetics*.
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Sets of protothetics formulas (i.e., propositional formulas with quantifiers on propositional variables) are considered. The sets under consideration are composed of formulas with bounded densities of quantifiers. Closed upper and lower time complexity bounds for resolving sets of protothetics formulas with bounded numbers of quantifiers are given for alternating Turing machines (ATM) [3] and stack register machines (SRM) [1]. Machines of both these classes are considered to have random access to input tape using special address tape or register.

THEOREM. *Let f be a nondecreasing function, $f(n) \leq n$. Let f be honest with respect to alternating time, i.e.,*

$$\mathrm{Time}(p(n)) \leq f(n)$$

for some Turing machine p that computes f with binary output. Let $\mathrm{Prot}(f)$ *be the set of all true protothetics formulas with number of quantifiers $f(n)$ where n is the length of the formula. Then*

$$\mathrm{Prot}(f) \in \mathrm{ATime}(f(n) + \log^2 n),$$

$$\mathrm{Prot}(f) \notin \mathrm{ATime}(f(n/\log^2 n)/\log f(n))$$

where $\mathrm{ATime}(g(n))$ *is the complexity class for* ATM *with time complexity bound $g(n)$, n is the length of input.*

Using methods of [2] we can obtain

$$\mathrm{ATime}(f) = \mathrm{SRMTime}(\exp(f(n)))$$

where $\mathrm{SRMTime}(g(n))$ is the complexity class for SRM with time complexity bound $g(n)$, n is the length of input. Therefore

$$\mathrm{Prot}(f) \in \mathrm{SRMTime}(\exp(f(n))n^{\log n}),$$

$$\mathrm{Prot}(f) \notin \mathrm{SRMTime}\big(\exp(f(n/\log^2 n)/\log f(n))\big).$$

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► MARCO BENINI, *Strong constructivity of second-order intuitionistic arithmetic*.
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The goal of this talk is to show that second-order intuitionistic arithmetic is strongly constructive. Although this results is by no means surprising, the proving technique we use is new, and far more general than the result we get.

We say that a logical theory Γ is naively constructive when:

- if $\Gamma \vdash A \vee B$ then $\Gamma \vdash A$ or $\Gamma \vdash B$ where $A \vee B$ is a closed formula (disjunction property).
- if $\Gamma \vdash \exists x.A(x)$ then there is a closed term t such that $\Gamma \vdash A(t)$, where $\exists x.A(x)$ is a closed formula (explicit definability property).

A theory is said to be strongly constructive [2, 3] if it is naively constructive and any proof of $A \vee B$ ($\exists x.A(x)$, respectively) contains enough information to build up a proof of A or a proof of B (or a proof of $A(t)$, for a suitable closed term t, respectively).

We will use the so-called Collection Method [5, 4, 1] to show that second-order intuitionistic arithmetic is strongly constructive.

The Collection Method is a proof theoretical instrument especially built to characterize the notion of strong constructivity. It has been used to give a computational meaning to constructive proofs, and to prove that many logics are constructive. Since this instrument is relatively new and it succeeds to prove constructivity for a logic (theory) even when semantical methods are not applicable (lacking a semantical characterization of models, for example), or when the logical system is not cut-free, we think it is worthwhile showing an application even if the result is well known.

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► NATASHA BOZOVIC, *Recursive (un)recognizability of properties of finitely presented groups and computational algebra*.
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We discuss here algorithmic recognizability of algebraic properties of finitely presentable groups. For each of the following properties of finitely presentable groups, we have proved that there is no algorithm that can decide for every finite presentation R whether the corresponding group $G(R)$ has this property or not: being decomposable into a free (or direct) product of groups having any Markov property; being a union of properties such that for at least one of these properties none of the groups enjoying it has k ($k > 1$) as the maximum number of nontrivial factors in a free decomposition of the groups; having a "context-free word problem"; being a poly-P property, where P is a strong hereditary property of finitely presentable groups, etc.

These results enable one to identify a number of new properties of universal finitely presentable groups (groups that contain, as a subgroup, an isomorphic copy of every finitely presentable group). For example, if N is a finitely generated normal subgroup of a finitely presentable group U, such that U/N has any strong hereditary property, then N is universal if and only if U is universal. This implies that for a finitely presentable universal group U, the following subgroups of U are also universal: every subgroup of finite index; the commutant K; every finitely generated subgroup containing K; every normal finitely generated subgroup N such that quotient U/N is solvable, nilpotent, torsion-free, one-relator, or has a solvable word problem, etc.

Starting from the few known examples of universal groups, it is now possible to construct presentations of many other finitely presentable universal groups, using methods and algorithms developed in computational algebra.

Finally, we will discuss the connections between purely algebraic notions such as group properties on one side, and decidability (recursiveness) on the other.

► MARK BROWN AND VALENTIN GORANKO, *An extended branching-time Ockhamist temporal logic*.

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For the branching-time temporal logic based on an Ockhamist semantics (where the truth of a formula is evaluated relative to possible future branches, or equivalently, at pairs ⟨moment, history through that moment⟩, and the temporal operators are relativized to the actual history of the evaluation) we explore a temporal language extended with two additional syntactic tools. For reference to the set of all possible futures at a moment of time we use syntactically designated "restricted variables" called fan-names. For reference to all possible futures alternative to the actual one we use a modification of a difference modality, localized to the set of all possible futures at the actual moment of time.

We argue that the extended language we introduce provides a natural, while very expressive, framework for formalizing and analyzing Ockhamist branching-time logics.

We construct an axiomatic system for that extended branching-time logic and prove its soundness and completeness with respect to bundle tree semantics. (A bundle in a tree is a set of all branches that belong to a collection of histories covering the whole tree.) Combining use of the fan-names and the difference operator in our system renders the language expressive enough to provide a relatively simple and perspicuous axiomatic system, and to enable an elegant development of the model theory of the logic we study, necessary for the proof of the completeness theorem. A technical advantage of our approach is that the canonical model construction is considerably simplified and much closer in style to the traditional one in modal logic (appropriately modified to deal with "Gabbay-style" rules), and eventually leads to a direct construction of a canonical tree-like frame, explicitly defining the set its moments, with the branches then defined in a standard manner.

Finally, we show that our axiomatic system can be easily extended with a variety of important additional operators, such as Since and Until, a global difference operator, operators for undivided and divided histories, reference pointers over paths, etc.

► I. C. BURGER AND J. HEIDEMA, *Data-modulated Boolean algebras and consequence relations*.

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Given any Boolean algebra $\mathcal{B} = (B, \leq, \wedge, \vee, \neg)$ and any (fixed) element $t \in B$, we define the function $m_t\colon B \to B,\ x \mapsto (x \leftrightarrow t)$, where

$$(x \leftrightarrow t) := (x \wedge t) \vee (\neg x \wedge \neg t).$$

Then m_t is a bijection; m_t is its own inverse: $[(x \leftrightarrow t) \leftrightarrow t] = x$; and $m_t(\neg x) = \neg m_t(x)$.

We now employ m_t to induce a new Boolean structure on the set B, i.e., to construct the Boolean algebra $\mathcal{B}_t = (B, \sqsubseteq_t, \sqcap, \sqcup, \neg)$ where

$$x \sqsubseteq_t y :\Longleftrightarrow m_t(x) \leq m_t(y);$$
$$x \sqcap y := m_t(m_t(x) \wedge m_t(y));$$
$$x \sqcup y := m_t(m_t(x) \vee m_t(y)).$$

(The complement operation of $\mathcal{B}_t$ is the same as that of $\mathcal{B}$, since $m_t(\neg m_t(x)) = m_t(m_t(\neg x)) = \neg x$.) Then m_t is an isomorphism: $m_t : \mathcal{B} \cong \mathcal{B}_t$.

What do the new meet and join operations, $\sqcap$ and $\sqcup$, look like when expressed in terms of the old operations? Well,

$$x \sqcap y = [x \wedge y] \vee [(x \vee y) \wedge \neg t] \qquad \text{and} \qquad x \sqcup y = [x \vee y] \wedge [(x \wedge y) \vee t].$$

If $\mathcal{B}$ has a bottom and a top, then $\mathcal{B}_t$ has $\neg t$ as bottom and t as top.

Consider now any propositional language and let B denote the set of logical equivalence classes of sentences of the language. Then $\mathcal{B} = (B, \models, \wedge, \vee, \neg)$ is the Tarski-Lindenbaum algebra which has the equivalence class $\bot$ of contradictions as bottom element and the equivalence class $\top$ of tautologies as top element. Suppose we have data (information) in the form of a sentence $t \in B$ which we want to employ in modulating $\mathcal{B}$ into a new Boolean algebra $\mathcal{B}_t$. In the latter, a sentence Y must lie above a sentence X if Y is closer to the data than X. This is achieved by the function m_t as described above. The resulting $\mathcal{B}_t$ (or $(B, \sqsubseteq_t)$) represents a data-dependent entailment relation—different from the usual t-expanded entailment relation:

$$X \models_t Y :\Longleftrightarrow X \wedge t \models Y.$$

The relation $\sqsubseteq_t$ is also a verisimilar relation when t represents the (complete knowledge of the) "truth" and the Tarski-Lindenbaum algebra is the special case $\mathcal{B}_\top$ in which "higher up" means closer to the "data" $\top$ (which contains no information). The data-modulated relation $\sqsubseteq_t$ is nonmonotonic in the following two ways:

(1) $X \sqsubseteq_t Y$ does not imply $Z \sqsubseteq_t Y$ when $Z \models X$, and
(2) $X \sqsubseteq_{t_1} Y$ does not imply $X \sqsubseteq_{t_2} Y$ when $t_2 \models t_1$.

In the relationship between the orders $\mathcal{B}$ and $\mathcal{B}_t$ the group structure of the Abelian group $(B, \leftrightarrow, \top)$ plays an interesting role.

► MARTIN BUNDER AND SACHIO HIROKAWA, *Classical formulas as types of $\lambda\nu$-calculus.*
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The formulas-as-types isomorphism tells us that every type of a closed λ-term can be interpreted as a theorem of intuitionistic implicational logic and every proof of such a theorem can be represented by a λ-term. Certain proof reductions correspond to reduction rules for the λ-calculus.

Extensions of the isomorphism to classical logic have been proposed by, e.g., Gabbay and de Quieroz, Griffin, Murthy, Parigot. We propose a $\lambda\nu$-calculus, which has a ν-introduction rule as follows:

$$\frac{\Gamma, x : a \to b \vdash M : a}{\Gamma \vdash \nu x.M : a}.$$

We consider the following three kinds of reduction rules, which follow naturally from the proof reductions.

(ν_g) $\quad C[\nu x.M] \to \nu x.C[M[x := \nu.xC[u]]],$

(ν_h) $\quad \nu x.C[xM] \to \nu x.M$ if any free variable in M is free in $C[XM]$,

(ν_n) $\quad \nu x.M \to M$ if x does not occur free in M.

The reduction rule (ν_g) represents the postponement of the ν-operator. If we postpone all ν-operators to the extreme left, we are in fact postponing all strictly classical inferences to be done last. Therefore, we can separate a proof figure into the intuitionistic part and the classical part.

However, the reduction rules (ν_g) and (ν_h) are too general to have the Church-Rosser Property. We analyze the cause of the failure of the Church-Rosser, and obtain a reduced $\lambda\nu$-calculus in which the context is restricted to the form $C[\,] = [\,]N$ for (ν_g) and $C[\,] = [\,]N_1 \dots N_n$ for (ν_h).

To prove the Church-Rosser Property for the reduced $\lambda\nu$-calculus, we introduce a substitution of the form $M[x^* := x(^*N)]$, where each subterm of the form xQ is replaced by $x(QN)$ inductively. It is a combination of (ν_g) reduction and a series of β-reductions. With this substitution, we formulate a Parallel Reduction and prove the Diamond Property for the Parallel Reduction. Then we prove the Church-Rosser Property for the reduced $\lambda\nu$-calculus.

► WALTER A. CARNIELLI, *Proof-theory for infinite-valued logics: A functional approach*.
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Finite-valued logics have been extensively treated based on tableau techniques (cf. [1] for a general treatment, and [3] for important subsequent developments). The case of infinite-valued logics is more complicated, and the question was postponsed in [1]. Meanwhile, connections between tableaux for infinite-valued logics and integer programming have been obtained, through the fundamental concept of signed formulas (see [5], [4] and the inside references).

In this paper we present a new general definition of *functional tableaux* (retaking some ideas of [2]) where formulas are signed with parameters in sets with some algebraic structure (as rings, fields, etc.). Tableau rules are thus described by means of functionals, and the existence of proofs is reduced to the existence of solutions in systems of equations.

For example, a complete set of tableau rules for all (finite and infinite) Łukasiewicz logics based on $\rightarrow$, $\neg$ is given by:

$$\frac{[a](X \rightarrow Y)}{[1-b]X, [a-b]Y} \quad \text{for } a < 1 \qquad\qquad \frac{[1](X \rightarrow Y)}{[b]X, [b+c]Y} \qquad\qquad \frac{[a](\neg X)}{[1-a]X}$$

where a, b, c are variables over $Q \cap [0,1]$ and thus, in this case, provability reduces to the decision problem of existence of rational solutions in bounded linear systems. Although this approach coincides in certain cases with results of [4] and latter refinements, the method is very general and can also be used to obtain simple decision procedures to certain infinite-valued logics, and to investigate the computational complexity of the corresponding satisfiability problems.

In particular, in cases of many-valued logics with p^n values, for p a prime number, it is possible to take profit of the combinatorial structure of finite fields, generalizing the method of "truth-values as signs" introduced by R. Hähnle (cf. [3]), with special interest for three-valued logics.

Tableaux of this sort are possible for a wide variety of finite and infinite-valued logics, provided the connectives are described by locally inversible functions. The simple and elegant formulation permits also to give sequent rules for infinite-valued logics, generalizing the methods in [2].

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► MARCELO DA S. CORRÊA AND EDWARD HERMANN HAEUSLER, *A lazy Lambek calculus*.

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The Lambek Syntactic Calculus (LSC), formerly Calculus of Syntactic Categories, has been designed to provide a mathematical characterisation for the sentence formation process of a language, by means of a type change system [3]. A Gentzen style presentation has been given for the Lambek Calculus, which lacks structural rules as exchange, weakening or contraction rules and, thus, has a noncommutative tensor product. Recently, it has been shown the relationship between LSC and a fragment of the noncommutative intuitionistic Linear propositional Logic.

On the other hand, several contributions and suggestions have been done in noncommutative linear logic to regain noncommutativity. Girard, for example, considers a (weaker) notion of commutativity given by a restricted form of exchange rule, called *cyclic exchange rule*, which allows circular permutations [2]. Yetter has improved this approach by introducing a new modality κ—*kappa* [4], which allows permutations between a modalized formula and the formulae placed at both sides of it. De Paiva has started with the Lambek Syntactic Calculus with additives and added only a modality like Yetter's κ [1].

In this work, we consider an extension of the (pure) Lambek Syntactic Calculus, called Lazy Lambek Calculus, obtained by introducing a modality $\circ$ similar to the Yetter's κ modality. Our aim is to capture the lazy evaluation process for an arbitrary λ-calculus. We also present a λ-calculus version for the new system.

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► PAOLA D'AQUINO, *Nonstandard finite fields in* $\mathrm{I}\Delta_0 + \Omega_1$.

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(Joint work with A. Macintyre.)

Let M be a model of $\mathrm{I}\Delta_0 + \Omega_1$ and K be the residue field of M for a nonstandard prime p in M.

If M is a model of PA then K is a pseudo-finite field (see [1]), i.e., it satisfies the following axioms of Ax

(1) there exists a unique extension of each degree n;
(2) every absolutely irreducible curve has a point in the field.

In the case of Open Induction, Macintyre and Marker proved in [2] that for any field L of characteristic 0 there is a model M of Open Induction and a prime $p \in M$ such that the residue field K is elementary equivalent to L. In particular, K can have infinitely many extensions of each degree.

Using results of bounded arithmetic and some Galois theory we can prove the following results.

THEOREM. *Let M be a model of $\mathrm{I}\Delta_0 + \Omega_1$, $p \in M$ a nonstandard prime and K the residue field. Suppose K contains the primitive n-roots of unity, for $n \in \mathbb{N}$. Then there exists a unique abelian extension of K of dimension n.*

COROLLARY. *For no prime $r \in \mathbb{N}$ and $k \in \mathbb{N}$, K has two distinct normal extensions of degree r^k.*

In the classical case every extension of a finite field is Galois and its Galois group is cyclic. In our setting we prove the following

THEOREM. *Let F be a finite normal extension of K with Galois group G. Then all Sylow subgroups of G are cyclic (i.e., G is a Z-group in the sense of Passman* [3]).

COROLLARY. *The Galois group of F over K is generated by two elements x, y such that $x^n = y^m = 1$, $x^{-1}yx = y^r$, $(r-1, m) = (n, m) = 1$ and $r^n \equiv 1 \pmod{m}$.*

In the classical case the Galois group of a finite extension is generated by a single element.

COROLLARY. *For each $n \in \mathbb{N}$ there is at most one normal extension of K of degree n.*

COROLLARY. *The norm map of any normal extension is surjective.*

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► JAIRO JOSE DA SILVA, *Husserl's two notions of completeness*.
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In 1901 Husserl addressed the Mathematical Society in Göttingen in order to present his views on two problems related to a question that had occupied him for at least ten years, i.e., since the publication of *Philosophie der Arithmetik* in 1891: how to justify the use of "imaginary" elements (like negative, rational, irrational and imaginary numbers) in mathematics? Husserl's choice of Hilbert's formalism to approach the problem of imaginary elements originated a series of subsidiary questions, such as: what sense does it make simply to add to a system of axioms a set of new axioms defining new "ideal" elements that are non-existing from the perspective of the old system? In order to prove facts about a certain domain of objects, are we justified in using elements that are provably non-existing from the perspective of the axiomatic system describing this domain? Under which conditions can we do this? Is consistency, besides being obviously a necessary, also a sufficient condition for the acceptance of imaginary elements in mathematics (as Hilbert claimed)? Can a statement that is consistent with, but unprovable in an axiomatic system be "true" in the "domain" of this system? These questions can be subsumed under two major problems, which according to my reading of the sketches prepared by Husserl for the Görttingen addresses (there were two of them), published in volume XII of *Husserliana*, are the following: to characterize what an "imaginary" object is from the perspective of a formal axiomatic system (the ontological problem), and to investigate the conditions for derivability to be an equivalent of truth in formal axiomatic systems (the epistemological problem). In order to deal with these questions Husserl introduced two related notions, which he called relative and absolute definiteness, that offered him the keys to the solution of both the ontological and the epistemological problems. The second notion, that of absolute definiteness, was identical with Hilbert's notion of deductive or syntactic completeness, whereas the notion of relative definiteness was a particular case of it. In this paper I intend to show how, and why, Husserl arrived at these notions and their relevance, according to Husserl, to the problem of imaginary elements in mathematics. I also argue, contrary to most commentators, that it was the first, not the second, notion that Husserl considered germane to the notion of completeness involved in Hilbert's axiom of completeness which, as we will show, shares with the property of relative definiteness the role of imposing a certain closure on the "domain" of a formal axiomatic system. This seemed to be enough reason to raise Husserl's suspicion, aired in a footnote to paragraph 72 of *Ideen I*, that Hilbert had actually found inspiration in his ideas for the axiom of completeness proposed in *Über den Zahlbegriff*. Most commentators find this suspicion based on a confusion which, according to them, Husserl makes between syntactic and semantic completeness. One of the conclusions of this paper is that such confusion cannot be so readily attributed to Husserl. This paper intends also, though only in passing, to throw some light on the question of the influence of Frege's criticism of *Philosophie der Arithmetik* on Husserl philosophical development, since I believe that the problem of imaginary elements in mathematics is more relevant to understand Husserl turning his back on "psychologism" than Frege's criticism of Husserl's earlier work.

► J. M. DAVOREN, *On continuous dynamics and modal logics*.
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An abstract transition system is a structure $\mathfrak{T} = (X, \{\xrightarrow{a}\}_{a \in \mathrm{Act}})$, where $X \neq \emptyset$ is the state space and for each $a \in$ Act, $\xrightarrow{a}$ is a binary relation on X. Such structures are used in many areas of computer science to model dynamic changes of state. They provide a Kripke semantics for propositional modal languages with box modalities $[a]$ for $a \in$ Act. Given an assignment ξ of a subset of X to each atomic proposition, extend the valuation to all formulas by interpreting the propositional connectives by their set-theoretic counterparts, and each modality $[a]$ by the universal pre-image operator

$$\tau_a(A) \stackrel{\circ}{=} \{\, x \in X \mid (\forall y \in X)\; x \xrightarrow{a} y \Longrightarrow y \in A \,\},$$

for $A \in \mathcal{P}(X)$. A formula φ is true in a model $(\mathfrak{T}, \xi)$ if and only if $\|\varphi\|_\xi = X$, and valid in a structure $\mathfrak{T}$ if and only if it is true for all valuations ξ for $\mathfrak{T}$. The Hilbert-style proof system for such a minimal modal action logic **AL** has a normality axiom scheme $K[a]$ and a rule of $[a]$-necessitation for each $a \in$ Act; **AL** is just the base of the modal μ-calculus $\mathbf{L}_\mu$ (see [1]).

My interest is in the topological notions of *continuity* for transition relations $\xrightarrow{a}$, and their expression in a modal logic. From Kuratowski's ***Topology***, §18, given a topology $\mathcal{T}$ on X, a relation $\xrightarrow{a}$ is said to be *upper-* (respectively, *lower-*) *semi-continuous* if and only if $\tau_a(Y)$ $(\sigma_a(U))$ is open whenver U is open in $\mathcal{T}$, where $\sigma_a(A) \stackrel{\circ}{=} (-\tau_a-)(A)$.

Let $\mathbf{TAL}_0$, (basic) Topological Action Logic, by the modal logic obtained from **AL** by extending the language with a new $\Box$ modality and adding the axioms and necessitation rule for the box modality of **S4**. From McKinsey and Tarski's work in the 40s, **S4** admits a topological semantics with $\Box$ interpreted by the topological interior operator. We take a *topological structure* to be a transition system $\mathfrak{S} = (X, \mathcal{T}, \{\xrightarrow{a}\}_{a \in \mathrm{Act}}, \mathcal{A})$, additionally equipped with a topology $\mathcal{T}$ on X and a *valuation algebra* $\mathcal{A}$: a Boolean algebra of subsets of X containing $\mathcal{T}$ and closed under each τ_a. The range of atomic assignments ξ for $\mathfrak{S}$ is restricted to $\mathcal{A}$. Then $\xrightarrow{a}$ is upper- (lower-) semi-continuous with respect to $\mathcal{T}$ if and only if the schema $[a]\Box\varphi \to \Box[a]\varphi$ ($\langle a\rangle\Box\varphi \to \Box\langle a\rangle\varphi$) is valid in $\mathfrak{S}$. Let **TAL** be the logic obtained from $\mathbf{TAL}_0$ by adding the two continuity axiom schemes for each $a \in$ Act. In proving completeness of **TAL**, one can proceed with the usual canonical model construction: the state space X is the set of all ultrafilters of the Lindenbaum algebra of **TAL**, and is equipped with a topology $\mathcal{T}_R$ defined by a *pre-order* R on X.

Pre-orders on a set X are in one-to-one correspondence with the class of *Alexandroff* topologies on X. Continuity with respect to an Alexandroff topology is intimately connected with the notion of a *bisimulation equivalence* on a transition system, and reveals a richer class of "nice" quotient transition systems. Applications to the theory of hybrid control systems (see [3]) are discussed.

Modal logics of continuous functions are investigated in [2], and independently considered in [4]. A paper will be available at `ftp://cam.cornell.edu/pub/davoren/davoren.html`.

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► WOLFGANG DEGEN AND JAN JOHANNSEN, *Cumulative higher-order logic as a foundation for set theory*.
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A cumulative hierarchy is of the form $H = \bigcup_{\alpha\in\Delta} H_\alpha$ where Δ is an initial segment of ON and $H_\alpha \subseteq H_\beta$ for all $\alpha, \beta \in \Delta$ with $\alpha < \beta$. Such cumulative hierarchies $\langle H, \in\rangle$ are usually used to model *first-order* set-theories with $\in$ for the membership relation as sole non-logical symbol. However, if one considers a cumulative hierarchy H as given primarily by its ranks H_α, a *typed* language L_Δ recommends itself as the most direct means to describe H. The language L_Δ has variables $a^\alpha, x^\alpha, \ldots$ of type α for $\alpha \in \Delta$ and predication relations $b^\beta\ (a^\alpha)$ between elements from H_β and H_α for $\beta > \alpha$. The relation $b^\beta\ (a^\alpha)$ is (*properly*) *cumulative* if $\beta > \alpha + 1$.

PROBLEM. Given a class $\mathcal{H}$ of cumulative structures of length Δ, find a logical system $\Sigma_{\mathcal{H},\Delta}$ in the language L_Δ which is complete with respect to $\mathcal{H}$.

In our paper, we give solutions to this problem for specific classes $\mathcal{H}$: We define the class of *cumulative Henkin structures* (CHS) by only stipulating that they satisfy certain impredicative comprehension principles. Our weakest logical systems K_Δ, which are generalizations of simple type theory to cumulative types up to Δ, are complete with respect to CHSs. The strongest notion we arrive at is the notion of an *extensional, normal and null-founded cumulative Henkin structure* (ECHS). The normality of an ECHS means, roughly, that every rank H_λ (λ a limit ordinal) contains exactly the entities from the smaller ranks. To capture normality syntactically, we assume an *infinitary* inference rule, called the *limit rule*, which allows to infer the sequent $\mathcal{S}[a^\lambda]$ from the premises $\mathcal{S}[a^\xi]$, $\xi < \lambda$. The arising infinitary system K_Δ^∞ are complete with respect to the class of ECHSs. Furthermore, every ECHS can be collapsed to a cumulative hierarchy *of sets*, i.e., each rank H_α consists of sets, $H_{\alpha+1}$ is a subset of the powerset of H_α, and predication is ordinary membership.

In order to embed set theory we define for each type α a type-homogeneous membership relation by

$$a^\alpha \in b^\alpha :\Longleftrightarrow \exists x^{\alpha+1}\ (x^{\alpha+1}(a^\alpha) \wedge \forall y^{\alpha+2}\ (y^{\alpha+2}(x^{\alpha+1}) \rightarrow y^{\alpha+2}(b^\alpha))).$$

Note that only in the last place we have made use of the properly cumulative predication relation $y^{\alpha+2}(b^{\alpha})$. Given now a *first-order* formula φ of set theory, we translate it into the L_{Δ}-formula $\varphi^{(a)}$ using the just defined $a^{\alpha} \in b^{\alpha}$ instead of $a \in b$ throughout. Then we show (among similar results): if φ is theorem of Zermelo's set theory Z, i.e., ZF without replacement, then $\varphi^{(\lambda)}$ is a theorem of K_{Δ}^{∞} for every limit ordinal $\lambda \geq \omega + \omega$ and $\Delta \geq \lambda + 3$. In this way, cumulative higher-order logic is a foundation of the set theory Z via the mapping $\varphi \mapsto \varphi^{(\lambda)}$. On the other hand, there are also first-order sentences φ inconsistent with ZF such that the translation $\varphi^{(\lambda)}$ is provable in K_{Δ}^{∞} for some $\lambda < \Delta$. We investigate conditions sufficient for the avoidance of such inconsistencies.

► AKIM DEMAILLE, *Commutativity of the exponentials in mixed linear logics*.
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Introduction. Mixed Linear Logics (MLL for short), are conservative extensions of both Linear Logic and Cyclic Non Commutative Linear Logic. As in Linear Logic, MLL have freedom in the management of the exponentials. Here we expose a system as strict with exponentials as it is for regular formulas.

Mixed linear logics. MLL are the meeting point between the works of J.-Y. Girard, and those of M. Abrusci. They enclose both commutative and non commutative multiplicative connectives: $\odot$ (*Then*) is the non commutative tensor, $\bigtriangledown$ (*Sequential*) is the non commutative counterpart of *Par*, and ';' is the non commutative ','. Such systems have been introduced by Philippe de Groote [2] (limited to the multiplicative intuitionistic fragment) and Paul Ruet [3] (who exhaustively studies the full classical system, and its intuitionistic pendant).

Yet the latter system has properties which we believe are nondesirable: commutativity between serialized formulas is enabled by the exponentials $!A \odot !B \vdash !B \odot !A$. Our point of view is that commutativity should simply never be regained in a non commutative context. A simple minded example will fairly describe our motivation. (Note that the fact that a sequent seems "stupid" with respect to a subjective reading means nothing but the fact that this particular reading is not tuned for the system.)

Exponentials and commutativity. Consider the provable sequent $!A \otimes !B \vdash !(A \,\&\, B)$ in Linear Logic. It says "*the result of taking a certain number of balls of type A or of type B can be reached by taking a certain number of balls A, and, concurrently, of balls B*". Common sense agrees.

$$\frac{\vdash \Gamma[?A, ?B]}{\vdash \Gamma[?A; ?B]}\,[? - \text{','}] \qquad\qquad \frac{\vdash ?\Gamma[A]}{\vdash ?\Gamma[!A]}\,[\textit{Promotion}].$$

Consider the equivalent sequent involving *Then*: $!A \odot !B \vdash !(A \,\&\, B)$. It is provable in Paul Ruet's system. It could be read as "*the result of writing a certain number of times As or Bs can be reached by writing a certain number of As, and* then *of Bs*". Common sense disagrees. Its origin lies in the rule $[? - \text{','}]$ which enforces commutativity between serialized modalized formulas.

In order to get rid of this rule while keeping the basic properties (such as cut elimination) [*Promotion*] must be controlled. Anyhow an unlimited [*Promotion*] is actually too strong for our purpose: it allows to prove $!A; !B \vdash !(A \odot B)$ from $!A; !B \vdash A \odot B$ which goes against our aims.

A mixed classical linear logic. In order to have a viable system, we only have to limit *Promotion* to the cases where the context Γ is *free*, i.e., has no ';'. It turns out that this system in which modalized formulas have no more properties than "plain" formulas is a natural object, as proved by the straightforward phase semantics design (see the full extent of this paper [1]). It is of course sound and complete with respect to phase spaces, and enjoys cut elimination.

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► DALIA DRAI, *Concepts of validity*.

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In most logic textbooks, the validity of an argument in natural language is defined as follows: an argument is *valid* if and only if there is no possible state of affairs in which the premises are true and the conclusion is false. This notion of validity is generally considered too broad for logical purposes and the following limited version is often suggested: an argument is *formally valid* if and only if it is valid in virtue of its logical form, i.e., every argument that has the same logical form is also valid. Of course, if an argument is formally valid, it is also valid. (I do not deal here with the reciprocal, i.e., whether an argument that is valid is also formally valid, see Sainsbury 1991.)

The first definition given above uses modal concepts such as possible states of affairs. The second definition uses, in addition, the concept of logical form and with it the distinction between logical and non-logical words. Both sets of concepts, the modal and the formal, raise difficult philosophical problems, but we still have to use at least one of them in order to define validity. I suggest that we should define validity with the aid of the formal concepts only without using the modal concepts, as follows: an argument is *substitutionally valid* if and only if there does not exist any argument with the same logical form with true premises and a false conclusion.

The body of the paper deals with the relationship between substitutional validity and formal validity. I raise two questions.

(1) If an argument is formally valid, is it also substitutionally valid?

To this question the answer is yes, and the reasoning is quite straightforward. The hard question is the second:

(2) If an argument is substitutionally valid, is it also formally valid?

I expose the assumptions about the richness of our language and the complexity of the world that are needed to support an affirmative answer to this question.

The conclusion I draw in this paper is that given these assumptions, we do not need the problematic modal concepts in order to understand the notion of formal validity in natural language.

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► ROY DYCKHOFF, *A deterministic terminating sequent calculus for propositional Dummett logic*.

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Sonobe [6] and Corsi [2] have given sequent calculi and tableau calculi for Dummett logic [3], i.e., intuitionistic logic with an axiom schema such as $(A \to B) \vee (B \to A)$. Recent work [1] by Avellone et al. has shown how, in the propositional case, such calculi can be modified so that root-first proof search terminates without use of a loop-checker, thus extending our own work [4] (cf. also [5, 7]) on the system G4ip (formerly LJT) for intuitionistic propositional logic.

The calculus of [1] has the disadvantage that some of the inference rules are non-invertible, thus forcing backtracking during proof search. In this paper we will make a further modification to this calculus, bringing it back closer to G4ip and making all the inference rules invertible. One reason for interest in such a calculus is that the invertibility of all rules is tied not to the classical nature of the logic but (we argue) to the linearity of the Kripke frames underlying the semantics.

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► MIRNA DŽAMONJA, *Some results related to universal models and Eberlein compacta*.
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With Saharon Shelah and others, I have been involved in a rather general project concerning the existence of universal models for a theory versus the classification of the theory in a certain classification scheme. The scheme is to fit in with the classification theory of Saharon Shelah. Although the problem comes from model theory, the project is essentially a set-theoretic one. The reason is that under *GCH* all first order countable theories have universal models in all uncountable ordinals, so one has to consider the situation under the failure of *GCH*. So we are drawn to the realm of consistency results and large cardinals on one side, and combinatorial principles of set theory, on the other.

This project branches off into various other subjects, for example the existence of universal models in theories which are not necessarily first order, like various objects from functional analysis, topology and boolean algebras, questions on which have been independently asked by mathematicians working in these areas.

I'll present a result due to Džamonja, which is an application of the general methods of the universality project, to a question about uniform Eberlein compacta.

► PHILIP EHRLICH, *Ordered fields with simplicity hierarchies: Generalization of Conway's ordered field No*.
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In his monograph ***On numbers and games*** [1], J. H. Conway introduced an ordered field *No* which contains (in a suitable sense that can be made precise) "all numbers great and small." However, in addition to its distinguished structure as an ordered field *No* has a rich algebraico-tree-theoretic structure, or *simplicity hierarchy*, that emerges from the recursive clauses in terms of which it is defined. In the present paper, we investigate a novel class of ordered fields with simplicity hierarchies whose properties generalize those of *No* and draw attention to some of the important relations that exist between *No* and this more general class of *s-hierarchical ordered fields* as we call them. In particular, we show that an ordered field together with a tree structure is an s-hierarchical ordered field if and only if it is isomorphic to an ordered field that is an initial subtree of *No*, and that every real-closed ordered field is a reduct of some s-hierarchical ordered field. We also generalize Conway's theories of *ordinals* and *omnific integers* by showing that every s-hierarchical ordered field A contains a cofinal, canonical subsemiring $\mathrm{On}(A)$—the *ordinal part* of A—which in turn is contained in a discrete, canonical subring $\mathrm{Oz}(A)$ of A—the *omnific integer part* of A—in which for each $x \in A - \mathrm{Oz}(A)$ there is a $z \in \mathrm{Oz}(A)$ such that $z < x < z + 1$ where 1—the multiplicative identity of A—is the least positive element of $\mathrm{Oz}(A)$. When A is a substructure of *No*, $\mathrm{Oz}(A)$ is a subring of *No*'s *omnific integers* and $\mathrm{On}(A)$ is a subsemiring of *No*'s subsemiring of *ordinals* (with sums and products defined naturally).

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► FRANCESC ESTEVA, LLUÍS GODO, PETR HÁJEK, AND MIRKO NAVARA, *Fuzzy logics with residuated implication and involutive negation*.

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Residuated fuzzy (many-valued) logic calculi are related to continuous t-norms, which are used as truth functions for the conjunction connective, and their residua as truth functions for the implication. Deep theoretical results, including completeness theorems, were proved for these logics in [2, 3]. In all residuated fuzzy logics, a negation is also definable from the implication and the truth constant 0. However, this negation behaves quite differently depending on the t-norm, in particular it is not involutive for t-norms different (up to isomorphisms) from the Łukasiewicz t-norm. In that case the resulting calculus is weak in some sense, for instance it lacks a strong disjunction.

From the basic logic defined by Hájek [2], we define the strict basic logic (SBL) which is a residuated logic whose definable negation is Gödel-like. Completeness results for the logic $\mathrm{SBL}_\sim$ defined by adding an involutive negation ($\sim$) as a new connective and for the corresponding predicate calculus are also proved. Standard and Pavelka-style (partial truth) completeness for product and Gödel logics with involutive negation are also studied [1].

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► PETER A. FEJER, *Every incomplete computably enumerable truth-table degree is branching*.
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If r is one of the reducibilities between sets of natural numbers studied in computability theory, one can form the structures consisting of all the r-degrees, the r-degrees of sets r-reducible to the 1-complete computably enumerable set K and the r-degrees containing computably enumerable sets. These structures are always partially ordered sets and are usually upper semi-lattices. Among the many algebraic questions one might ask about these structures, a basic one is whether every element not equal to the greatest element is branching, where we call a degree *branching* if it is meet-reducible, i.e., it is the meet of two degrees strictly above it. In the case of the computably enumerable r-degrees and the most commonly studied reducibilities r (namely, many-to-one (m), truth-table (tt), weak truth-table (wtt) and Turing (T)), the answer to this question is known for all reducibilities except truth-table. (In fact, every incomplete c.e. m- and wtt-degree is branching, while both the branching and nonbranching c.e. T-degrees are dense among all the c.e. T-degrees.) We give the solution to the last of the problems just mentioned by showing that every incomplete computably enumerable truth-table degree is branching.

The fact that every Turing-incomplete truth-table degree is branching can be shown using a technique due to Ambos-Spies. (We call a c.e. truth-table degree *Turing complete* if its members belong to the Turing degree of K.) Although the result has not been published, it follows from a slight strengthening of a result of Nies and Shore that has appeared in the literature. The argument is an infinite injury one using a tree of strategies.

Our contribution is the argument that every Turing-complete, tt-incomplete c.e. truth-table degree is branching. Although one might suspect that the Turing-complete case would be more complicated than the Turing-incomplete one, we are able to exploit the Turing-completeness of the given degree in a novel way and our construction is in fact a finite-injury one.

(This is joint work with Richard Shore.)

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The idea to generalize possible world semantics to a many-valued context is not new. Thomason, Morgan, Ostermann, Morikawa and others have defined many-valued modal logics. The most advanced treatment of the topic consists in a series of papers by M. C. Fitting. All authors consider the generalization of classical (i.e., two-valued) evaluation of formulas in possible worlds to many-valued evaluations. However, they only consider rather straightforward counterparts of the classic modal operator $\Box$ ("necessarily") and its dual $\Diamond$ ("possibly"). In contrast, we want to emphasize that modal operators that do *not* correspond to such modalities arise naturally in different many-valued contexts. In consequence we introduce the concept of "distribution modalities" in analogy to distribution quantifiers.

Moreover, we aim at a very general, uniform and modular representation. Prefixed signed tableaux, as presented by Fitting, turn out to be an almost perfect tool for this purpose. We provide corresponding soundness and completeness theorems.

This work can also be considered another exercise in the very topical subject of "combining logics", most inspiringly propagated, e.g., by D. Gabbay. Indeed we like to view the introduced class of logics as the space of all possible combinations of the following three building blocks:

- an arbitrary finite-valued "base logic",

- any possible worlds semantics with standard accessibility relation, and
- (most importantly:) an arbitrary collection of distribution modalities.

Once the particular choice for these three parameters is made, a sound, complete and even optimized tableau based calculus for the corresponding logic can (in principle) be generated automatically using procedures like those implemented in the system MULTLOG. The many-valued modal logics described by Thomason, Morgan, Ostermann, and others appear as simple instances of our general frame.

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In this paper we reconstruct various versions of the classical *ontological argument* within a general axiomatic framework, based on the primitive concepts of *quality* and *relation*. This framework has been recently introduced by E. De Giorgi, in order to accomodate the basic principles of any "sufficiently clear" theory (see [3]). Four principles inspire and inform this foundational programme:

- *non-reductionism*: there are objects of many qualitatively different kinds;
- *self-description*: the basic properties, relations, operations considered by the framework theory are themselves objects of the theory;
- *open-endedness*: any sufficiently clear concept should be "engraftable" in a natural way in the framework;
- *semi-formal axiomatic presentation*: scientific theories and the framework itself should be presented clearly and rigorously in terms of elementary primitive concepts, albeit not in a given formal system.

The present paper introduces, in a fragment of the framework theory of [4], the concepts of *contingent* and of *totally positive* (perfective) quality, and the *modal* concept of *necessary existence*, thus allowing us to present various forms of St. Anselm's ontological argument. The goal of this argument is to establish the *necessary* existence of an *ens perfectissimum*. This can be achieved by introducing the notion of *totally positive* quality (*perfectio*) and isolating suitable principles. Our presentation is inspired by recent work on the subject by Magari [7], Anderson [1], and Hájek [6], which elaborates on Gödel's manuscript proof (see [5]). Other modern reconstructions and analyses of Gödel's argument which are worth comparing are those of Sobel [9], or Anderson and Gettings [2] and of Orilia [8].

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► ÀNGEL J. GIL AND JORDI REBAGLIATO, *A characterization of the Gentzen systems satisfying the cut rule*.

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We show that, in Gentzen systems, there is a close relation between two of the main characters in algebraic logic and proof theory respectively: protoalgebraicity and the cut rule. Protoalgebraic deductive systems were introduced in [2] and have been widely studied in the context of *algebraic logic*. Roughly speaking, a deductive system is protoalgebraic if a pair of formulas which cannot be distinguished on the basis of a certain theory cannot be distinguished on the basis of any larger theory either.

In Gentzen systems the concept of protoalgebraicity is enhanced by a new proof-theoretical content, to which we devote the main part of this study. First of all, it is easy to see that a Gentzen system containing the identity axiom and the exchange and the cut rules is protoalgebraic. For the m-sided Gentzen systems—those whose sequents are m-tuples of finite sequences of formulas, considered among others in [1, 4]—the same result holds, but the contraction rule is required: if a Gentzen system contains the identity axiom, the exchange and contraction rules and *all* the cut rules—there may be a cut rule for each pair of components—then it is protoalgebraic. The necessity of the exchange and the contraction rules can be avoided by introducing a slight modification in the definition of the cut rule.

The main goal of [3] is the study of the converse of this result, i.e., must a protoalgebraic Gentzen system contain the cut rule, either as a primitive or as a derived rule? If we admit arbitrary rules the answer is negative. However, in sequent calculi rules are usually divided into structural rules and logical rules. For these reasons we restrict our discussion to what we call *regular sequent calculi*, i.e., sequent calculi consisting of the identity axiom, some structural rules and some *logical rules*, in a sense we make precise and which is intended to be general enough to include all the usual rules. An important feature of the logical rules is the presence of side sequents, which are sequents that are not altered by the rule and are arbitrary, except for their length.

We prove that a protoalgebraic regular sequent calculus must contain, at least, one of the cut rules. Thus, in the usual case of 2-sided sequents, a regular sequent calculus that contains the exchange rule is protoalgebraic if and only if it contains the cut rule. The many-sided case is still to be resolved at this point. In this case the situation is more complicated because the problem remains of determining whether the calculus has to contain *all* the cut rules, at least as derived rules. By analysing the way variables behave through a formal proof we prove that given a regular many-sided sequent calculus that contains the weakening, exchange and contraction rules, it is protoalgebraic if and only if all the cut rules are derivable, provided that there is no restriction on the length of the side sequents of the logical rules.

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In this talk we present some results of measure theory in the context of Reverse Mathematics.

Part of the results were obtained in collaboration with Professor S. G. Simpson ([1, 2]).

The purpose of Reverse Mathematics is to study the role of set existence axioms, trying to establish the weakest subystem of second order arithmetic in which a theorem of ordinary mathematics can be proved. The basic reference is Simpson's monograph [3].

Historically, the subject of measure theory developed hand in hand with the nonconstructive, set-theoretic approach to mathematics. Errett Bishop has remarked that the foundations of measure theory present a special challenge to the constructive mathematician. Although our program of Reverse Mathematics is quite different from Bishop-style constructivism, we feel that Bishop's remark implicitly raises an interesting question: "Which nonconstructive set existence axioms are needed for measure theory?"

WWKL_0 seems to be the right subsystem for measure theory, since most of the results in measure theory are indeed equivalent to it.

WWKL_0 consists of $\mathsf{RCA}_0 + \mathsf{WWKL}$, where WWKL is a very weak version of König's lemma.

WWKL_0 was introduced about 10 years ago by Yu and Simpson ([4]) who introduced the fundaments of measure theory in this context and proved the equivalence of WWKL_0 with the countable additivity of the Lebesgue measure.

Here we present the equivalence between WWKL_0 and a couple of versions of the classical result of Vitali's covering theorem.

Moreover we present an RCA_0 version of disjoint countable additivity which contrasts with the result by Yu-Simpson.

Then we introduce the concept of measurable function in the context of Reverse Mathematics giving the proof of a classical result about the measurability of continuous functions using a new and interesting technique.

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In the abstract possible world semantics the worlds are objects without any internal structure and the accessibility relations between them satisfy some abstract conditions (like reflexivity, transitivity, etc.). Some applications, however, need semantic structures in which possible worlds have an internal structure and the accessibility relations between possible worlds depend on that structure.

We present a natural example of a modal logic (called here a hyperboolean modal logic) HBML with possible world semantics based on "hyperframes" in which the worlds have the structure of sets (or, more abstractly, elements of a Boolean algebra) and the accessibility relations between possible worlds correspond to the Boolean operations. Respectively, the algebraic semantics for HBML is given by Boolean algebras with operators, constructed as algebras of complexes (or, power structures) of Boolean algebras, called here *hyperboolean algebras*.

Such a logic can have various interpretations, one of them related to an idea of Vakarelov and Gargov to study a "logic of expert groups" which can be simply presented as follows: groups of experts have their "opinions" (knowledge, beliefs, judgments, intuitions, etc.) on a certain matter, and these opinions are to be put together and coordinated in a way which would enable some logical analysis on the "integrated opinions", in particular performing logical operations on them. These opinions can be simply presented as "yes-no", or "true-false", but they can have a more fine-grained Boolean structure. A reasonable formal approach seems to be to represent the groups' opinions as sets of Boolean values and adopt internal Boolean operations for Boolean constructions on those opinions, e.g., form a conjunction of two group opinions by taking all conjunctions of opinions of individual agents from each group, etc.

In this paper we construct a complete axiomatizations of HBML and show that it lacks the finite model property. The technique of axiomatization hinges upon the fact that a "difference" operator is definable in hyperboolean algebras, and makes use of additional inference rules. That technique can be generalized to axiomatize other "hyper-structures" such as those for groups, rings, etc.

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During the 1960s and the 1970s, researchers in generalized recursion theory carried out an in-depth investigation of both *positive* and *nonmonotone* inductive definability. Most of this work was *local*, in the sense that it studied inductively definable relations on some fixed infinite structure of mathematical significance, such as the integers, the reals, or initial segments of the universe (see [5, 6, 1]). In the late 1970s, Barwise and Moschovakis [2] initiated a study of *global* inductive definability, and showed that it arises naturally in several different areas of mathematics. Instead of definability of relations on a fixed infinite structure, the global theory is concerned with the inductive definability of *queries* (that is, global relations) on some infinite class of (finite or infinite) structures. Since the early 1980s, researchers in finite model theory have carried out a detailed study of global inductive definability on classes of finite structures. Specifically, positive inductive definability and nonmonotone inductive definability on classes of finite structures have been studied under the names of *least fixed-point* logic and *inflationary fixed-point* logic (see [3]). Moreover, they have been shown to be intimately connected to polynomial-time computability on classes of finite structures. Nonetheless, global inductive definability on classes of infinite structures has remained rather unexplored, even though the initial investigation of Barwise and Moschovakis [2] was aimed in this direction.

Our goal is to further explore global inductive definability on classes of infinite structures. For this, we introduce and study the concept of the *global closure ordinal* of an inductive definition. More precisely, let $\varphi(\bar{x}, X)$ be a first-order formula such that the arity of the relation symbol X is equal to the number of the free variables $\bar{x}$. On each structure A, the formula φ gives rise to an increasing sequence $(\varphi_A^\xi)_{\xi \in \mathrm{ON}}$ of relations on A defined by the transfinite induction

$$\varphi_A^\xi = \varphi_A^{<\xi} \cup \{\, \bar{a} \mid A \vDash \varphi(\bar{a}, \varphi_A^{<\xi})\,\},$$

where $\varphi_A^{<\xi} = \bigcup_{\eta<\xi} \varphi_A^\eta$. Note that if the formula $\varphi(\bar{x}, X)$ is positive in X, then $\varphi_A^\xi = \varphi(\bar{a}, \varphi_A^{<\xi})$ for every ξ. The *closure ordinal* $\mathrm{cl}(\varphi, A)$ *of* φ *on* A is the least ordinal ξ such that $\varphi_A^\xi = \varphi_A^{<\xi}$. This is a classical local notion that has been extensively studied in the literature (see [5, 6, 1]). In contrast, here we are interested in a counterpart global notion. If $\mathcal{C}$ is a class of structures, then the *global closure ordinal* $\mathrm{gcl}(\varphi, \mathcal{C})$ of φ on $\mathcal{C}$ is the supremum of the closure ordinals $\mathrm{cl}(\varphi, A)$ taken over all structures $A \in \mathcal{C}$, if such an ordinal exists, or ∞, otherwise. In this abstract, we focus our attention on $\mathrm{gcl}(\varphi) = \mathrm{gcl}(\varphi, \mathcal{A})$, where $\mathcal{A}$ denotes the class of all structures.

Grilliot [4] observed that if φ is an existential first-order formula, then $\mathrm{gcl}(\varphi) \leq \omega$. Our first result asserts that if $\xi = \mathrm{gcl}(\varphi)$ for some arbitrary first-order formula φ, then there are a universal first-order formula ψ and a positive existential-universal formula χ such that $\xi = \mathrm{gcl}(\psi) = \mathrm{gcl}(\chi)$. In other words, universal formulas and positive existential-universal formulas can realize every global closure ordinal. This result sharply contrasts with the state of affairs in local inductive definability; indeed, there are infinite structures on which for every $n \geq 2$ there is a Σ_{n+1}-formula whose closure ordinal cannot be realized by any Σ_n-formula (see [6]). We also obtain a characterization of the class of global closure ordinals of first-order formulas in terms of a certain Π_1^1-describability condition. As a consequence, we show that the first weakly inaccessible cardinal is a global closure ordinal, whereas each global closure ordinal is smaller than the first strongly inaccessible cardinal.

We also study the uniform closure ordinals of inductively definable queries. The *uniform closure ordinal* of a query Q is the infimum over the global closure ordinals of all possible inductive definitions of Q. Our main result here is that a query Q has uniform closure ordinal less than or equal to ω if and only if it is inductively definable by an existential first-order formula. This provides a partial solution to a problem raised by Barwise and Moschovakis, who asked for a characterization of all positive first-order formulas having global closure ordinal less than or equal to ω. Finally, we show that a countable admissible ordinal is a global closure ordinal if and only if it is the uniform closure ordinal of a query inductively definable by a universal first-order formula.

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PPC presents a non-classical three-valued predicate calculus ([1]), where for a sequent $\alpha_1, \ldots, \alpha_n \vDash \beta$ the formulae at the left-hand side of the turnstile are considered to be **true**, and the formula at the right-hand side of the turnstile is either **true** or **undefined**. In this talk we extend this notion to sequents $\alpha_1, \dots, \alpha_n \models \beta_1, \dots, \beta_n$, which mean that there are no counterexamples, i.e., models for which the expressions at the left-hand side of the turnstile are **true** and the expressions at the right-hand side are **false**. Note that the latter is a classical definition, but it gets a different interpretation in a three-valued logic.

The three-valued predicate calculus PPC is built on the Kleene operators *negation* ($\neg$) and *conjunction* ($\wedge$) and a new non-monotone operator Δ to express the definedness of a formula α (i.e., α being true or false). The *universal quantifier* ($\forall$) extends the conjunction in a classical manner. Carnielli pointed out that the propositional part of PPC can be seen as a subset of the logic $\mathcal{L}_3$ of Łukasiewicz.

Using the non-monotone operator Δ ($\Delta\alpha \equiv$ **true** if and only if α is defined and $\Delta\alpha \equiv$ **false** if and only if α is undefined), we present a tableau method for predicates without operational signature, as a systematic search for a counterexample. Note that our approach is not based on a translation from signed tableaux to sequents, as suggested by W. Carnielli for reason that PPC has not a connective C such that for each truth-value i: $C(i) \neq i$ [2].

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Gentzen games were introduced in [1] as a combinatorial core of the Gentzen cut elimination in sequential calculus. There the Gentzen games were given for predicate logic and Peano arithmetic essentially using the lexicographic path ordering in rewrite systems. Here we extend the Gentzen games to other systems as infinitary logic, iterated inductive definitions and Π^1_1 CA.

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In his monograph [3, Ch. 12] Cantini defines a truth theory VF over total applicative theories based on van Fraassen's concept of *supervaluation* (for a corresponding theory over Peano arithmetic cf. [2]). It allows to conclude the truth of (the representing term of) a formula φ if φ is a tautology, independently of its logical complexity. So this theory is an example of a non-reductive approach to truth. Cantini proves that VF has the same proof-theoretic strength as the well-known theory of positive inductive definitions ID_1 by embedding the proof-theoretically equivalent, but syntactically weaker theory $\mathrm{ID}_1(\mathrm{acc})$ (for these theories cf. [1]). We improve this result by giving an interpretation of ID_1 itself, showing that VF has strong syntactical expressive power, cf. [4]. Also, we address a modification of VF based on *partial* applicative theories which uses the methods of [5].

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We examine the standard form of the axiom of choice. It is well known that this form is inconsistent with ZFI [1]. But some partial variants of this form of the axiom of choice can be consistent with ZFI and one can try to investigate these variants. For example, let us examine such a variant: there exists a function of choice for the countable family of nonempty disjunctive sets of natural numbers. This affirmation is unprovable and independent in ZFI (we proved this fact). Moreover, it is not possible to reinforce this result somehow. For example, let us examine such variant of the axiom of choice: there exists a function of choice for the countable family of nonempty disjunctive countable sets. Such form of the axiom of choice is not consistent with ZFI (and with other very weak systems with intuitionistic logic).

We submit the consistency and independent proofs of partial axiom of choice of the intuitionistic set theory with two kinds of variables. The same result takes place for the set theory with only one kind of variable. The language of our theory contains two kinds of variables (numerical and set-theoretic), symbols for the predicates (a natural number belongs to a set and a set belongs to a set), the logical connectives and the quantifiers for all kinds of variables.

The body of axioms consists of all standard set-theoretic axioms with usual boundary and the axiom of double complement of sets.

[1] ***Referativnyj Zhurnal Matematika*** (1979), no. 5, 5A59 (in Russian).

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We define a truth function $T\colon On \to 2$ by

$$T(\alpha) = 1 \qquad \text{if and only if} \qquad (\alpha, <, \mathrm{Synt}, T\restriction\alpha) \vDash \alpha.$$

On the right-hand side, a Gödelisation of the appropriate language by ordinals is assumed and *Synt* is a collection of syntactical functions for that coding. The function T has the same information content as the constructible universe L since in $(On, <, \mathrm{Synt}, T)$ we can define a structure isomorphic to $(L, \in)$. The canonical Skolem function $h\colon On \to On$ picks a least witness $H(\alpha) < \alpha$ for α if α is an existential statement. For $\alpha \in On$ let $T_\alpha = (\alpha, <, \mathrm{Synt}, T\restriction\alpha, h\restriction\alpha)$, and for $X \subseteq \alpha$ let $T_\alpha[X]$ be the smallest subalgebra of T_α containing X. The T_α-hierarchy and the associated hull operator $T_\alpha[\cdot]$ satisfy basic laws of hyperfinite structure theory:

- *Condensation*. If X is a subalgebra of T_α then there is a unique $\bar{\alpha} \le \alpha$ and a unique isomorphism
$$\pi\colon (X, <, \mathrm{Synt}, T\restriction X, h\restriction X) \backsimeq T_{\bar{\alpha}}.$$
- *Monotonicity*. If $\alpha \le \beta$, $X \subseteq Y$, $X \subseteq \alpha$, $Y \subseteq \beta$, then $T_\alpha[X] \subseteq T_\beta[Y]$.
- *Continuity*. If α is a limit ordinal and $X \subseteq \alpha$, then
$$T_\alpha[X] = \bigcup_{\beta<\alpha} T_\beta[X \cap \beta].$$

- *Finiteness*. For all α, there exists a finite $z \subseteq \alpha$ such that

$$\forall X \subseteq \alpha + 1 \colon T_{\alpha+1}[X] \subseteq T_\alpha[(X \cap \alpha) \cup p] \cup \{\alpha\}.$$

- *L-Adequacy*. If α is singular in L, then there is $\beta \geq \alpha$, a finite $p \subseteq \beta$ and $\gamma < \alpha$ such that $T_\beta[\gamma \cup p]$ is cofinal in α.

These laws are known from Silver machines (see [1]) or from fine structure with restricted Skolem functions [2]. They allow simplified proofs of $\Box$ and other fine-structural principles in L.

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Many diverse problems of compatibility of structures can be unified by generalizing the concept of a homomorphism. In 1977 Bandler and Kohout introduced *generalized homomorphism*, *proteromorphism* and *amphimorphism*, forward and backward compatibility of relations, and non-associative and pseudoassociative products (compositions) of relations [1]. In the original papers of Bandler and Kohout the logic of relations was crips. The purpose of this presentation is to show that the concepts of generalized morphisms, compatibility, etc., can be extended to relations based on any system of fuzzy logic in which the axioms of the BL family of fuzzy logics of Petr Hájek [2] hold.

Motivation: Crisp generalized morphisms. Let A, B, C, D be sets with relations R, S upon them—R from A to B and S from C to D, where each relation determines some structure. In addition, we have homomorphism mappings F and G. F is from A to C and G is from B to D. There are two points of departure that stem from this fundamental algebraic notion of *homomorphism*: (i) the design or checking mappings which will "preserve" or "respect" certain given relations, and on the other hand (ii) the design or checking of relations which "absorb" or "validate" certain given mappings. For example let $A = B$, $C = D$ and R, S be orders. Given A and C we wish to find one or all of the mappings from A to C that preserve orders—this illustrates the case (i). An example of (ii) is, given a mapping from A to C, how to match the order on A given by R, with some other order on C, or vice versa—so that some given mapping will preserve or co-preserve them. Another example is where $A = B \times B$, $C = D \times D$ and R and S determine some groupoids.

In this situation, the conventional homomorphism yields a commuting diagram of arrows such that $R \circ G = F \circ S$, where, of course, the morphisms F and G are the relations which are both covering and univalent (i.e., functional). To obtain the constructions that solve the problems (i) or (ii) requires us to solve the above relational equation with respect to one of the relations R, S, F or G. When the mappings (functional relations) F and G are replaced by general relations, the equation is no longer valid but has to be replaced by two inequalities. The notion of a homomorphism splits into two independents notions, generalized morphism and generalized proteromorphism.

Generalized morphisms in fuzzy BL-logics. All fuzzy relations in the sequel are defined in the interval [0, 1] where & is a continus t-norm and $\rightarrow$ is its residuum. In this set-up Hájek's formalization of BL-logics [2] applies. $\mathcal{R}(X \rightsquigarrow Y)$ denotes the lattice of all fuzzy relations from X to Y; $\Longleftrightarrow$ represents the meta-logical statement "if and only if".

Definition. Let F, R, G, S be heterogenous relations between the sets A, B, C, D such that $F \in \mathcal{R}(A \rightsquigarrow C)$, $R \in \mathcal{R}(A \rightsquigarrow B)$, $G \in \mathcal{R}(B \rightsquigarrow D)$, $S \in \mathcal{R}(C \rightsquigarrow D)$.

1. The conditions that (for all $a \in A, b \in B, c \in C, d \in D$) $(aFc \& aRb \& bGd) \rightarrow cSd$ will be expressed in any of the following ways: (i) $FRG : S$ is forward compatible, (ii) F, G respect R, S forwards, (iii) R, S absorb F, G forwards, (iv) F, G are generalized homomorphisms from R to S.
2. The conditions that (for all $a \in A, b \in B, c \in C, d \in D$) $(aFc \& cSd \& bGd) \rightarrow aRb$ will be expressed in any of the following ways: (i) $FRG : S$ is backward compatible, (ii) F, G respect R, S backwards, (iii) R, S absorb F, G backwards, (iv) F, G are generalized proteromorphisms from R to S.

DEFINITION. For arbitrary fuzzy relations in $[0, 1]$, R from the set X to Y, S from Y to Z, define:

1. $R \circ S = (\forall x)(\forall z)(\exists y)(x\ R\ y\ \&\ y\ S\ z)$;
2. $R \lhd S = (\forall x)(\forall z)(\forall y)(x\ R\ y \rightarrow y\ S\ z)$;
3. $R \rhd S = (\forall x)(\forall z)(\forall y)(x\ R\ y \leftarrow y\ S\ z)$;

THEOREM (Compatibility Theorem).

1. *$FRG : S$ are forward compatible* $\iff F^T \circ T \circ G \sqsubseteq S \iff R \sqsubseteq F \lhd S \rhd G^T$.
2. *$FRG : S$ are backward compatible* $\iff F \circ S \circ G^T \sqsubseteq R \iff S \sqsubseteq F^T \lhd R \rhd G$.

$FRG : S$ are both-ways compatible if and only if they are both forward and backward compatible. The Rs of forward compatibility constitute a lower ideal, while those of backward compatibility constitute an upper ideal or filter; the both-ways problem has a solution if and only if their intersection is non-empty. The conventional homomorphism is a special case of *both-ways* compatibility. The generalized morphisms presented here are relevant not only theoretically, but have also an important practical use in solving systems of relational inequalities and equations.

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[2] P. HÁJEK, ***Fuzzy logic from the logical point of view***, Lecture Notes in Computer Science, no. 1012, Springer-Verlag, Berlin, pp. 31–49.

► K. V. KOROVIN, *Some properties of compositions of permutations with respect to algorithmic reducibilities*.
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Properties of groups

$$G_d = \{\, f \mid f \text{ is a permutation of } \omega \text{ and } f \leq_T d \,\}$$

where d is a Turing degree are well studied [1]. In [1] A. S. Morozov has formulated the following problem: is it possible to extend these results to other reducibilities? In our paper we demonstrate that for some wide class of algorithmic reducibilities the answer for this question is negative.

THEOREM 1. *For wtt-reducibility or arbitrary reducibility stronger than wtt-reducibility, there exists a set $F \leq_T 0'$ such that the set*

$$\{\, f \mid f \text{ is a permutation of } \omega \text{ and the graph of } f \text{ reduces to } F \,\}$$

is not closed under composition.

THEOREM 2. *Let A be an arbitrary set such that $A \leq_T 0'$. There exists a permutation f such that the graph of f is 2-limit computable and the graph of f^2 is not wtt-reducible to A.*

COROLLARY 1. *There exists a permutation f such that the graph of f is tt-reducible to $\emptyset'$ but the graph of f^2 is not wtt-reducible to $\emptyset'$.*

Corollary 2. *The set*

$$\{ f \mid f \text{ is a permutation and the graph of } f \text{ is } \omega\text{-limit computable} \}$$

is not closed under composition.

The following notion of reducibility on functions was introduced and well studied by M. M. Arslanov [2].

Definition. A function f is ma-reducible to a set A if there exist three total recursive functions a, b, c, such that

$$f(x) = \begin{cases} a(x), & \text{if } c(x) \in A, \\ b(x), & \text{if } c(x) \notin A. \end{cases}$$

For this reducibility the following theorem holds.

Theorem 3. *There is a c.e. set A, such that the set*

$$\{ f \mid f \text{ is a permutation and } f \leq_{\text{ma}} A \}$$

is not closed under composition.

[1] A. S. Morozov, *Groups of computable automorphisms*, in ***Handbook of recursive mathematics***, American Mathematical Society, to appear.

[2] M. M. Arslanov, ***Computable enumerable sets and degrees of unsolvability***, Kazan University Publications, 1986.

► PIOTR KULICKI, *Axiomatic rejection in first order theories.*
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Among a few discovered methods of refutation for logical systems, axiomatic rejection is probably the least known. It was J. Łukasiewicz who, drawing his inspiration from Aristotle, conceived the idea of an axiomatic refutation calculus describing the set of non-theorems of a logic (see [1]). The first calculus which received such a formulation was Aristotle's syllogistic, which is a first-order theory, but then most effort was devoted to propositional logics. The results include axiomatic refutation systems for many-valued, modal, intuitionistic and intermediate propositional logics. In my presentation I would like to return to axiomatic rejection in the case of first order theories with a special interest in its relation to models and logic programs.

First, we consider the relation between a system of axiomatic rejection for a theory and a model theoretic structure that can be used for decision procedure for that theory. For theories fulfilling certain conditions it holds that the size of the domain of the smallest model that can be used to refute all rejected axioms of a theory limits the size of such domain for any other formulas. Application of that result to Aristotle's syllogistic leads to the conclusion that any formula of syllogistic enriched by classical connectives is decidable with the use of models in a two-membered domain.

Furthermore we consider the recursive rule schema that is used in the system of axiomatic rejection for Aristotle's syllogistic. The rule schema express the following disjunction property:

$$A \to B \vee C \in \text{Th} \qquad \text{if and only if} \qquad A \to B \in \text{Th} \quad \text{or} \quad A \to C \in \text{Th}$$

where A is a conjunction of atomic formulae and B and C are disjunctions of atomic formulae. Surprisingly, that property is equivalent to the postulate of decomposition of a formula to Horn clauses, which is normally used to obtain a correct logic program from any first order theory.

[1] J. ŁUKASIEWICZ, ***Aristotle's syllogistic from the standpoint of modern formal logic***, Clarendon Press, 1951.

► OLIVER KULLMANN, *Restricted versions of extended resolution*.
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Extended Resolution (ER), introduced by Tseitin [7], is among the most powerful propositional proof systems. Motivated by improved SAT algorithms (see [5]), in [6] the notion of "Blocked Clauses" has been introduced, generalizing the introduction rule of ER (see also [4]).

This concept has been studied in [3] in order to get "approximate insights" for Extended Resolution (by means of "Generalized Extended Resolution" (GER)). [1] considered (strongly) restricted versions of GER and tried to give simulation results.

We now want to study the combination of full GER with some known resolution restrictions.

[1] ALLEN VAN GELDER, *Propositional search with k-clause introduction can be polynomially simulated by resolution*, in ***Fifth international symposium on artificial intelligence and mathematics***, January 1998, `http://rutcor.rutgers.edu/~amai/`.

[2] OLIVER KULLMANN, *Methods for* 3-SAT-*decision in less than* $2^{0.59\cdot n}$ *steps*, this JOURNAL, vol. 1 (1995), no. 1, pp. 96–97, Abstracts of contributed papers of the Logic Colloquium '93, Keele, England, July 20–29, 1993.

[3] ———, *On a generalization of extended resolution*, ***Discrete Applied Mathematics***, 1996, to appear in the special edition on the satisfiability problem.

[4] ———, *Some remarks on extended resolution*, this JOURNAL, vol. 3 (1997), p. 267, Abstracts of contributed talks of the Logic Colloquium '96, Donostia-San Sebastián, Spain, July 9–15, 1996.

[5] ———, *Worst-case analysis,* 3-SAT *decision and lower bounds*: *Approaches for improved* SAT *algorithms*, ***DIMACS Series in Discrete Mathematics and Theoretical Computer Science***, vol. 35 (1997), pp. 261–313.

[6] ———, *New methods for* 3-SAT *decision and worst-case analysis*, ***Theoretical Computer Science***, to appear; first version announced in [2].

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► JAN KUPER, $\mathbf{Y}\Omega_3$ *is almost easy*.
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In [2] Berarducci and Intrigila raise the question whether $\mathbf{Y}\Omega_3$ is easy, i.e., whether for all closed terms M the equation $\mathbf{Y}\Omega_3 = M$ can be consistently added to the lambda calculus. They give a partial answer to this question (notation: we write $\mathbf{Con}(A = B)$ if the equation $A = B$ can be consistently added to the lambda calculus; $\mathrm{BT}(X)$ denotes the Böhn tree of X):

- $\mathbf{Y}\Omega_3$ is n.f.-easy, i.e., $\mathbf{Con}(\mathbf{Y}\Omega_3 = N)$ for every closed normal form N,
- if $\mathrm{BT}(M) \neq \mathrm{BT}(\omega_3)$, then $\mathbf{Con}(\mathbf{Y}\Omega_3 = M)$.

In this paper we only slightly strengthen this result. The more important part is the simplicity of the proof technique. We prove the following theorem (notation: $\boldsymbol{\lambda}$ denotes the untyped lambda calculus, $\boldsymbol{\lambda}^+$ is $\boldsymbol{\lambda} + \Omega_3 M = M$):

THEOREM. *If* $\boldsymbol{\lambda}^+ \nvdash M = \omega_3$, *then* $\mathbf{Con}(\mathbf{Y}\Omega_3) = M$. *Furthermore*, $\mathbf{Con}(\mathbf{Y}\Omega_3 = \omega_3)$.

REMARK. There do exist terms for which the condition is not fulfilled, e.g.,

$$M \equiv \mathbf{Y}\,(\lambda u.\lambda x.x(\Omega_3 u(\lambda yz.x))(\Omega_3 u(\lambda yz.x))).$$

On the other hand, the condition of the theorem is fulfilled for, e.g., $M \equiv \lambda x.x\Omega\Omega$. Notice that for both terms M we have $\mathrm{BT}(M) \subseteq \mathrm{BT}(\omega_3)$.

PROOF TECHNIQUE. The technique to prove the above theorem is simple and consists of two stages.

First we extend the lambda calculus with the following reduction rule:

$$\textit{if } \boldsymbol{\lambda}^+ \vdash M = M', \textit{ then } \Omega_3' M' \xrightarrow{+} M, \textit{ where } \Omega_3' \textit{ is of the form } \Omega_3, \Omega_3\omega_3, \Omega_3\omega_3\omega_3, \dots,$$

and we show that the resulting calculus has the Church-Rosser property (straightforward).

Second, we use the Jacopini technique (see [1, 3]) for this extended calculus in the formulation as given in [4]. That is to say, in order to show $\mathbf{Con}(\mathbf{Y}\Omega_3 = M)$, we only have to show (straightforward) that

- $\mathbf{Y}\Omega_3$ is *operationally less defined* then M, i.e., if $F(\mathbf{Y}\Omega_3)$ has a normal form, then FM has the same normal form (see [5]),
- $\mathbf{Y}\Omega_3$ is *proof replaceable* by M, i.e., if $F(\mathbf{Y}\Omega_3) = F'(\mathbf{Y}\Omega_3)$, then there is a G such that $G(\mathbf{Y}\Omega_3)M = FM$ and $GM(\mathbf{Y}\Omega_3) = F'M$ (see [4]).

[1] J. BAETEN and B. BOERBOOM, *Ω can be anything it should not be*, ***Indagationes Mathematicae***, vol. 41 (1979), pp. 111–120.

[2] A. BERARDUCCI and I. INTRIGILA, *Some new results on easy lambda terms*, ***Theoretical Computer Science***, vol. 121 (1993), pp. 71–88.

[3] G. JACOPINI, *A condition for identifying two elements of whatever model of combinatory logic*, in ***Lambda calculus and computer science*** (C. Böhm, editor), Springer-Verlag, Berlin, 1975, pp. 213–219.

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[5] G. D. PLOTKIN, *LCF considered as a programming language*, ***Theoretical Computer Science***, vol. 5 (1977), pp. 223–255.

► HANNES LEITGEB, *Truth predicates defined by translations*.
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According to Tarski [2], one could call a (classical) model $\mathfrak{M}$ a *truth model* for an object language $\mathcal{L}_1$ in a metalanguage $\mathcal{L}_2$ (with truth predicate) if and only if

$$\mathfrak{M} \models \mathrm{True}(s) \leftrightarrow \varphi'$$

for all s, φ' such that:

- s is a name of a sentence $\varphi \in \mathcal{L}_1$,
- φ' is the translation of φ into $\mathcal{L}_2$.

I.e., $\mathrm{True}(s)$ is (up to equivalence) nothing but the *translation* of the sentence denoted by s into the metalanguage. Since translation is usually left undefined, one generally chooses $\mathcal{L}_1$ to be a sublanguage of $\mathcal{L}_2$ and the identity function as translation.

Instead, we will define translation formally and thereby get a more general concept of truth models, where the truth predicate is defined by a translation (note that we will not stick to classical models but rather look at lattice-valued models including Boolean-valued models).

We will show that such general truth models even exist in the case of $\mathcal{L}_1 = \mathcal{L}_2$ with arbitrary self-reference concerning denotation, i.e.: *if truth predicates are defined by translations, languages can contain their own truth predicate*.

This connects our approach to more recent accounts of truth theories like Gupta [1].

[1] A. Gupta and N. Belnap, ***The revision theory of truth***, MIT Press, Cambridge, Massachusetts, 1993.

[2] A. Tarksi, *Der Wahrheitsbegriff in den formalisierten Sprachen*, ***Studia Philosophica***, vol. 1 (1935), pp. 261–405.

► PAOLO LIPPARINI, *Regular ultrafilters and* $[\lambda, \lambda]$*-compact products of topological spaces*.
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A topological space X is $[\mu, \lambda]$*-compact* if and only if every cover by λ open sets has a subcover by $< \mu$ sets. X is *finally* μ*-compact* if and only if X is $[\mu, \lambda]$-compact for all $\lambda \geq \mu$. *Compact* means finally ω-compact.

Theorem 1. *If a product of topological spaces is* $[\lambda^+, \lambda^+]$*-compact and* λ *is regular then all but at most* λ *factors are* $[\lambda, \lambda]$*-compact*.

Corollary 1. *If a product is finally* ω_{n+1}*-compact then all but at most* ω_n *factors are compact*.

Corollary 2. *If* λ *is regular, and* X^{λ^+} *is* $[\lambda^+, \lambda^+]$*-compact then* X *is* $[\lambda, \lambda]$*-compact*.

Theorem 2. *Suppose that* λ *is a singular cardinal and* $\lambda = \sup\{ \lambda_\alpha \mid \alpha \in \operatorname{cf} \lambda \}$. *If* $Y \times \prod_{\alpha \in \operatorname{cf}\lambda} Y_\alpha$ *is* $[\lambda, \lambda]$*-compact then either* Y *is* $[\operatorname{cf}\lambda, \operatorname{cf}\lambda]$*-compact or some* Y_α *is* $[\lambda_\alpha, \lambda]$*-compact. In particular, if* $X^{\operatorname{cf}\lambda}$ *is* $[\lambda, \lambda]$*-compact then* X *is either* $[\operatorname{cf}\lambda, \operatorname{cf}\lambda]$*-compact or* $[\lambda_\alpha, \lambda]$*-compact for some* $\alpha \in \operatorname{cf}\lambda$.

From now on, suppose $\mu \leq \lambda$, $\mu' \leq \lambda'$ and $\kappa \geq \sup\{\lambda, \lambda'\}$.

Definition. We say that an ultrafilter D *covers* $S_\mu(\lambda)$ if and only if D is over $S_\mu(\lambda)$ and $\{ x \in S_\mu(\lambda) \mid \alpha \in x \} \in D$, for every $\alpha \in \lambda$. We write $(\lambda, \mu) \overset{\kappa}{\Longrightarrow} (\lambda', \mu')$ to mean that there are κ functions $(f_\alpha)_{\alpha\in\kappa}$ from $S_\mu(\lambda)$ to $S_{\mu'}(\lambda')$ such that whenever D covers $S_\mu(\lambda)$ then for some $\alpha \in \kappa$ $f^*_\alpha(D)$ covers $S_{\mu'}(\lambda')$.

The above notion is equivalent to the one introduced in [2] (the parameter κ is from [2, Remark 0.21(c)]).

Theorem 3. *Suppose that* $(\lambda, \mu) \overset{\kappa}{\Longrightarrow} (\lambda', \mu')$ *holds. If* $(X_\alpha)_{\alpha\in\kappa}$ *are topological spaces and no* X_α *is* $[\mu', \lambda']$*-compact then* $\prod_{\alpha\in\kappa} X_\alpha$ *is not* $[\mu, \lambda]$*-compact*.

Theorem 3 is proved by extending the methods of [1]. Theorems 1 and 2 follow using the results in [2, Section 0]. Many more results can be obtained in this way, under special set-theoretical assumptions.

[1] X. Caicedo, *The abstract compactness theorem revisited*, to appear in ***Logic in Florence*** (A. Cantini, E. Casari, and P. Minari, editors).

[2] P. Lipparini, *Ultrafilter translations I*, ***Archive for Mathematical Logic***, vol. 35 (1996), pp. 63–87.

► LARISA MAKSIMOVA, *Projective Beth property in superintuitionistic logics*.
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In 1960 G. Kreisel proved that each superintuitionistic logic has the Beth property. We say that a logic L has the Projective Beth Property (PBP) if the condition $\vdash_L A(\mathbf{p}, \mathbf{q}, x)\ \&\ A(\mathbf{p}, \mathbf{q}', y) \to (x \leftrightarrow y)$ implies that there exists a formula $B(\mathbf{p})$ such that $\vdash_L A(\mathbf{p}, \mathbf{q}, x) \to (x \leftrightarrow B(\mathbf{p}))$ (here $\mathbf{p}$, $\mathbf{q}$, $\mathbf{q}'$ are disjoint lists of variables that do not contain x and y).

We say that a class V of algebras has the property SES if for each $\mathbf{A}$, $\mathbf{B}$ in V, and for every monomorphism $\alpha\colon \mathbf{A} \to \mathbf{B}$ and for every $x \in \mathbf{B} - \alpha(\mathbf{A})$ there exist $\mathbf{C} \in V$ and monomorphisms $\beta\colon \mathbf{B} \to \mathbf{C}$, $\gamma\colon \mathbf{B} \to \mathbf{C}$ such that $\beta\alpha = \gamma\alpha$ and $\beta(x) \neq \gamma(x)$.

For any superintuitionistic logic L, let $V(L)$ be a variety of Heyting algebras associated with L. A Heyting algebra is subdirectly irreducible if and only if it has an opremum, i.e., the greatest among the elements different from $\top$.

THEOREM 1. *For every superintuitionistic logic L the following are equivalent*:

(i) *L has the Projective Beth Property,*
(ii) $V(L)$ *has* SES,
(iii) *the class* $\mathrm{SI}(V(L))$ *of all subdirectly irreducible algebras in* $V(L)$ *has the property* SES *and, moreover, for every* $\mathbf{A}, \mathbf{B}, \mathbf{C} \in \mathrm{SI}(V(L))$, *such that* $\mathbf{A}$ *is a common subalgebra of* $\mathbf{B}$ *and* $\mathbf{C}$ *and all three algebras have the same opremum, there exists a* $\mathbf{D} \in \mathrm{SI}(V(L))$ *and monomorphisms* $\beta\colon \mathbf{B} \to \mathbf{D}$ *and* $\gamma\colon \mathbf{C} \to \mathbf{D}$ *such that* β *and* γ *coincide on* $\mathbf{A}$.

It is well known that Craig's Interpolation Property CIP implies PBP on the class of superintuitionistic logics. The converse does not hold. Let L_4 be a super-intuitionistic logic characterized by 4-element linear ordered Heyting algebra; it is the greatest logic of the third slice [1]. It is known from [2] that CIP fails for L_4.

THEOREM 2. L_4 *possesses the Projective Beth Property.*

THEOREM 3. *For each* $4 \leq n < \omega$, *there is no superintuitionistic logic of* n*th slice with* PBP.

THEOREM 4. *If a superintuitionistic logic L has* PBP *then L satisfies the Principle of Variable Separation* (*see* [3] *for definition*).

[1] T. HOSOI, *On intermediate logics I*, ***Journal of the Faculty of Science, University of Tokyo***, vol. 14 (1967), pp. 293–312.

[2] L. MAKSIMOVA, *Craig's theorem in superintuitionistic logics and amalgamable varieties*, ***Algebra and Logic***, vol. 16 (1977), no. 6, pp. 643–681.

[3] ———, *On variable separation in modal and superintuitionistic logics*, ***Studia Logica***, vol. 55 (1995), pp. 99–112.

► GIAN ARTURO MARCO, *Some remarks on orbit isolation and analytic sets*.
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The omitting types property in the Banach-Mazur game version is a central tool we have to classify the orbits of countable models of theories by a direct control of the complement in the Cantor space of (coded) interpretations. Omitting types can be replaced in the general framework of Polish group actions by the Effros theorem; this more general descriptive set theoretical notion is necessary to characterize Borel equivalence relations in Polish group spaces in terms of the Borelness of the map associating to the points of the space their stabilizer [1]: the question whether (or not) we can generalize the orbit isolation property to general orbital analytic equivalence relations is the problem the present paper addresses.

We show that for invariant analytic sets in Polish spaces, the isolation property and the c.c.c. property for analytic s-ideals are equivalent, and, on the basis of suitable determinacy hypothesis we improve a result of A. S. Kechris and S. Solecki [2] by showing that the Borel approximation property for analytic sets implies orbital isolation.

By making use of Stern models [3] we prove the general failure of the orbital isolation property for components of analytic sets, and we conclude giving some evidence to the fact that in the generic extensions of the universe generated by a "bounded" notion of forcing, analytic equivalence relations are not induced by Polish group actions, whenever the Borel approximation property fails to hold; the nature of "thin" equivalence relations is essential to the argument.

[1] H. BECKER and A. S. KECHRIS, ***The descriptive set theory of Polish group actions***, Cambridge University Press, 1996.

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[3] J. Stern, *On Lusin's restricted continuum problem*, ***Annals of Mathematics***, vol. 120 (1984), pp. 7–37.

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(Work in collaboration with Alessandro Andretta and Camillo Costantini.)

If X is a countable topological space the descriptive set-theoretic complexity of $C_p(X)$ (i.e., the set of real-valued functions on X with the topology of pointwise convergence)—viewed as a subspace of the Polish space $\mathbb{R}^X$—has been studied in depth. Much less is known if X is uncountable (in this case $\mathbb{R}^X$ is not metrizable, and hence not Polish). We study $C_p(X)$ by itself and, if X is Polish, almost completely characterize when it is a standard Borel space (this means that its Borel σ-algebra coincides with the Borel σ-algebra of a Polish topology):

Theorem 1 (Andretta-M.). *Let X be Polish*:

1. *if X is locally compact then $C_p(X)$ is standard Borel*;
2. *if X is not the countable union of compact sets then $C_p(X)$ is not standard Borel but it is coanalytic.*

We are interested in how a function space behaves with respect to the topology of a different function space. For example we prove:

Theorem 2 (Andretta-M.). *Let X be locally compact Polish, μ a non-atomic Borel measure on X which is positive on open sets and finite on compact sets. Then $C(X) \cap L^p(X, \mu)$ is $\mathbf{\Pi}^0_3$-complete in $L^p(X, \mu)$.*

In particular $C([0,1])$ is $\mathbf{\Pi}^0_3$-complete in $L^p([0,1])$ and $C(\mathbb{R}) \cap L^p(\mathbb{R})$ is $\mathbf{\Pi}^0_3$-complete in $L^p(\mathbb{R})$ (here both L^ps are taken with respect to Lebesgue measure). On the other hand $\mathbb{N}^\mathbb{N}$ is the prototype of non-locally compact Polish spaces and we have:

Theorem 3 (Andretta-M.). *If μ is a finite non-atomic Borel measure on $\mathbb{N}^\mathbb{N}$ which is positive on open sets then $C(\mathbb{N}^\mathbb{N}) \cap L^p(\mathbb{N}^\mathbb{N}, \mu)$ is $\mathbf{\Pi}^1_1$-complete in $L^p(\mathbb{N}^\mathbb{N}, \mu)$.*

A corollary is that the set of real-valued functions on $[0,1]$ which are continuous on $[0,1] \setminus \mathbb{Q}$ is $\mathbf{\Pi}^1_1$-complete in $L^p([0,1])$: this holds because every continuous real-valued function on $[0,1] \setminus \mathbb{Q}$ can be extended to a function on $[0,1]$ which is still continuous at every irrational. One can ask more general questions about the existence of extensions of continuous functions that preserve the continuity on the original domain. Some of our results have a topological flavour:

Theorem 4 (Costantini-M.). *Let X be a topological space, $A \subseteq X$ dense, and Y locally compact. Every continuous $f : A \to Y$ has an extension $\tilde{f} : X \to Y$ which is continuous at every point of A. (This answers a question of Arhangel'skii's.)*

Others are more descriptive set-theoretic:

Theorem 5 (Costantini-M.). *Let X be an uncountable Polish space and Y a separable metrizable space. The following are equivalent*:

(i) *for every $A \subseteq X$, every continuous $f : A \to Y$ has an extension $\tilde{f} : X \to Y$ which is continuous at every point of A*;
(ii) *Y is Polish.*

Theorem 6 (Costantini-M.). *Let X be metrizable, $A \subseteq X$, Y completely metrizable and at least one of X and Y be separable. Every continuous $f : A \to Y$ has an extension $\tilde{f} : X \to Y$ of Baire class 1 which is continuous at every point of A.*

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We consider questions about simultaneous convergence of bounded sequences that immediately lead us into the set theory of the reals.

We consider families $\mathcal{F} \subseteq 2^\omega$.

$\mathcal{F}$ is called *splitting* if and only if $\forall X \in [\omega]^\omega\ \exists f \in \mathcal{F}\ f \restriction X$ is not almost constant.

$\mathcal{F}$ is called *chaotic* (see [2]) if and only if for every Toeplitz matrix $(m_{i,j})_{i,j\in\omega}$ there is some $f \in \mathcal{F}$ such that

$$\lim_{i\to\infty} \sum_{j\in\omega} m_{i,j} f(j)$$

does not exist. A matrix $(m_{i,j})_{i,j\in\omega}$ is called Toeplitz (roughly spoken), if each row sums up to 1, and if the rows are absolutely summable with the same bound for all rows, and if each column converges to 0.

$\mathcal{F}$ is called *finitely splitting* (see [1]) if and only if for every partition $\{ X_n \mid n \in \omega \}$ of ω into finite sets there is some $f \in \mathcal{F}$ such that $\exists^\infty n\ f\restriction X_n \equiv 0 \wedge \exists^\infty n\ f \restriction X_n \equiv 1)$.

It is quite easy to see that finitely splitting families are chaotic and that chaotic families are splitting.

We show that for either reverse inclusion it is consistent that it does not hold, and we show some results on the cardinal characteristics that are given by the smallest cardinality of a family of each kind.

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A Boolean algebra is *recursive* if its universe is the natural numbers and the quantifier-free formulae uniformly denote recursive relations. It is *n-recursive* if, in addition, the Σ_n formulae uniformly denote recursive relations, and *decidable* if this is true of all formulae.

Two natural questions thrown up by these definitions are:

1. Is there, for each n, an n-recursive Boolean algebra that is not $(n+1)$-recursive, or, better still, has no n-recursive copy?
2. Is there a Boolean algebra that is n-recursive for all n but has no decidable copy?

In this talk I answer these questions, both in the positive.

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In [4] the contraction-free and cut-free sequent calculus **G3ip** for intuitionistic propositional logic was extended by rules for theories of apartness and order. The logical content of the axioms of these theories is expressed by the geometry of sequent calculus rules, which have only atomic formulas as active and principal. In this way also such extensions are contraction-free and cut-free. Cut elimination permits structural proof analysis, and syntactic proofs of conservativity results.

The results of [4] are generalized as follows: We show how to present a wide class of elementary intuitionistic theories as systems of rules, with all structural rules admissible.

By using the full invertibility of classical rules, the result can be strengthened: All classical elementary theories can be presented as contraction-free and cut-free systems of rules. Details will appear in [5].

We also consider extensions of the terminating calculus **G4ip**, introduced in [1, 3]. For details and an extensive bibliography we refer to [2].

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► KARL-HEINZ NIGGL, *Characterising polytime through higher type recursion*.
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It is well-known that by a single use of higher type recursion on notation one can define proper Kalmar-elementary functions. This is due to a certain non-linear use of the "previous function" in the step term of the recursion.

It is shown how to restrict recursion on notation in all finite types so as to characterise the polynomial time computable functions. The restrictions are obtained by enriching the type structure with $!\sigma$, and by adding linear concepts to the lambda calculus.

For this purpose, a simply typed term system RNA in the style of Gödel's system T is presented, where RNA indicates the essential restrictions to the formation of terms: *Relevance*, *Necessitation*, and *Affinability*. RNA-terms are built from variables and the "binary" constants 0, s_0, s_1, p by restricted application accounting for R, by a simplification of Prawitz's rule for introducing "!" in his formulation of S4 accounting for N, by elimination of "!", by restricted lambda abstraction accounting for A, and by rules for introducing case analysis and recursion on notation in all finite safe types. *Safe types* are those types which do not contain "!". Similar to Gödel's T, terms are interpreted over the set-theoretical function spaces.

Affinability formalises the underlying concept of linearity which is central to the whole system. It is designed such that it is decidable whether or not a given general term is in RNA.

It is shown that RNA is closed under reduction. Strong normalisation for RNA and uniqueness of normal forms then follow. In particular, every closed RNA term of ground type reduces to a numeral denoting the value of the term. Thus, RNA can be considered a simple functional programming language where computation is normalisation.

As for the expressive power of RNA, a kind of sharing concept implemented within RNA is employed to prove that every RNA program computing a number theoretical function can be evaluated in time polynomial in the lengths of the given inputs. That all polynomial time computable functions are definable in RNA simply holds by embedding the BC functions. (Joint work with S. Bellantoni and H. Schwichtenberg.)

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A formal theory of first-order fuzzy logic with evaluated syntax is presented in [3, 5] and elsewhere (cf. also [1]). The system of logical connectives and truth values is the Łukasiewicz one. We deal with evaluated formulas a/A, i.e., i.e., formulas A evaluated by an evaluation a (on the syntactical level!). Generalizations of the classical concepts of provability of a formula A in a theory T, which is given by a fuzzy set of axioms, $T \vdash_a A$ and its truth, $T \models_a A$ are introduced.

In [6], a constructive proof of the following theorem, originally initiated by McNaughton [2] is given.

THEOREM 1. *The class of functions representable by formulas of the propositional fuzzy logic coincides with the class of piecewise linear functions with integer coefficients.*

It allows us to represent each piecewise linear function by its normal form—a formula analogous to the perfect disjunctive normal form.

The following is a fuzzy analogue of the classical Herbrand theorem.

THEOREM 2. *Let T be a fuzzy predicate calculus and $A \in F_{J(T)}$ a closed formula in prenex form. Then $T \vdash_a A$ if and only if to every $b < a$ there are $p_1, \dots, p_n$ such that $\models_d^Q p_1 A_H^{(1)} \bigtriangledown \cdots \bigtriangledown p_n A_H^{(n)}$ is a fuzzy quasitautology, where $d >^* (p_1 + \cdots + p_n)b$ and $A_H^{(i)}$ are instances of the matrix of the Herbrand existential formula A_H.*

($\bigtriangledown$ is the Łukasiewicz disjunction; $\models_a^Q A$ is a fuzzy quasitautology in the degree a; $a >^* b$ if and only if $a > b$ for $b < 1$ or $a = 1$ otherwise).

Some consequences of both results characterizing properties of the contradictory theories of fuzzy logic are demonstrated. For example, the following shows that a direct generalization of the Hilbert-Ackermann theorem (cf. [4]) is impossible.

THEOREM 3. *Let T be a fuzzy predicate calculus. Then there is a contradictory open fuzzy theory T' such that there are special axioms $A_1, \dots, A_n$, $a_i = \mathrm{Ax}^{\mathrm{S}}(A_i)$, and a fuzzy quasitautology*

$$\models_b^Q \neg\bar{A}_1 \bigtriangledown \cdots \bigtriangledown \neg\bar{A}_n$$

where $\bar{A}_i$ are instances of the special axioms and $b \leq \neg(a_1 \otimes \cdots \otimes a_n)$.

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► LUIZ CARLOS PEREIRA AND LUDMILLA FRANKLIN, *Multiple-conclusion natural deduction for intuitionistic logic*.

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Intuitionistic sequent-calculi systems are usually obtained from their classical counterparts by means of a "cardinality restriction": intuitionistic systems should forbid "multiple-consequents". This strong cardinality restriction can be replaced by local ones: unary consequences do not have to be imposed on the concept of sequent, but can rather be restricted to the application of certain inference rules. In this way the intuitionistic system LJ$'$ is obtained from Gentzen's LK. We know now that even this weak cardinality restriction is not essential, since it can be replaced by "explicit-dependency" restrictions. The intuitionistic system FIL was obtained from LK through the use of this idea. The aim of the present work is to introduce a Natural Deduction version NFIL of FIL. NFIL is a natural deduction multiple conclusion system for intuitionistic propositional logic. We prove weak normalization for NFIL and we show how NFIL can be used as an adequate intuitionistic basis in the formalization of (1) some intermediate logics, and of (2) an intuitionistic multiplicative disjunction. In the final part of the work we compare our approach to multiple conclusion natural deduction with other approaches as well as with other attempts to formalize full intuitionistic linear logic.

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► JAN VON PLATO, *Complete system of natural deduction rules and full normal form for classical propositional logic*.

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A rule of excluded middle is given that concludes the sequent $\Gamma \Longrightarrow C$ from the premises $P, \Gamma \Longrightarrow C$ and $\sim P, \Gamma \Longrightarrow C$, with P atomic and Γ and C arbitrary. Its addition to the usual contraction- and cut-free intuitionistic sequent calculus gives a classical single-succedent calculus for propositional logic in which the structural rules are admissible. The rule of excluded middle is admissible for arbitrary formulas, and P can always be taken from atoms of C.

With just one succedent formula, translation of the above to natural deduction is immediate. The new rule gives a generalization of indirect proof for atoms and a complete system of natural deduction for classical propositional logic, with fully normal derivations, disjunction included. The normal form has purely intuitionistic subderivations followed by applications of the new rule, the subderivations identifying the computational content of the classical proof. The results extend to the universal quantifier, but existence remains negatively translated.

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In linaer logic, there are special connectives ('!' and its dual '?'), often called modalities, such that given a formula F, $!F$ indicates the potentiality of using F as often as one needs (i.e., an arbitrary number of times, including zero). In the linear sequent calculus, see [2], one encounters four types of rules dealing with modally decorated formulas, the restricted weakening and contraction among others. However, none of them may serve as an explicit introduction of either '!' or '?', as opposed to all the other linear logic connectives and the respective introduction rules. Thus the linear modalities discussed in fact act as a meta-device to bookkeep the formulas with the intended particular freedom in use. And, it is this object-meta level formulation of linear logic that causes serious difficulties in constructing a natural complete semantics for it.

We feel, there might be a good way out, however. To this end, we shall below discuss affine logics with bounded contraction (i.e., an extension of linear logic) where the linear modalities are definable. To gain the cut-elimination property we shall move from ordinary sequent to the hyper-sequent formulation, along the lines of [1]. Relying on the results gathered in [3] and [4], we will finally fix the corresponding complete semantics, in terms of games and a suitably modified Whitman's free lattice construction.

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Lee considered in [1] a satisfiability notion for first-order fuzzy logic and proved that a formula ϕ is satisfiable (unsatisfiable) in fuzzy logic if and only if it is satisfiable (unsatisfiable) in two-valued classical logic. In his paper a formula ϕ was defined as valid in a model if and only if its meaning was between 0.5 and 1, and as not valid in a model if and only if its meaning was between 0 and 0.5. In our paper we present more a general result and simpler arguments to clarify why satisfiability in fuzzy logic coincides with satisfiability in two-valued logic.

The language of the first-order fuzzy logic $\mathcal{L}$ is the same as the language of classical logic. In this logic predicate symbols are interpreted by fuzzy relations and logical connectives are defined as in Zadeh's fuzzy logic [4], i.e., conjunction of two formulae $\phi \wedge \psi$, negation of a formula $\neg\phi$ and quantifier formula $(\forall x)\phi$ are interpreted by $\min(\phi, \psi)$, $1 - \phi$ and $\inf_{d \in D}\{\phi(d)\}$ respectively. We abbreviate $\neg(\neg\phi \wedge \neg\psi)$ to $\phi \vee \psi$, $\neg\phi \vee \psi$ to $\phi \to \psi$ and $\neg(\forall x)\neg\phi$ to $(\exists x)\phi$.

Let α be a number from $[0, 1]$, $\text{False} = [0, \alpha]$, $\text{True} = [1 - \alpha, 1]$. A formula ϕ is said to be α-valid in the model $\mathcal{M}$ (written $\vDash_{\mathcal{M}} \varphi \in \text{True}$) provided we have $\tau(\phi) \in \text{True}$. A formula ϕ is said to be α-not valid in the model $\mathcal{M}$ (written $\vDash_{\mathcal{M}} \phi \in \text{False}$) provided we have $\tau(\phi) \in \text{False}$. A formula ϕ is said to be α-valid just in case it is α-valid in every model. A formula ϕ is said to be α-satisfiable just in case there is a fuzzy logic model $\mathcal{M}$ in which it is α-valid. A formula ϕ is said to be α-unsatisfiable just in case for any fuzzy logic model $\mathcal{M}$ it is α-not valid.

Theorem. *Let* $0 < \alpha \leq 0.5$, *then*:

1. α-*(un)satisfiability in fuzzy logic coincides with the (un)satisfiability in two-valued logic.*
2. α-*validity in fuzzy logic coincides with the validity in two-valued logic.*

In [1] it has been proved only for the case $\alpha = 0.5$ that a first-order fuzzy logic formula ϕ is α-unsatisfiable if and only if it is unsatisfiable in two-valued logic.

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► REINERT A. RINVOLD, *Denotation systems and relations*.
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The denotation systems or dilators of Girard are generalized and defined in categories of sets with structure, CSS. Categories of sets with structure is an axiomatization of a category with structured sets as objects, and structure preserving functions as morphisms.

Equivalences between denotation systems, normal functors and functors having the normal form property, are shown for an important class of CSS. Also weak denotation systems which corresponds to functors preserving direct limits and pullbacks, but not equalizers, are defined. Similar equivalences are shown for weak denotation systems. Important examples of weak denotation systems exist in the qualitative domains of Girard.

When $(\mathcal{K}, U)$ is a CSS, the forgetful functor $U : \mathcal{K} \to \mathrm{SET}$ gives interesting examples of denotation systems. This is the functor which forgets the structure of the objects. When all morphisms of $\mathcal{K}$ are embeddings, the functions $U^n : \mathcal{K} \to \mathrm{SET}$ have denotation systems. In every CSS there is an internal language of relations and operations. A representation theorem for relations of $(\mathcal{K}, U)$ is proved. This representation theorem uses the trace of U^n and shows that we can consider the trace of this functor as the basic $\mathcal{K}$-relations of arity n.

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Consider the first-order language with equality, with one binary predicate $\prec$, and infinitely many unary predicates. Our base theory Γ is axiomatized by the transitivity axiom for $\prec$, and the persistence axiom for all unary predicates, that is Px and $x \prec y$ imply Py, for all unary P. So Γ essentially is the theory of transitive Kripke models. For this class of models a completeness theorem for Basic Propositional Calculus BPC holds. (BPC is a proper subsystem of Intuitionistic Propositional Calculus.) For each proposition-logical formula B we can find a natural first-order formula $I(B, x)$ with intended meaning that a transitive Kripke model forces B at node x. All formulas $I(B, x)$ are persistent and preserved under bisimulations. We use this fact as a starting point to syntactically characterize, up to provable equivalence, the first-order formulas satisfying versions of persistence and bisimulation. Our results generalize some previously known characterizations of this kind.

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Visser's propositional logic was first considered in Visser [2] as the propositional logic embedded into the modal logic **K4** by Gödel's translation. He gave a natural deduction system $\vdash_{\mathbf{VPL}}$ for the consequence relation of Visser's propositional logic. A main difference from the consequence relation $\vdash_{\mathbf{IPL}}$ of intuitionistic propositional logic is $\{p, p \supset q\} \nvdash_{\mathbf{VPL}} q$

while $\{p, p \supset q\} \vdash_{\mathbf{IPL}} q$. In other words, in $\vdash_{\mathbf{VPL}}$, modus ponens does not hold in general. It makes Hilbert style formalization for $\vdash_{\mathbf{VPL}}$ difficult. This difficulty was pointed out in Suzuki, Wolter and Zakharyaschev [1], where the problem was formulated to find a finite Hilbert style formalization for $\vdash_{\mathbf{VPL}}$. Here we consider formalizations for $\vdash_{\mathbf{VPL}}$ with a restricted form of modus ponens and try to solve the problem in [1].

First, we give a formalization for $\vdash_{\mathbf{VPL}}$ by using adjunction and restricted modus ponens. Let Γ be a set of formulas. We define the consequence relation $\vdash_{\mathbf{VPL}^*}$ inductively as follows:

(1) if $A \in \mathbf{A} \cup \Gamma$, then $\Gamma \vdash_{\mathbf{VPL}^*} A$,
(2) if $\Gamma \vdash_{\mathbf{VPL}^*} A$ and $\emptyset \vdash_{\mathbf{VPL}^*} A \supset B$, then $\Gamma \vdash_{\mathbf{VPL}^*} B$,
(3) if $\Gamma \vdash_{\mathbf{VPL}^*} A$ and $\Gamma \vdash_{\mathbf{VPL}^*} B$, then $\Gamma \vdash_{\mathbf{VPL}^*} A \wedge B$,

where $\mathbf{A}$ is the set of all substitution instances of the following axioms:

$$\begin{array}{lll}
(\supset_1) a \supset a, & (\supset_2) a \supset (b \supset a) & (\supset_3)(b \supset c) \wedge (a \supset b) \supset (a \supset c), \\
(\wedge_1) a \wedge b \supset a, & (\wedge_2) a \wedge b \supset b, & (\wedge_3)(c \supset a) \wedge (c \supset b) \supset (c \supset a \wedge b), \\
(\vee_1) a \supset a \vee b, & (\vee_2) b \supset a \vee b, & (\vee_3)(a \supset c) \wedge (b \supset c) \supset (a \vee b \supset c), \\
(\bot) \bot \supset a, & (D) a \wedge (b \vee c) \supset (a \wedge b) \vee (a \wedge c). &
\end{array}$$

And we prove

THEOREM. $\Gamma \vdash_{\mathbf{VPL}^*} A$ *if and only if* $\Gamma \vdash_{\mathbf{VPL}} A$.

We also consider consequence relations with only one restricted modus ponens. Let $\mathbf{S}$, $\mathbf{S}_1$ and $\mathbf{S}_2$ be sets of formulas. And let $\mathbf{MP} = \mathbf{S}_1 \times \mathbf{S}_2$. We define the consequence relation $\vdash_{\mathbf{S},\mathbf{MP}}$ inductively as follows:

(1) if $A \in \mathbf{S} \cup \Gamma$, then $\Gamma \vdash_{\mathbf{S},\mathbf{MP}} A$,
(2) for any pair $(X, Y) \in \mathbf{MP}$,

$$\text{if } \Gamma \vdash_{\mathbf{S},\mathbf{MP}} X \text{ and } \Gamma \vdash_{\mathbf{S},\mathbf{MP}} X \supset Y, \text{ then } \Gamma \vdash_{\mathbf{S},\mathbf{MP}} Y.$$

THEOREM. *There exists no pair* $(\boldsymbol{S}, \boldsymbol{MP})$ *such that*

$$\Gamma \vdash_{\mathbf{VPL}} A \quad \textit{if and only if} \quad \Gamma \vdash_{\mathbf{S},\mathbf{MP}} A.$$

From the above theorem, we can see the necessity of the inference rule (3) in the definition of $\vdash_{\mathbf{VPL}^*}$.

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► MARCUS SCHAEFER, *A guided tour of minimal indices and shortest descriptions*.

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The set of minimal indices of a Gödel numbering φ is defined as $\mathrm{MIN}_\varphi = \{e : (\forall i < e)[\varphi_i \neq \varphi_e]\}$ [2]. It was proved by Meyer [3] in 1972 that $\mathrm{MIN}_\varphi \equiv_{\mathrm{T}} \emptyset''$, but beyond this MIN_φ has remained mostly uninvestigated. This paper collects the scarce results on MIN_φ from the literature and adds some new observations including that MIN_φ is autoreducible, but neither regressive nor $(1, 2)$-computable. We also study several variants of MIN_φ that have been defined in the literature like size-minimal indices [1], shortest descriptions, and minimal indices of decision tables [4]. Some challenging problems remain open.

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► P. A. SCHREINER, *Continua of superintuitionistic predicate logics without Beth's property*.
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It is well known that there exist only seven consistent propositional superintuitionistic logics with interpolation property (L. Maksimova, 1977). G. Kreisel (1960) proved that the Beth's property is shared by all propositional superintuitionistic logics. But the problem of describing predicate superintuitionistic logics with the interpolation property or Beth's property remains open.

L. Maksimova has proved in [1] that there exists a continuum of predicate superintuitionistic logics with equality that have the interpolation property.

Let J_{fd} denote the logic which is characterized by the class of all Kripke frames whose domains are finite for each world. Let J^*_{fd} denote logic which is characterized by the class of all Kripke frames whose domains are finite for each non-maximal world.

THEOREM ([2], [3]). *J_{fd} and J^*_{fd} have neither Beth's property nor interpolation property.*

We note that the logic J^*_{fd} is the first example of predicate intermediate logic without Beth's property.

THEOREM. *Let L be any propositional superintuitionistic logic. There exists a continuum of predicate superintuitionistic logics with equality, whose propositional fragment is L, which are not contained in the classic predicate logic and have not Beth's property.*

THEOREM. *Let L be any propositional superintuitionistic logic which is contained in Dummet's logic LC. There exists a continuum of predicate intermediate logics with equality, whose propositional fragment is L, without Beth's property.*

[1] L. L. MAKSIMOVA, *Interpolation in superintuitionistic predicate logic with equality*, ***Algebra and Logic***, vol. 36 (1997), no. 5, pp. 319–329.

[2] P. A. SCHREINER, *Failure of interpolation property and Beth's property in some predicate superintuitionistic logic*, ***Algebra and Logic***, vol. 35 (1996), no. 1, pp. 59–65.

[3] ———, *Intermediate predicate logic without Beth's property*, ***Algebra and Logic***, vol. 37 (1998), no. 1, pp. 107–117.

► JOHN H. SEREMBUS, *A generalized functional completeness proof for a propositional logic*.
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The author, in [1] and [2] offered a novel proof of the functional (in)completeness of sets of connectives of a Propositional Language PL using a hexadecimal representation of the information contained in the truth tables for the binary connectives of PL he developed in [1]. He offered a decision procedure, which determines in every case, which subsets of the sixteen binary connectives of PL are adequate for expressing the remaining binary connectives of PL and which subsets are inadequate. Along the way, interesting aspects of truth tables with two variables were highlighted and explored.

In the present paper, the author extends the results of these two papers to apply to all n-ary connectives of PL (for n greater than or equal to 0). That is, the paper offers a decision procedure, which determines in every case, which subsets of the n-ary connectives of PL are adequate for expressing the remaining connectives of PL and which subsets are inadequate.

This is also accomplished by using the hexadecimal notation introduced earlier. It provides a way of correlating n-ary connectives of PL (for n other than 2) with the information contained in the hexadecimal representation of the binary connectives. Along the way, interesting aspects of truth tables in general will be highlighted and explored. For example, a procedure will be offered that will deliver definitions of any n-ary connective in terms of other n-ary connectives when such a definition exists.

[1] JOHN H. SEREMBUS, *Functional completeness revisited*, presented to the European meeting of the Association for Symbolic Logic, Vesprem, Hungary, August 9–15, 1992.

[2] ———, *A study of truth table notation of propositional logic*, presented to the Conference on Notational Engineering, George Washington University, Washington, D. C., May 23–24, 1996.

► MAMORU SHIMODA, *A natural interpretation of fuzzy sets and relations*.
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We present a new and natural interpretation of fuzzy sets and relations in a Heyting-valued model for intuitionistic set theory, where the basic notions have quite natural meanings and various properties of fuzzy sets and relations are easily obtained.

Let H be a complete Heyting algebra and $V^{(H)}$ be the H-valued model in [2]. The *Heyting-value* $\|\varphi\| \in H$ and the *check set* $\check{x}$ are defined as usual. Basic operations such as *intersection*, *union*, and *complement* of sets, *composition* and *inverse* of relations, etc., are naturally defined in $V^{(H)}$.

Every $A \in V^{(H)}$ is called an *H-fuzzy set*, and every subset of $\check{X}$ in $V^{(H)}$ is called an *H-fuzzy subset of X*. The *membership function of A on X* is the mapping $\mu_A\colon X \to H;\ x \mapsto \|\check{x} \in A\|$. Membership functions are extensions of ordinary fuzzy sets in [3] and instances of L-fuzzy sets in [1]. For $\mu, \nu\colon X \to H$, the relation $\mu \leq \nu$ and the mappings $\mu \wedge \nu,\ \mu \vee \nu,\ \neg\mu\colon X \to H$ are pointwise defined by the values.

THEOREM 1. *Let X be a set and the membership functions be defined on X.*

(1) *For every $\mu\colon X \to H$, there is an H-fuzzy subset A of X such that $\mu = \mu_A$.*
(2) *If A, B are H-fuzzy subsets of X, then $\|A \subseteq B\| = \mathbf{1}$ if and only if $\mu_A \leq \mu_B$.*
(3) *If A, B are H-fuzzy sets, then $\mu_{A\cap B} = \mu_A \wedge \mu_B,\ \mu_{A\cup B} = \mu_A \vee \mu_B$, and $\mu_{\check{X}\setminus A} = \neg\mu_A$.*

The theorem shows that there is a natural correspondence between H-fuzzy subsets of X and mappings from X to H, which preserves order and basic set operations. If $R \in V^{(H)}$ is an H-fuzzy subset of $X \times Y$, R is called an *H-fuzzy relation from X to Y* and we write $R \sqsubseteq_H X \times Y$.

THEOREM 2. *Let the membership functions be defined on the suitable cartesian products.*

(1) *If $R \sqsubseteq_H X \times Y$, then $R^{-1} \sqsubseteq_H Y \times X$, and $\mu_{R^{-1}}\langle yx\rangle = \mu_R\langle xy\rangle$ $(\forall x \in X,\ \forall y \in Y)$.*
(2) *If $R \sqsubseteq_H X \times Y$ and $S \sqsubseteq_H Y \times Z$, then $S \circ R \sqsubseteq_H X \times Z$ and $\mu_{S\circ R}\langle xz\rangle = \bigvee_{y\in Y}(\mu_R\langle xy\rangle \wedge \mu_S\langle yz\rangle)$ $(\forall x \in X,\ \forall z \in Z)$.*
(3) *If $R \sqsubseteq_H X \times X$, then R is reflexive if and only if $\mu_R\langle xx\rangle = \mathbf{1}$ $(\forall x \in X)$, symmetric if and only if $\mu_R\langle xy\rangle = \mu_R\langle yx\rangle$ $(\forall x, y \in X)$, and transitive if and only if $\mu_R\langle xy\rangle \wedge \mu_R\langle yz\rangle \leq \mu_R\langle xz\rangle$ $(\forall x, y, z \in X)$.*

[1] J. A. GOGUEN, *L-fuzzy sets*, ***Journal of Mathematical Analysis and Applications***, vol. 18 (1967), no. 1, pp. 145–174.

[2] M. SHIMODA, *Categorical aspects of Heyting-valued models for intuitionistic set theory*, ***Commentarii Mathematici Universitatis Sancti Pauli***, vol. 30 (1981), no. 1, pp. 17–35.

[3] L. A. ZADEH, *Fuzzy sets*, ***Information and Control***, vol. 8 (1965), no. 3, pp. 338–353.

[4] ———, *Similarity relations and fuzzy orderings*, ***Information Sciences***, vol. 3 (1971), no. 2, pp. 177–200.

► NOEMIE SLAATS AND ALBERT HOOGEWIJS, *Semantic tableaux for LPF and the adequacy theorem*.
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In this talk we introduce a semantic tableaux method for the three-valued Logic for Partial Functions (LPF) [1] and sketch the adequacy theorem for a reduced predicate calculus.

LPF is a three-valued predicate calculus with the truth-tables for the logical *negation* and *disjunctions* corresponding to Kleene's notation. The logical operators are extended with a non-monotone operator Δ to express the *definedness* of a fomrula. The equality is considered to be weak equality.

	$\neg$	Δ
T	F	T
F	T	T
U	U	F

$\vee$	T	F	U
T	T	T	T
F	T	F	U
U	T	U	U

$=$	P	Q	U
P	T	F	U
Q	F	T	U
U	T	U	U

The alphabet also contains the existential binder $\exists$ and the symbol ':' for declaring the type of a term.

To introduce the notion of *semantic tableau*, we need the definition of *validity* and *consequence*. In LPF, a valuation is a mapping $\mathcal{V}\colon \text{Form}_{\text{LPF}} \mapsto \{T, F, U\}$ with Form_{LPF} being the well-formed formulae of LPF. $\mathcal{M}$ is a *model* for a list of formulae L if and only if for all $\gamma \in L$: $\mathcal{V}(\gamma) \equiv T$. A formula α is *valid* for a model $\mathcal{M}$ ($\mathcal{M} \vDash \alpha$) if and only if $\mathcal{V}(\alpha) \equiv T$. A formula α is a *consequence* of a list of formulae L ($L \vDash \alpha$) if and only if for all models $\mathcal{M}$ of L, $\mathcal{M} \vDash \alpha$. A list of formulae R is a consequence of a list of formulae L ($L \vDash R$) if and only if for at least one formula $\alpha \in R$, $L \vDash \alpha$. Let $L \rhd R$ denote a *sequent*. A model $\mathcal{M}$ is a *counterexample* for a sequent $L \rhd R$ if and only if $\mathcal{M} \vDash \alpha$ for all $\alpha \in L$ and $\mathcal{M} \nvDash \beta$ for all $\beta \in R$ (note that this means $\mathcal{V}(\beta) \in \{F, U\}$ for all $\beta \in R$). We introduce reduction rules for non-atomic and non-Δ-atomic formulae, to transform the problem of finding a counterexample for a sequent, into the problem of finding a counterexample for a sequent with formulae of reduced complexity. The resulting tableau of sequents is called a *semantic tableau*. If every branch of the tableau ends with the application of a *closure rule*, then the tableau is *closed*, else the tableau is *open*.

The adequacy theorem shows that $L \vDash R$ if and only if $L \rhd R$ is the top-sequent of a closed semantic tableau.

[1] C. Jones and C. Middelburg, *A typed logic of partial functions reconstructed classically*, ***Acta Informatica***, vol. 31 (1994), pp. 399–430.

[2] C. Jones, K. Jones, P. Lindsay, and R. Moore, ***Mural: A formal development support system***, Springer-Verlag, 1991.

► OTMAR SPINAS, *Ramsey theorems for Polish planes*.
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Suppose X is a Polish space which is not K_σ. We prove that for every Borel colouring of X^2 by countably many colours, there exists a rectangle with both sides closed and not K_σ, whose all points have the same colour. We also prove that every Borel colouring of $[X]^2$ by finitely many colours has a homogeneous set which is closed and not K_σ. As a corollary to the proofs we obtain that the product forcing of two copies of superperfect tree forcing does not add a Cohen real. This answers negatively a question of Goldstern.

The methods of proof can also be used to answer questions of Brendle about the existence of superperfect free sets for Borel functions on the plane and of superperfect sets of mutually non-constructible reals.

► D. E. TISHKOVSKY, *On algebraic counterpart of Beth property of superintuitionistic predicate logics*.
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A notion of quasicylindric algebra was defined in [1]. For each superintuitionistic predicate logic L there was constructed a variety $V(L)$ of quasicylindric algebras. There was also proved that L is strong complete with respect to $V(L)$. The aim of the paper is to translate uniformly Beth Property and Projective Beth Property of each superintuitionistic predicate logic L to the language of $V(L)$.

Logic L has Beth Property if and only if for each formula $A(p, q_1, \dots, q_k)$ (where p, q_1, …, q_k are all predicate symbols in A) the condition

$$A(p, q_1, \dots, q_k), A(p', q_1, \dots, q_k) \vdash_L \forall x_0 \dots \forall x_{m-1}\, (p(x_0, \dots, x_{m-1}) \equiv p'(x_0, \dots, x_{m-1}))$$

implies the existence of a formula $B(q_1, \dots, q_k)$ such that

$$A(p, q_1, \dots, q_k) \vdash_L \forall x_0 \dots \forall x_{m-1}\, (p(x_0, \dots, x_{m-1}) \equiv B).$$

Logic L has Projective Beth Property if and only if for each formula $A(p, q_1, \dots, q_k, r_1, \dots, r_l)$ (where p, q_1, …, q_k, r_1, …, r_l are all different predicate symbols in A) the condition

$$A(p, q_1, \dots, q_k, r_1, \dots, r_l), A(p', q_1, \dots, q_k, r_1', \dots, r_l') \vdash_L \forall x_0 \dots \forall x_{m-1}\, (p(x_0, \dots, x_{m-1}) \equiv p'(x_0, \dots, x_{m-1}))$$

implies the existence of a formula $B(q_1, \dots, q_k)$ such that

$$A(p, q_1, \dots, q_k, r_1, \dots, r_l) \vdash_L \forall x_0 \dots \forall x_{m-1}\, (p(x_0, \dots, x_{m-1}) \equiv B).$$

A class K of algebras has ES^* if and only if for each algebras $\mathcal{A}, \mathcal{B} \in K$ if $\mathcal{A}$ is subalgebra of $\mathcal{B}$, $b \in |\mathcal{B}| - |\mathcal{A}|$ and the set $|\mathcal{A}| \cup \{b\}$ generate $\mathcal{B}$ then there exist an algebra $\mathcal{C}$ in K and homomorphisms $g, h\colon \mathcal{B} \to \mathcal{C}$ such that $g|_{\mathcal{A}} = h|_{\mathcal{A}}$ and $gb \neq hb$.

A class K of algebras has *SES* if and only if for each algebras $\mathcal{A}, \mathcal{B} \in K$ if $\mathcal{A}$ is subalgebra of $\mathcal{B}$ and $b \in |\mathcal{B}| - |\mathcal{A}|$ then there exist algebra $\mathcal{C}$ in K and homomorphisms $g, h\colon \mathcal{B} \to \mathcal{C}$ such that $g|_{\mathcal{A}} = h|_{\mathcal{A}}$ and $gb \neq hb$.

THEOREM. *Let L be a superintuitionistic predicate logic. The following equivalences hold*:

1. *L has Beth Property if and only if $V(L)$ has ES^**;
2. *L has Projective Beth Property if and only if $V(L)$ has SES.*

[1] D. E. TISHKOVSKY, *On algebraic semantics for superintuitionistic predicate logic*, in ***Minisemester "Logic, algebra and computer science" (Helena Rasiowa in memoriam)***, Warsaw, December 1996.

► CARLO TOFFALORI, *o-minimality and expansions of Boolean algebras*.
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We study a notion of o-minimality for partially ordered structures, in particular for lattice ordered structures expanding Boolean algebras. This notion generalizes in a natural way the well known definition for totally ordered structures. It is easy to see that some significant properties of totally ordered o-minimal models do not hold in this extended setting; for instance, the Exchange Lemma fails. However we show the existence of prime models over arbitrary subsets, and we characterize ω-categorical o-minimal structures. Finally we classify o-minimal structures among Boolean algebras and measure spaces. In the former case we show that the o-minimal Boolean algebras are just those admitting only finitely many atoms. In the latter, we see that a measure-space (A, F, m) (where A is a Boolean algebra, F is an ordered field and m is the measure function) is o-minimal if and only if both A and F are (so A has only finitely many atoms and F is real closed) and m is "almost everywhere" 0.

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BL-algebras were introduced by Hájek [2] in order to provide an algebraic proof for the completeness theorem of a class of $[0, 1]$-valued logics familiar in fuzzy logic framework. BL-algebras rise as Lindenbaum algebras from certain logical axioms in a similar manner as MV-algebras (cf. [1]) do from the axioms of Łukasiewicz logic. In fact, a BL-algebra becomes an MV-algebra if we adjoin to the axioms the double negation law $x = x^{**}$. Thus, BL-algebras are in some intuitive sense 'non-double negation MV-algebras'. Hence, the theory of MV-algebras, now in the scope of intensive investigation, becomes one of the guides to the development of the theory of BL-algebras. Generalizations from MV-algebra theory to BL-algebra theory also shows us which concepts are proper and genuine BL-algebra concepts and which are not. In [4] we started this analysis and proved, e.g., that semi-simple BL-algebras are necessarily MV-algebras, thus semi-simplicity is not a proper BL-algebra concept. The same holds for locally finite BL-algebras, too.

Our basic tools in the investigation of a BL-algebra L are deductive systems, i.e., subsystems D of L such that $\mathbf{1} \in D$ and if $x, x \to y \in D$ then $y \in D$. In particular, in MV-algebra theory deductive systems (also called filters in the literature) and ideals are dual notions.

In this paper we study Boolean deductive systems and implicative deductive systems. It turns out that these deductive systems coincide. We prove that a BL-algebra L has a proper Boolean deductive system if and only if L is bipartite, moreover, there are BL-algebras which are bipartite and are not MV-algebras. We also study local BL-algebras and prove more generally some results which are known to hold for MV-algebras. Finally, we show that there are local BL-algebras which are not MV-algebras.

[1] C. C. CHANG, *Algebraic analysis of many-valued logics*, ***Transactions of the American Mathematical Society***, vol. 88 (1958), pp. 467–490.

[2] P. HÁJEK, *Metamathematics of fuzzy logic*, ***Technical report 682***, Institute of Computer Science, Academy of Science of the Czech Republic, 1996.

[3] C. S. HOO, *MV-algebras, ideals and semisimplicity*, ***Mathematica Japonica***, vol. 34 (1989), pp. 563–583.

[4] E. TURUNEN, *BL-algebras and basic fuzzy logic*, submitted.

► HELMUT VEITH, *Interpolation in fuzzy logic*.
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We survey results about interpolation in fuzzy and many-valued propositional logics [1, 4, 6] whose truth functions are defined by continuous triangular norms [3]; continuous triangular norms have been characterized as cardinal sums of the triangular norms in Łukasiewicz, Gödel and Product Logic [5].

Emphasis is given on elimination of fuzzy quantifiers (i.e., supremum and infimum quantifiers); fuzzy quantifiers extend the known correspondence between quantified Boolean formulas and uniform interpolation from classical to fuzzy truth values.

In case of failure of interpolation, we characterize minimal interpolating extensions of the languages. For finite-valued logics, we count the number of interpolating extensions by Fibonacci and related sequences [2].

[1] M. BAAZ and H. VEITH, *Interpolation in fuzzy logic*, ***Archive of Mathematical Logic***, to appear.

[2] H. W. GOULD, *Binomial coefficients, the bracket function, and compositions with relatively prime summands*, ***Fibonacci Quarterly***, vol. 2 (1964), no. 4, pp. 241–260.

[3] P. HÁJEK, ***Metamathematics of fuzzy logic***, book in preparation.

[4] P. S. KRYSTEK and S. ZACHOROWSKI, *Łukasiewicz logics have not the interpolation property*, ***Reports on Mathematical Logic***, vol. 9 (1977), pp. 39–40.

[5] H. C. LING, *Representation of associative functions*, ***Publicationes Mathematicae Debrecen***, vol. 12 (1965), pp. 182–212.

[6] L. MAKSIMOVA, *Craig's interpolation theorem and amalgamable varieties*, ***Doklady Akademii Nauk SSSR***, vol. 237 (1977), no. 6, pp. 1281–1284.

► WIM VELDMAN, *The Borel hierarchy theorem in intuitionistic mathematics*.
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In constructive mathematics one defines the class of the Borel subsets of a Polish space as the least class containing the closed and the open sets that is closed under the operations of countable union and countable intersection. The question if the resulting class forms a "real" hierarchy is nontrivial. How to prove, for instance, that some intersections of countably many open sets like the subset of $\mathbb{R}$ consisting of the positively irrational numbers do not coincide with any union of countably many closed sets? Brouwer, although using another example, gave such a proof. He invokes an axiom of intuitionistic analysis, called (by Kleene) Brouwer's principle or the continuity principle, that is unacceptable for classical mathematicians and even for some constructivists. We show that this continuity principle, together with an axiom of countable choice, enables one to prove, with some effort, an intuitionistic Borel hierarchy theorem.

► PETER VOJTÁŠ, *Fuzzy logic programming*.
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We present a mathematical model for fuzzy logic programming. The language can contain several different conjunctions, disjunctions and implications. This connectives are flexible subject of change in order to fit real data. Rules are implications equipped with a truth value. We base our procedural semantics on many-valued modus ponens. We generalize classical fixpoint theory to this case and prove some completeness results.

► MICHAEL B. WRIGLEY, *Wittgenstein's pre-"Tractatus" philosophy of mathematics*.
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Wittgenstein was first drawn into philosophy by the ideas of Frege and Russell, in particular their logicist philosophy of mathematics. Yet almost from the beginning, he identified fundamental problems with this account of mathematics, and in the *Tractatus* presents an account of mathematics that is very different from logicism. In this paper I focus in detail on the period before the *Tractatus*, and show that there is clear evidence that although he criticized the specific versions of logicism put forward by Frege and by Russell, Wittgenstein himself continued to accept the basic logicist thesis, that mathematics is part of logic, throughout this period. In particular, although by November 1913 Wittgenstein had already seen clearly that Russell's Axiom of Infinite and Axiom of Reducibility are not logical propositions, a careful analysis of the *Notebooks* and other pre-*Tractatus* texts shows clearly that he did not take this to undermine the basic logicist thesis, but only the specific version proposed by Russell. I argue that as late as June 1915 there is clear evidence of Wittgenstein's continued adherence to logicism, and indeed in the surviving pre-*Tractatus* texts, the latest of which dates from January 1917, there is no clear, unequivocal evidence of a rejection of logicism. I chart the modifications of Wittgenstein's version of logicism and conclude by making some suggestions concerning how he came to finally reject logicism completely and propose a radically different account of mathematics in the *Tractatus*.

Abstracts presented by title

► V. R. BAIRASHEVA, *One example of applying the priority method in the theory of finite-state automata*.

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Following Rayna [1], we call two infinite words $X = \{x_i\}_{i=0}^{\infty}$ and $Y = \{y_i\}_{i=0}^{\infty}$ "equivalent by automata (being involved in one degree of automaton reducibility)", if there exist two initial finite-state automata (T_1, s_1) and (T_2, s_2) so that automaton (T_1, s_1) puts out the infinite words Y (maybe with some delay) if getting X as an input value, and automaton (T_2, s_2) puts out the infinite word X (also with possible delay) if getting Y as an input.

The partially ordered set V of degrees of automaton transformation is being naturally generated.

Referring to [1], there is the minimal element of V which is the degree consisting of periodic infinite word. Also it has shown the existence of atoms in the set V. Studying atoms of V has been continued in some other works (see [2] showing the existence of atoms with different properties).

Following [3], let us define logical metalanguage $\mathcal{U}$. Atom formulae of language $\mathcal{U}$ are $x(n) = a$, where x is a variable for infinite words, n is a variable for numbers and a is a constant-letter of some alphabet. Atom formula means the next statement: infinite word x has a letter a in the case of nth occurrence. Formulae of language $\mathcal{U}$ are being constructed of atom ones using rules of predicate calculus, moreover quantifiers can be applied to both types of variables.

The set of all formulae of language $\mathcal{U}$ containing one variable x for infinite word that gets true in case of substitution of an infinite word X for variable x, will be called "monadic theory of infinite word X" [4].

THEOREM. *There exists decidable infinite word with undecidable monadic theory.*

This infinite word belongs to the atom of structure of V.

Proof synthesizes Rayna's construction [1] of atom building with use of priority method developed in the recursive function theory. (It maintains undecidability of monadic theory of infinite word being decidable.)

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[1] G. RAYNA, *Degrees of finite-state transformability*, ***Information and Control***, vol. 24 (1974), pp. 144–154.

[2] V. R. BAIRASHEVA, *Degrees of finite-state transformability of random sequences*, ***Ph.D. Thesis***, Kazan, 1990.

[3] B. A. TRAHTENBROT and IA. M. BARZDIN', ***Finite-state automata***, Nauka, Moscow, 1970.

[4] L. M. SEMENOV, *Logical theory of one-place functions on the set of natural numbers*,

Izvestie Akademiya Nauk SSSR, Seriya Matematicheskaya, vol. 47 (1983), no. 3, pp. 1175–1195.

► OLGA N. BUSHMAKINA, *Problem of metalogical basis of being in contemporary philosophy*.
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Hermeneutics and postmodernism are offered the problem of basis of being in two different approaches. Absolute positiveness of being is determined through the ontological difference or ambiguousness of being. Being is divided into the being as reality and being as notion. Transcendental basis is united being into whole, but it is eliminated out the limits of knowledge's system. The basis of system is stated metalogical. At postmodernism being as absence (vacant) exists as absolute negativeness until any possibility of mentality. We do not think it. It exists as metalogical one. Absolute negativeness is determined over the absolute positiveness, and on contrary. The basis of knowledge's system is eliminated out of its limits. The logical basis of system is not over. If system have itself basis, then it is over. Basic principle must be undoubted, or tautological. It exists into limits of this system. Paradoxical affirm unites positive and negative statements together, and therefore it is eliminated out of the limits of knowledge's system. Tautology affirms basis and except moreover. The being as basic principle of system exists as tautological principle. It is identity with existence and mentality. The being as mentality may be moved "from" itself "to" itself "into" itself. Indefinite existence of being may be demonstrated as Dasein or being into "place". The being of jointness is coincidence of being itself. The location of being exists as "place-of-determination". For description of being as whole "place-of-determination" must exists as final infinite and as defnite indefiniteness, that is point. Self-coincidence of being in point is realised, when it is vacant and full simultaneously. The being passes as full through the "pierceness" of being as vacant one identity of being as full one and being as vacant one as are affirmed into self-coincidence of being as self-definition. The being is self-defined without transcendental entity. It is determined, when it passed through the its "pierceness" ("chink"). Therefore, basis of being exists into limits itself. Knowledge's system becomes logical discourse.

► MARCELO E. CONIGLIO, *An extension of categorical semantics*.
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In the usual categorical semantics (in the sense of [4]) the constants of sort A are interpreted as elements $\mathbf{1} \to M(A)$. In $\mathbf{Sh}(\mathcal{H})$, this gives a global section of $M(A)$; therefore, a sheaf B has elements (constants) only if $EB = \top$. So we cannot syntactically describe an algebraic theory in a category in which the objects with structure have different extents.

We propose here an extension of categorical semantics by considering the *extent* of the objects. This will be accomplished by the definition of a subobject $E(B)$ of $\mathbf{1}$ for each object B of a logical category $\mathcal{C}$. If $\mathcal{C} = \mathbf{Set}$, then $E(B) = \mathbf{1}$ if B is nonempty, and $E(B) = \emptyset$ if $B = \emptyset$; if $\mathcal{C} = \mathbf{Sh}(\mathcal{H})$, then $E(B)$ will be the restriction of $\mathbf{1}$ to EB, identified in a natural way with EB.

Definition. Given an object B in a logical category $\mathcal{C}$, and $b : B \to \mathbf{1}$, we define the extent of B as $E(B) = \exists_b(B)$, i.e., the image of B under b. $\mathrm{el}(B) = \mathrm{Hom}(E(B), B)$ will denote the elements of B.

In $\mathbf{Sh}(\mathcal{H})$, $\mathrm{el}(B)$ is the set of global sections of B. We have that $E(M(X))$ is the interpretation of $(\exists x)(x = x)$ if $x : X$. By interpreting a constant $a : A$ as $M(a)$ in el$(M(A))$, we have the following consequences:

1. The constants must be considered in the contexts of interpretation. So, if $Q = (x, \dots, z; a, \dots, d)$ is a context with $x : X, \dots, d : D$, then $M(Q) = M(X) \times \cdots \times M(Z) \times (E(M(A)) \wedge \cdots \wedge E(M(D)))$.
2. The provisos about the preservation of all the free variables in the rules of the logic G (cf. [4]) must be extended to constants. In fact, it suffices to require the preservation of the sorts of the free variables and constants, and ($\Longrightarrow \exists$) has no proviso if the left-side of the sequents are empty.
3. We can now distinguish between the sequents $\top \Longrightarrow F$ and $\Longrightarrow F$. The first keeps its original meaning; the second denotes that the finite set of formulas F reaches its greatest value GV(F, Q), which is defined recursively in the obvious way according to the syntax of the formulas in F (Q is the context of F formed by all the free variables and constants occurring in F).

This distinction makes the sequents $\Longrightarrow (\exists x)(x = x)$ valid (and provable) as intended (although it is not true for $\top \Longrightarrow (\exists x)(x = x)$), since we have:

THEOREM. *With the new definitions, a sequent $m := F \Longrightarrow F'$ is valid in all the models of a theory* (*set of sequents*) *K if and only if m is provable from K in the modified system G'.*

Thus we have a conservative extension which expands the horizon of categorical logic to a more comprehensive class of objects. That is, we have enriched the class of models by a simultaneous refinement, at the syntactic and semantic levels. The categorical semantics with extents has been successfully used in the construction of the logic for sheaves over quantales (cf. [1]).

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► CARLOS AUGUSTO DI PRISCO, *Preservation of partition properties of $L(\mathbb{R})$*.
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This is a progress report on work done in collaboration with Stevo Todorcevic. We study several kinds of partition relations defined in structures related to the real line. Although the most general versions of these partition relations contradict the axiom of choice, some of them are consistent with the existence of ultrafilters on ω. The approach followed is to examine the preservation of properties of $L(\mathbb{R})$ to a generic extension obtained by adding an ultrafilter on ω. In some cases the preservation result is equivalent to a parametrized partition property.

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Bishop easily defined the inverse number x^{-1} of a single number x when it is apart from 0 (in symbols, $x \# 0$) [2, p. 21]. However, he was unable to decide for whatsoever number x not equal to 0 whether there exists its inverse number. In order to develop constructive algebra—in particular, for defining the inverse element—the dominant Bishop's school relies on this notion of apartness [10]. However, some constructivists define respectively "apartness" as the mere Brouwer's definition, and "tight apartness" as including the following property (ii) [10, pp. 8, 30], [3, p. 7]. From a historical viewpoint, this notion was suggested by Brouwer and then reiterated by Heyting: "Definition 1: For real number-generators a and b, a lies apart from b, means that n and k can be found such that $|a_{n+p} - b_{u+p}| > 1/k$ for every p." [9, p. 19] Brouwer first [4] and then both Heyting [9, p. 17] and Dummett [7, pp. 40–41] "proved" the following three properties of this notion: "(i) If $a \# b$, $a = b$ is impossible. (ii) If $a \# b$ is impossible, $a = b$. (iii) If $a \# b$, then for any element c of S, either $a \# c$, or $b \# c$." [3, p. 49] By the definition, property (i) is trivial. On the contrary, a counterexample for invalidating property (iii) is easily given by means of a "fugitive" property. Let the number c be $c = 0, c_1 c_2 \ldots$, where the digit c_i is defined as a_i if Goldbach's conjecture holds true for all $n < i$, otherwise it is b_i. Moreover, property (ii) too appears to be a nonconstructive one; by negating a positive notion—the existence of "a k such that ..."—it appeals to a (negative) existence proof which is possible in few cases only (it leaves out fugitive numbers). As a fact, both "proofs" of such a property argue on an $n°$-approximation depending on a particular k; then, they state that "for each k we can find such an $n°$"; [9, p. 17], [7, p. 41] yet, this statement by ranging on all k, implies an unbounded capability of calculation; whereas the whole argument appeals to the excluded middle law about "each $n°$". (Moreover, Heyting's Th. 2.2.2 implies the double negation law.) In sum, the common notion of apartness is an essentially non-constructive notion. Rather, by involving a quantifier only, both properties (ii) and (iii) pertain to a non-constructive mathematics—as Weyl's, which is bounded in such a way. [8] However, apartness is an ambiguous notion [11] and moreover it is not a so much efficient notion for finding out counterparts of theorems of classical group theory. [1] Thus, the question arises: How to develop a constructive group theory? Past history suggests some answers: (i) to require the existence of the inverse elements by means of some physical, external condition, as the inventor of both the technique and the method of group theory—Lazare Carnot—did in his books of mechanics; [5], [6] (ii) to consider—as Bishop did—Abelian compact groups; where inverse elements results from a continuous mapping $x \to x^{-1}$, which disregards a single element; [2, p. 299] (iii) to consider those groups only where the inverse element is trivially obtained, i.e., either groups on decidable elements—as those interesting Galois—or additive groups—as one-parameter Lorentz group is. In sum, constructive algebra seems to need ingenuity in an essential way.

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The talk gives the essentials of a second-order-logic-based explanation of modal incompleteness. The leading concept is that modal incompleteness is to be explained in terms of the incompleteness of standard second-order logic, since modal language is basically a second-order language. I investigate a normal incomplete sentential modal system due to Van Benthem, and address both the formal and the philosophical facets of modal incompleteness from the vantage point that modal systems can be analyzed in terms of structures with a domain of second-order individuals (subsets) that are assigned under an interpretation to propositional variables within languages of sentential modal logic. Quine's animadversions upon second-order logic and in particular his views that second-order logic is 'set theory in sheep's clothing' are then examined. Against Quine's stance I will seek to show that a vindication of second-order logic can be gotten provided a proper due is given to the sharp distinction between the logical (Fregean) notion of set which is the concern of second-order logic and the iterative notion of set which lies within the realm of set theory.

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We introduce finite type extensions TUB and $\mathrm{TUB}^* \supset \mathrm{TUB}$ of (a function variable version of) the second-order system ACA_0 which are based on strong principles of uniform boundedness UB. Whereas TUB is a subsystem of classical simple type theory, TUB^* proves also non-classical principles like the uniform continuity of every functional $2^{\mathbb{N}} \to \mathbb{N}$. Nevertheless both systems have a proof-theoretic interpretation in Gödel's calculus T in the sense that every provable Π^1_1-sentence has a no-counterexample interpretation by functionals of type level ≤ 2 in T which can be extracted from a given proof (see also [1], [2]).

TUB^* allows us to carry out classical analysis of continuous functions between Polish spaces in a very easy way avoiding complicated codings since continuity conditions (and moduli of continuity, etc.) need not to be formalized.

We also consider subsystems TUB^*_n of TUB^* (based on some restrictions of UB, see [1]) starting from a system TUB^*_0 whose provably recursive functions are bounded by polynomials (see [3] for the latter). For $n \geq 1$ the provably recursive functionals of type ≤ 2 of TUB^*_n are just the ones definable in the fragment T_{n-1} of T with recursion up to type $n-1$ only.

Principles of uniform boundedness allow us to give short proofs of analytical principles with restricted use of arithmetical comprehension (taking specific instances of the principles into account). Proofs relying on such instances can then be analyzed by proof-theoretic methods resulting in a faithful description of the provably recursive function(al)s of the particular instances of these principles used in the given proof. Many analytical principles can be obtained already in TUB^*_0.

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The concept of majorant-computability over the reals which integrates methods and ideas of continuous mathematics and the modern mathematical logic is investigated. A characteristic property of this approach is that we do not use terminating algorithms in the definitions. The result of computation is defined by a nonterminating process computing approximations closer and closer to the result. This approach does not depend on the way of representing the reals. The use of nonstandard models of the first-order theory of the reals enables us to investigate properties of computability of partial real-valued functions. In our approach, the majorant-computable functions include an interesting class of real-valued total functions that possess meromorphic extensions onto $\mathbb{C}$. This class, in particular, contains functions that are solutions to known differential equations.

Let $\langle \mathbb{R}, 0, 1, +, \cdot, \leq \rangle$ be the standard model of the reals, denoted also by $\mathbb{R}$. Bold face indicates sequences, in particular $\mathbf{x} = x_1, \ldots, x_n$, $\mathbf{y} = y_1, \ldots, y_k$. We use the definitions of the set of hereditarily finite sets $\mathrm{HF}(M)$ over a model $\mathbf{M}$, and the definitions of Σ-, Π-definability proposed in [1]. We recall the notion of majorant-computability for real-valued functions (see [5]).

Definition. A function $f : \mathbb{R}^n \to \mathbb{R}$ is called *majorant-computable* if there exist effective sequences of Σ-formulas $\{\Phi_s(a, \mathbf{x}, y)\}_{s\in\omega}$ and $\{G_s(a, \mathbf{x}, y)\}_{s\in\omega}$, with a parameter a, an elementary proper extension $\bar{\mathbb{R}}$ of $\mathbb{R}$ such that the following conditions hold.

1. There exists $t \in \bar{\mathbb{R}}$ such that $t > n$ for every natural number n;
2. For all $s \in \omega$, the formulas $\Phi_s(t, \mathbf{x}, y)$ and $G_s(t, \mathbf{x}, y)$ define total functions $f_s : \bar{\mathbb{R}}^n \to \bar{\mathbb{R}}$ and $g_s : \bar{\mathbb{R}}^n \to \bar{\mathbb{R}}$;
3. For all $\mathbf{x} \in \bar{\mathbb{R}}^n$, the sequence $\{f_s(\mathbf{x})\}_{s\in\omega}$ of the functions defined in 2 monotonically increases; the sequence $\{g_s(\mathbf{x})\}_{s\in\omega}$ monotonically decreases;
4. For all $s \in \omega, \mathbf{x} \in \mathrm{dom}(f)$, $f_s(\mathbf{x}) \leq f(\mathbf{x}) \leq g_s(\mathbf{x})$ and, for all $\mathbf{x} \in \bar{\mathbb{R}}^n$, $f_s(\mathbf{x}) \leq g_s(\mathbf{x})$;
5. $f(\mathbf{x}) = y \leftrightarrow \lim_{s\to\infty} \mathrm{sp}(f_s(\mathbf{x})) = y$ and $\lim_{s\to\infty} \mathrm{sp}(g_s(\mathbf{x})) = y$.

The following theorem connects the graph of a majorant-computable function with validity of a finite formula in the set of hereditarily finite sets, $\mathbf{HF}(\bar{\mathbb{R}})$ (where $\bar{\mathbb{R}}$ is an elementary proper extension of the standard real numbers).

Theorem 1. *For all functions $f : \mathbb{R}^n \to \mathbb{R}$, the following assertions are equivalent*:

1. *The function f is majorant-computable.*
2. *There exist a prime extension $\bar{\mathbb{R}} \succ \mathbb{R}$ and a Π-formula that determines the function F in the model $\boldsymbol{HF}(\bar{\mathbb{R}})$ with the property $F|_{\mathbb{R}} = f$.*
3. *There exists a Π-formula that in any elementary proper extension $\bar{\mathbb{R}} \succ \mathbb{R}$ determines the function F with the property $F|_{\mathbb{R}} = f$.*

For real-valued functions let us denote the class of Σ-*definable* functions as $\mathbf{\Sigma}$, the class of computable functions that introduced by Moschovakis (see [4]) as **M**, class of computable functions that introduced by Blum, Shib and Smale (see [2]) as **BSS**, class of computable functions that introduced by Pour-El and Richards (see [5]) as **PR**, class of computable functions that introduced by Edalat and Sünderhauf (see [3]) as **ES**, class of majorant-computable functions as **M.-C**.

THEOREM 2.

1. *For continuous total real-valued functions, we have the following inclusions*:

$$\mathbf{BSS} \subset \mathbf{M} = \mathbf{\Sigma} \subset \mathbf{PR} = \mathbf{ES} = \mathbf{M}.\text{-}\mathbf{C}.$$

2. *For partial real-valued functions, we have the following inclusions*:

$$\mathbf{BSS} \subset \mathbf{M} = \mathbf{\Sigma} \subset \mathbf{ES} \subset \mathbf{M}.\text{-}\mathbf{C}.$$

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► SERGEJ MARDAEV, *Least fixed points of modal formulas*.
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A modal formula $\varphi(p, q_1, \dots, q_n)$ is called positive (modalized) in p when each occurrence of p in φ is positive (within the scope of modality).

The well-known Fixed Point Theorem [1] for the logic GL states that each modalized in p formula have unique and definable fixpoint in GL. GL is characterized by strictly partially ordered Kripke models with the ascending chain condition.

A strictly partially ordered Kripke frame $\langle W, < \rangle$ is called an SC-frame when each infinite ascending chain $x_1 < x_2 < \dots$ is cofinal, i.e., $\forall a \in W\ \exists n\ a < x_n$.

EXAMPLES. Each strictly partially ordered Kripke frame with the ascending chain condition is an SC-frame. The natural numbers $\langle \mathbb{N}, < \rangle$ and the integers $\langle \mathbb{Z}, < \rangle$ are SC-frames.

THEOREM. *For each positive in p formula $\varphi(p, q_1, \dots, q_n)$ there exists a formula $\omega(q_1, \dots, q_n)$ which defines the least fixed point of the formula φ in each SC-model.*

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► ALEKSEI V. MOLCHANOV, *On endomorphism semigroups of weak p-hypergraphs*.
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A hypergraph [1] H is called weak p-hypergraph if

(1) any vertex of H is contained at least in one of its edges,
(2) any p vertices of H are contained in not more than one edge,
(3) any edge of H contains at least $p + 1$ vertices,
(4) there exist $p + 1$ vertices of H which don't belong to an edge.

For example, any p-hypergraph [2, 3] is a weak p-hypergraph, any affine plane and any projective plane are weak 2-hypergraphs too.

The purpose of this talk is to state recent results of investigation of the interplay between weak p-hypergraphs and its endomorphism semigroups. Main idea of our approach is to study the concrete characterization problem for these semigroups. This result permits us to construct a relatively elementary interpretation of weak p-hypergraphs in semigroups and obtain the following results:

(1) we prove that any weak p-hypergraph H is determined up to isomorphism by its endomorphism semigroup End H,
(2) we investigate the abstract characterization problem for the endomorphism semigroups of weak p-hypergraphs,
(3) we describe weak p-hypergraphs for which the endomorphism semigroups are elementary equivalent,
(4) we study a connection between unsolvable elementary theories of weak p-hypergraphs and semigroups.

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► VLADIMIR A. MOLCHANOV, *Axiomatization of classes of algebraic systems with the help of nonstandard analysis methods*.
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The first results in topological model theory obtained by A. Mal'cev [2] and numerous investigations of the Eilenberg's correspondence [1] between varieties of finite semigroups, rational languages and automata show that the usual language of the lower predicate calculus has limited possibilities, which are insufficient for consistent developing of both topological model theory and finite model theory. On the other hand, results of the papers [3], [4] show that some of these problems can be successfully solved with the help of methods of Robinson's nonstandard analysis.

The purpose of this talk is to give a nonstandard characterization of some well-known classes of algebraic systems which cannot be axiomatized by formulas of the lower predicate language.

Let Ω denote any fixed type of algebraic system with only a finite number of predicate symbols. To describe algebraic properties of algebraic Ω-systems we use a nonstandard formal language $\mathcal{L}_X$ over an alphabet X such that *nonstandard terms* of $\mathcal{L}_X$ are elements of the nonstandard extension ${}^*W$ of the Ω-algebra $W = W_\Omega(X)$ of Ω-words over X. Formulas of $\mathcal{L}_X$ are built up from *nonstandard identities* (i.e., atomic formulas of the form $t_1 = t_2$, $P(t_1, \dots, t_n)$, where $t_1, \dots, t_n \in {}^*W$ and P is an n-ary predicate symbol of Ω) with the help of propositional connectives and quantifiers. An interpretation of $\mathcal{L}_X$ in an algebraic Ω-system A is defined with the help of a mapping $\theta\colon X \to A$, which is canonically extended to the homomorphism ${}^*\theta\colon {}^*W \to {}^*A$.

Definition. A class of algebraic Ω-systems $\mathbf{K}$ is called *a nonstandard variety* (*nonstandard quasivariety*) if it is axiomatizable by a class of nonstandard identities (nonstandard implications).

The following result solves the problem on description of hereditary formations (i.e., $\mathbf{HSP}_{\text{fin}}$-closed classes of algebraic systems) on the base of the nonstandard approach raised by E. Palutin.

Theorem. *A class of algebraic Ω-systems $\mathbf{K}$ is a nonstandard variety if and only if it is closed* *under the formation of homomorphic images, subsystems and finite direct products, i.e.,* **K** *is a hereditary formation.*

The following result solves the problem on axiomatization of pseudoquasivarieties (i.e., $\mathbf{ISP}_{\text{fin}}$-closed classes of finite algebraic systems) by nonstandard implications raised by J. Almeida.

THEOREM. *A class of finite algebraic* Ω*-systems* **K** *is a nonstandard quasivariety if and only if it is closed under the formation of isomorphic images, subalgebras and finite direct products, i.e.,* **K** *is a pseudoquasivariety.*

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Let *FFL* be the fragment of first-order logic introduced in the section 4.2 of van Oosten [2] for a language consisting of function symbols together with logical symbols, equality $=$, conjunction $\wedge$, and existential quantifier $\exists$. A categorical interpretation (Definitions 4.5–4.7) of *FFL* is provided in a regular category, and the soundness of *FFL* is proved (Theorem 4.10). Examining his arguments, one notices that his soundness proof of *FFL* works for a substructural fragment of *FFL*, and it can be adopted to a multiplicative extension of *FFL*.

(A) Let FFL' be a formal system resulting from *FFL* by the restriction: "the assumption ψ has to be always present whenever the rule (iii) of Definition 4.9 is applied." In FFL', the left-rule for $\wedge$ is pareserved, but the weakening rule (for empty assumptions) does not hold. Van Oosten's proof of the soundness (Theorem 4.10) works for FFL'.

(B) Let *MFFL* be an extension of *FFL* resulting from the addition of multiplicative conjunction. Interpretation of *MFFL* is obtained from that of *FFL* (van Oosten [2], Definition 4.5–4.7) through the replacement of cartesian product $\times$ by tensor product $\otimes$. (In particular, $[\![FV(t)]\!]$ is then re-defined to be $[\![s_1]\!] \otimes \cdots \otimes [\![s_n]\!]$ in Definition 4.5.) Van Oosten's proof for the soundness of *FFL* carries over to *MFFL*. For this particular extension *MFFL*, resource to fibrations is not necessary.

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MIM—The IM Morphed Computing Logics for multimedia are new projects with important computing applications since [4]. The basis is a mathematical logic where a Gentzen or natural deduction [3] system is defined by taking arbitrary structures and multimedia objects and are encoded by diagram functions. By trans-morphing hybrid picture's corresponding functions a new hybrid picture is deduced. The techniques can be applied to arbitrary topological structures. Multimedia objects are viewed as syntactic objects defined by functions, to which the deductive system is applied. Thus we define syntactic morphings to be technique by which multimedia objects and hybrid pictures are homomorphically mapped via their defining functions to new hybrid pictures. Functorial structure morphings can be new application areas. The deduction rules are a Gentzen system augmented by Morphing and Trans-morphing. The logical language has function names for hybrid pictures. The MIM Morph Rule—An object defined by the function n-type $\langle f_1, \dots, f_n \rangle$ can be Morphed to an object defined by the functional n-tuple $\langle h(f_1), \dots, h(f_n) \rangle$, provided h is a homomorphism of abstract signature structures [5]. The MIM TransMorph Rules—A set of rules whereby combining hybrid pictures $p_1, \dots, p_n$ defines an Event $\{p_1, p_2, \dots, p_n\}$ with a consequent hybrid picture p. Thus the combination is an impetus event. The languages and MIM rules are applied to algebraic structures. The deductive theory is a Gentzen system in which hybrid pictures are named by parameterized functions; augmented by the MIM morph and transmorph rules. The Model theory is defined from Intelligent Syntax Languages [5]. A computational logic for intelligent languages is presented in brief with a soundness and completeness theorem in [5]. The idea is to do it at abstract models syntax [1] trees without specifics for the shapes and topologies applied. We start with $L_{\omega_1,\omega}$, and further on might apply alternate well-behaved infinitary languages.

THEOREM. *MIM Logic is sound and complete.*

[1] C. F. NOURANI, *Automatic models from syntax*, ***Scandinavian Linguistics***, Oslo, Norway, January 1995.

[2] G. GENTZEN, *Beweisbarkeit und Unbeweisbarket von Anfangsfallen der transfiniten Induktion in der reinen Zahlentheorie*, ***Mathematische Annalen***, vol. 119 (1943), pp. 140–161.

[3] D. PRAWITZ, ***Natural deduction: A proof theoretic study***, Stokhom, Almqvist and Wiksell, 1965.

[4] C. F. NOURANI, *Intelligent multimedia*, 1996, poster announcement to the International Conference on Computational Intelligence, Monash University, Victoria, Austrialia, February 1998.

[5] ———, ***Slalom tree computing***, Artificial Intelligence Communications, December 1996, IOS Press, Amsterdam.

► D. E. PAL'CHUNOV, *Strongly constructivizable prime I-algebras*.
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We consider Boolean algebras with $l \in \omega$ distinguished ideals, called as *I-algebras*. We study prime models of the I-algebra theory. One can find definitions related to I-algebras in [1].

A model $\mathcal{A}$ is called *non-vanishing* if for any decomposition $\mathcal{A} = \mathcal{B} \times \mathcal{C}$ we have $\mathcal{A} \equiv \mathcal{B}$ or $\mathcal{A} \equiv \mathcal{C}$; $\mathcal{A}$ is called *basic* if $\mathcal{A}$ is non-vanishing and finitely axiomatizable.

For an I-algebra $\mathcal{A}$ and an element $a \in \mathcal{A}$ the set $\hat{a} = \{b \in \mathcal{A} \mid b \le a\}$ defines corresponding I-algebra denoted as $(\hat{a})$.

In [1] the sequence of formulas $V_n(x)$, $n \in \omega$, together with related characteristic $r_{\mathcal{A}}(n)$ are introduced. An I-algebra $\mathcal{A}$ is basic if and only if there exists a number n with $\mathcal{A} \vDash V_n(1)$. If $\mathcal{A} \vDash V_n(1)$ and $\mathcal{B} \vDash V_n(1)$ then $\mathcal{A} \equiv \mathcal{B}$. For an element $a \in \mathcal{A}$ the statement $\mathcal{A} \vDash V_n(a)$ holds if and only if $(\hat{a}) \vDash V_n(1)$.

We have $r_{\mathcal{A}}(n) = 0$ if and only if $\mathcal{A} \vDash \neg\exists x\ V_n(x)$; $r_{\mathcal{A}}(n) = k$, $k \in \omega$, if and only if k is the number of mutually disjoint elements $a \in \mathcal{A}$ such that $\mathcal{A} \vDash V_n(a)$; $r_{\mathcal{A}}(n) = \infty$ if and only if this number is as much as desired.

An I-algebra $\mathcal{A}$ is said to be *local* if the set $M(\mathcal{A}) \rightleftharpoons \{\, n \in \omega \mid r(n) \neq 0 \,\}$ is finite. An I-algebra $\mathcal{A}$ is local if and only if $\mathcal{A} \times \mathcal{B}$ is finitely axiomatizable for some I-algebra $\mathcal{B}$. An I-algebra $\mathcal{A}$ is called *prime* if $\mathcal{A}$ is the prime model of the elementary theory $\mathrm{Th}(\mathcal{A})$.

THEOREM. *For any recursive characteristic r there exists a strongly constructivizable prime I-algebra $\mathcal{A}$ with $r_{\mathcal{A}} = r$.*

It is proved [1] that

(a) if an I-algebra $\mathcal{A}$ is local then the elementary theory $\mathrm{Th}(\mathcal{A})$ has the prime model;
(b) if $\mathcal{A}$ is a superatomic Boolean algebra with one distinguished ideal then the elementary theory $\mathrm{Th}(\mathcal{A})$ has the prime model.

COROLLARY 1. *If a prime I-algebra $\mathcal{A}$ is local then $\mathcal{A}$ is strongly constructivizable.*

COROLLARY 2. *Every prime superatomic Boolean algebra with one distinguished ideal is strongly constructivizable.*

For models $\mathcal{A}$ and $\mathcal{B}$ we denote $\mathcal{B} \leq \mathcal{A}$ if $\mathcal{A} = \mathcal{B} \times \mathcal{C}$.

Models $\mathcal{A}$ and $\mathcal{B}$ are called *finitely equivalent* if for any finitely axiomatizable $\mathcal{C} \leq \mathcal{A}$ and $\mathcal{D} \leq \mathcal{B}$ there exist $\mathcal{M} \leq \mathcal{A}$ and $\mathcal{N} \leq \mathcal{B}$ such that $\mathcal{C} \equiv \mathcal{N}$ and $\mathcal{D} \equiv \mathcal{M}$. Thus $\mathcal{A}$ and $\mathcal{B}$ are finitely equivalent if and only if they have the same (up to elementary equivalence) finitely axiomatizable direct factors.

COROLLARY 3. *For any I-algebra $\mathcal{A}$ there exists a prime strongly constructivizable I-algebra $\mathcal{B}$ such that $\mathcal{A}$ and $\mathcal{B}$ are finitely equivalent.*

[1] D. E. PAL'CHUNOV, *Prime and countably saturated models of the theory of Boolean algebras with distinguished ideals*, ***Siberian Advances in Mathematics***, vol. 4 (1994), no. 3, pp. 83–108.

► S. G. PYRKIN, *On m-equivalence of superatomic I-algebras*.
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Boolean algebras with a distinguished ideal (*I-algebras*) are considered in this paper. We study m-equivalence of superatomic boolean algebras with a distinguished ideal. The elementary theories of superatomic I-algebras were completely described in [1]. One can find definitions on I-algebras in [1]; on m-equivalence of I-algebras in [2].

DEFINITION. Let $\leq$ be a sign for the lexigraphic order. We denote

$$r(\mathcal{A}, I) \leq_{3,2,1} r(\mathcal{B}, J)$$

if

$$(r_3(\mathcal{A}, I), r_2(\mathcal{A}, I), r_1(\mathcal{A}, I)) \leq (r_3(\mathcal{B}, J), r_3(\mathcal{B}, J), r_3(\mathcal{B}, J)).$$

Denote

$$\varphi(t) = \max\{\, k \mid t \geq 2^k \,\},$$
$$\psi(t) = \min\{\, k \mid t \leq 2^1 + \cdots + 2^k \,\}, \quad \psi(0) \rightleftharpoons 0,$$
$$\chi(t) = \max\{\, k \mid t \geq 2^1 + \cdots + 2^k \,\}, \quad \chi(1) \rightleftharpoons 0, \quad \chi(0) \rightleftharpoons 0.$$

THEOREM. *Let $(\mathcal{A}, I)$ and $(\mathcal{B}, J)$ be superatomic I-algebras,*

$$r(\mathcal{A}, I) = (n_1, n_2, n_3), \quad r(\mathcal{B}, J) = (m_1, m_2, m_3), \quad r(\mathcal{A}, I) <_{3,2,1} r(\mathcal{B}, J)$$

and $n_3 \geq 5$. *The following diagram shows the maximal* m *such that* $(\mathcal{A}, I) \equiv_m (\mathcal{B}, J)$:

<table>
<tr><td rowspan="3">$n_3 < m_3$</td><td>$n_2 = 0$</td><td colspan="3"></td><td>$m = n_3 - 2$</td><td>(1)</td></tr>
<tr><td rowspan="2">$n_2 \neq 0$</td><td colspan="3">$n_2 = 1$</td><td>$m = n_3 - 2$</td><td>(2)</td></tr>
<tr><td colspan="3">$n_2 \geq 2$</td><td>$m = n_3 - 1$</td><td>(3)</td></tr>
<tr><td rowspan="8">$n_3 = m_3$</td><td>$n_2 = 0$,
$m_2 = 0$</td><td colspan="3"></td><td>$m = \varphi(n_1) + n_3 - 2$</td><td>(4)</td></tr>
<tr><td>$n_2 = 0$,
$m_2 \neq 0$</td><td colspan="3"></td><td>$m = n_3 - 2$</td><td>(5)</td></tr>
<tr><td rowspan="6">$n_2 \neq 0$,
$m_2 \neq 0$</td><td colspan="3">$n_2 = m_2$</td><td>$m = \psi(n_1) + n_3 - 2$</td><td>(6)</td></tr>
<tr><td rowspan="5">$n_2 < m_2$</td><td rowspan="2">$n_1 < m_1$</td><td>$2n_1 \leq n_2$</td><td>$m = \psi(n_1) + n_3 - 2$</td><td>(7)</td></tr>
<tr><td>$2n_1 > n_2$</td><td>$m = \chi(n_2) + n_3 - 2$</td><td>(8)</td></tr>
<tr><td colspan="2">$n_1 = m_1$</td><td>$m = \chi(n_2) + n_3 - 2$</td><td>(9)</td></tr>
<tr><td rowspan="2">$n_1 > m_1$</td><td>$2m_1 \leq n_2$</td><td>$m = \psi(m_1) + n_3 - 2$</td><td>(10)</td></tr>
<tr><td>$2m_1 > n_2$</td><td>$m = \chi(n_2) + n_3 - 2$</td><td>(11)</td></tr>
</table>

[1] D. E. Pal'chunov, *On the undecidability of theories of Boolean algebras with a distinguished ideal*, ***Algebra i Logika***, vol. 25 (1986), no. 3 pp. 326–346 and 364.

[2] ———, *Finitely axiomatizable Boolean algebras with a distinguished ideal*, ***Algebra i Logika***, vol. 26 (1987), no. 4, pp. 435–455 and 525.

► IRINA A. SEMAKINA, *Existential structures of language and sense*: *The problem of description*.
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Philosophical aspect of logic proposes connection of notion "being" as the basic category of philosophy with notion "language". With according to this proposition it is necessary to consider them in identity. It gives us a possibility to investigate the sense's being as a movement from being to language and from language to being. This movement is the description of "language of being" as "being of language". The working out of notion "being" by M. Heidegger, which understood Language as home of Being, leads us to description of existential structure of language. M. Heidegger found the given structure in grammatics. But he only projected the way of such research. We suggest that this way permits us to consider language as language's reality but not sign's system which leads us to problems of sensible discourse. Idea of language as sign's system refers to limit existing between language and objective world, and all attempts to understand entity of language have results representation about language as about empty form, nonsensible form. Within the framework of hermeneutics language's being or language's reality is represented as space of time (time's space) which can be definite by grammatical terms. The problem of sense which we have in logic and in philosophy is connected with definition of place where sense exists. We suggest that place of sense can be described by means of two categories: "text" and "proposition". These notions will characterise sense's way of existence which will be understood as language's activity. "Text" can be considered as activity of deriving, "proposition" as activity of proposition (offer). Thus, sense as foundation of language reality derives and proposes itself. It derives itself because sense is connected with predicted core of proposition. Sense is active subject, setting on foundation language, which tells about itself.

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(Joint work with G. Cherlin and S. Shelah.)

Rado first observed that there is a universal countable group, that is a countable group G such that every countable group is isomorphic to an induced subgraph of G. Since then many similar universal graph problems have been considered by P. Komjath, J. Pach, A. Mekler, Z. Furedi, M. Goldstern, M. Kojman, G. Cherlin, and N. Shi. These researches give either a positive or a negative solution for the existence of a countable universal graph for a class of graphs which forbid finitely many subgraphs.

We apply model theoretic methods to this universal graph problem and show that to a large extent the question reduces to one of local finiteness of an associated "algebraic closure" operator. We have the following

THEOREM. *Let C be a finite set of connected finite graphs, $\mathcal{G}_C$ be the class of all countable graphs omitting C, $\mathcal{E}_C$ be the class of all existential complete graphs in $\mathcal{G}_C$, T_C^* be the theory of $\mathcal{E}_C$. Then the following are equivalent*:

(a) *T_C^* is $\aleph_0$-categorical.*
(b) *The Stone space of all n-existential types $S_n(T_C^*)$ is finite for each n.*
(c) *For $A \subseteq M \models T_C^*$ finite, the algebraic closure of A is finite.*

These conditions imply that $\mathcal{G}_C$ contains a universal countable graph.

For the nonexistence of universal graphs our model theoretic methods are very close to those which have been used in practice, although that is not explicit in those papers. In this case we consider the minimal size of A such that the algebraic closure of A is infinite.

In the second half of this paper we apply the criterion of the existence for universal graphs obtained in this paper to find a large group of new classes of graphs that have a universal graph.

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Reducibilities order sets by information contained in them (that is being taken by using algorithmic methods). At the same time the theory of algorithms does not study questions concerning structure of information allocation (in sets or infinite sequences). For example, does the information involved in the sequence is doubled or contained in some subsequences of original sentence. Such questions were, perhaps, firstly formulated in [1].

We will consider the possibility of reconstruction (using a reducibility) of all information contained in the set, though having indefinitely small part of it. It appears to be similar to holographic effect in optics. The investigation is being done within subject scope of automaton reducibility [2].

Following Rayna [2], we call two infinite sequences $X = \{X(i)\}$ and $Y = \{Y(i)\}$ "equivalent by automata (being involved in one degree of automaton reducibility)", if there exist two initial finite-state automata (T_1, s_1) and (T_2, s_2) so that automaton (T_1, s_1) puts out the sequence Y (maybe with some delay) if getting the sequence X as an input, and automaton (T_2, s_2) puts out the sequence X (also with probable delay) if getting the sequence Y as an input.

The partially ordered set V of degrees of automaton reducibility is being naturally generated.

DEFINITION. Infinite sequence X is called *holographic* if for any periodic sequence a in the alphabet $\{0, 1\}$ and containing infinite number of 1s X is reducible by automaton to the sequence Y, defined by condition if $a(i) = 1$ then $Y(i) = X(i)$ else $Y(i) = 0$.

This definition means that all information contained in holographic sequence is included (and able to be reconstructed by finite-state automaton) in any indefinitely small (and periodic) part of the holographic sequence.

THEOREM. *Holographic sequences exist (and belongs to the atom of V).*

PROBLEM. Will this effect take place in case of change of reducibility?

HYPOTHESIS. The existence of sequences with such properties does not depend on chosen reducibility. It just shows deep laws of information packing in infinite sequences.

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[1] V. D. SOLOVYEV, *Structure of information allocation in infinite sequences*, ***Discretnaia Matematica***, vol. 8 (1996), no. 2.

[2] G. RAYNA, *Degrees of finite-state transformability*, ***Information and Control***, vol. 24 (1974), pp. 144–154.

► VLADIMIR SOTIROV, *All syllogistic arithmetized à la Leibniz*.
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The first Leibniz trial to arithmetize the Aristotelian syllogistic [1, pp. 42–92 and 245–247] explored divisibility of integers; it was unsuccessful. The second one used pairs of co-prime numbers and was successful, as Slupecki proved [2, §34]. However, this second translation was sophisticated, moreover, it did not envelop syllogistic of term negation or that of term conjunction. In this paper we justify the viability of the earlier (and less complicated) Leibniz idea and propose two translations into arithmetic which are appropriate for the extended syllogistic as well.

Arithmetizations of the traditional syllogistic. The language of the classical propositional calculus is expanded with term variables together with two binary term relations: A and I. Syllogistic atoms are all formulae of the kind $s\,A\,p$ ('Every s is a p') or $s\,I\,p$ ('Some s is a p') with s and p being terms. A syllogism is any propositional formula with all propositional letters replaced by syllogistic atoms. In the standard semantics of the Aristotelian syllogistic, if S and P are arbitrary non-empty sets, $s\,A\,p$ is translated as 'S is a subset of P', $s\,I\,p$ as 'the intersection of S and P is not empty', and the formal propositional connectives are replaced with the informal ones. This semantics we name Scholastic following Leibniz himself.

The second semantics in set theory we name Leibnizian being partially accepted by him. When a non-empty set U is given, term variables are evaluated by subsets of U different from U, $s\,A\,p$ is interpreted as 'S includes P', and $s\,I\,p$ as 'the union of S and P is different from U'. In both (equivalent) semantics a syllogism is said to be true when its translation is a true sentence about sets.

Now, in the Scholastic arithmetical interpretation terms will be evaluated by integers greater than 1. If b and c are the values of s and p, $s\,A\,p$ is replaced with 'b is a divisor of c', and $s\,I\,p$ with 'g. c. d.(b, c) is greater than 1'. For the second interpretation named Leibnizian an arbitrary integer u greater than 1 is introduced and terms are evaluated by their arbitrary proper divisors (less than u). $s\,A\,p$ is replaced by 'b is divisible by c', and $s\,I\,p$ by 'l. c. m.(b, c) is less than u'. Formal connectives are replaced by their informal analogues. Call a syllogism arithmetically true in the Scholastic (respectively, in the Leibnizian) sense when the sentence so obtained is an arithmetical truth.

THEOREM. *A syllogism is true if and only if it is arithmetically true in the scholastic as well as in the Leibnizian sense.*

Arithmetizations of syllogistic with negative terms. Expand the language of the syllogistic by a term operation $-$; $-t$ is read 'non-t'. In both set-theoretical semantics, a universal set U is introduced. Terms are evaluated by subsets of U different from the empty set and U. If a term t is evaluated by a set T, the value of $-t$ is the complement of T to U. In both arithmetical interpretations a Universe number u greater than 1 without multiple factors is introduced together with the following rules: (1) all evaluating integers are divisors of u different from 1 and u; (2) if term t is evaluated by an integer d then $-t$ is evaluated by the integer u/d.

THEOREM. *A syllogism is true if and only if it is arithmetically true in the scholastic as well as in the Leibnizian sense.*

Arithmetizations of syllogistic with term composition. The composition will be denoted by $\circ$. The class of terms is the smallest class including term variables, and closed under negation and composition. So all Boolean operations over terms are defined. Given a Universe U, an evaluation of a term t in U is a set T obtained after replacing all all term variables in t with arbitrary subsets of U (possibly empty or U) as well as all term operations with their corresponding set-theoretical operations. Namely, in the Scholastic semantics $\circ$ is interpreted as an intersection, and in the Leibnizian semantics it is a union.

THEOREM. *A syllogism is true if and only if it is arithmetically true in the scholastic as well as in the Leibnizian sense.*

Note. The full text of the paper is to appear in the ***Journal of Applied Non-Classical Logics***.

[1] L. COUTURAT, ***Opuscules et fragments inédits de Leibniz, éxtraits des manuscrits de la Bibliothèque royale de Hanovre***, Paris, 1903.

[2] J. ŁUKASIEWICZ, ***Aristotle's syllogistic from the standpoint of modern formal logic***, 2nd ed., Clarendon Press, Oxford, 1957.

► D. VAN HEULE AND A. HOOGEWIJS, *Tableaux in automated theorem proving: Isabelle and PPC.*
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Tableaux methods offer elegance, flexibility and analytic proof theory for a wide variety of logics. Some theorem provers based on tableaux like HARP [3] are very sophisticated while others such as 3TAP [1] offer a great flexibility. Isabelle [4] is a generic, interactive theorem prover based upon a form of resolution. Automation is provided by means of a "Classical Reasoner", which searches for proofs using the tableau approach. This will be demonstrated on examples, using the Partial Predicate Logic (PPC) [2].

In the first section, the object logic LK, a classical first-order logic through Gentzen's sequent calculus, will be used to construct an automatic tautology checker for PPC.

In the second part we will discuss the Classical Reasoner. Because of the correspondence between natural deduction and sequent calculus, the tableaux method is adapted to a natural deduction style, which seems easier to automate. *Fast_tac*, the most important tactic of Isabelle and *Blast_tac*, coded directly in ML, faster and more powerful than fast_tac but with some limitations will be demonstrated in PPC.

[1] R. HÄHNLE, B. BECKERT, and S. GERBERTDING, *3TAP*: *The many-valued theorem prover*, 3rd Edition, University of Karlsruhe, Institute for Logic, Complexity and Deduction Systems, September 1994, http://il2www.ira.uka.de/~threetap.

[2] A. HOOGEWIJS, *Partial predicate logic in computer science*, ***Acta Informatica***, vol. 24 (1987), pp. 381–393.

[3] R. OPPACHER and E. SUEN, *HARP: A tableau-based theorem prover*, ***Journal of Automated Reasoning***, vol. 4 (1988), pp. 69–100.

[4] L. PAULSON, ***Isabelle: A generic theorem prover***, Lecture Notes in Computer Science, Springer-Verlag, 1994.

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Modularity (preservation of faithfulness) is important for stepwise software development. We examine modularity and interpolation in caregories of specifications over a general π-institution and provide conditions for one in terms of the other. These conditions, which are local to a subcategory, are of two kinds: special cases of modularity and connective-free interpolation-like properties of the consequence relations.

We relax π-institutions [1] to *general* π-institutions (GPI, for short) by requiring only structurality and consider the category of specifications over a GPI $G = \langle \mathrm{Sgnt}, \mathrm{Snt}, \vdash \rangle$. We use $e\colon \Gamma \leq \Sigma$ for a translation $e\colon I \to J$ in Sgnt that conserves specifications $\langle I, \Gamma \rangle$ and $\langle J, \Sigma \rangle$, i.e., $\Gamma \vdash_I \tau$ whenever $\Sigma \vdash_J \tau^e$, with $\tau \in \mathrm{Snt}(I)$. Given a pair of translations $e\colon I \to J$ and $f : I \to K$, we use $f/e\colon J \to e \oplus f$ and $e/f : K \to e \oplus f$ for their pushouts in Sgnt. We also use $\emptyset_K$ for $\langle K, \emptyset \rangle$.

Given a translation $f : I \to K$ and specifications $Q = \langle J, \Sigma \rangle$ and $R = \langle K, \Theta \rangle$, we call them *modular* over G ($\langle Q, f, R \rangle \in \mathrm{Mdl}[G]$) if and only if

$$e/f : \Gamma^f \cup \Theta \leq \Sigma^{f/e} \cup \Theta^{e/f}$$

whenever $e\colon \Gamma \leq \Sigma$, and *interpolable* over G ($\langle Q, f, R \rangle \in \mathrm{Int}[G]$) if and only if whenever $\Sigma^{f/e} \cup \Theta^{e/f} \vdash_{e \oplus f} \tau^{e/f}$, with $\tau \in \mathrm{Snt}(K)$, there exists $\Omega \subseteq \mathrm{Snt}(I)$ such that $\Omega^e \subseteq \mathrm{Cn}_J(\Sigma)$ and $\Omega^f \cup \Theta \vdash_K \tau$.

We now define presentation, language and axiom *modularity*: $G \in \mathrm{PM}$ if and only if $\langle Q, f, R \rangle \in \mathrm{Mdl}[G]$ for all Q, f and R, $G \in \mathrm{LM}$ if and only if $\langle Q, f, \emptyset_K \rangle \in \mathrm{Mdl}[G]$ for all Q and f, and $G \in \mathrm{AM}$ if and only if $\langle Q, 1_I, R \rangle \in \mathrm{Mdl}[G]$ for all Q and R. We define the *interpolation-like* properties: $G \in \mathrm{DI}$ if and only if $\langle Q, f, R \rangle \in \mathrm{Int}[G]$ for all Q, f and R, $G \in \mathrm{SI}$ if and only if $\langle Q, f, \emptyset_K \rangle \in \mathrm{Int}[G]$ for all Q and f, and $G \in \mathrm{CT}$ if and only if $\langle Q, 1_I, R \rangle \in \mathrm{Int}[G]$ for all Q and R.

Our results for modularity fall into three classes.

Characterization for modularity: $G \in \mathrm{PM}$ if and only if $G \in \mathrm{LM}$ and $G \in \mathrm{AM}$.

Sufficient conditions (assuming transitivity): $\mathrm{DI} \subseteq \mathrm{PM}$, $\mathrm{SI} \subseteq \mathrm{LM}$, $\mathrm{CT} \subseteq \mathrm{AM}$.

Necessary conditions (assuming reflexivity): $\mathrm{PM} \subseteq \mathrm{DI}$, $\mathrm{LM} \subseteq \mathrm{SI}$, $\mathrm{AM} \subseteq \mathrm{CT}$.

Thus, the special cases provide simpler tests for modularity. Our interpolants are flexible since they are sets of sentences in lieu of sentences (with some special form). Also, necessity is established by means of uniform interpolants (a single Ω depending only on Σ, but not on Θ or τ). This imposes strong requirements for modularity of a reflexive GPI.

We thus have conditions for interpolation-like properties of consequence relations, being sufficient for reflexive ones and necessary for transitive ones. Monotonicity serves only to show that $\langle e \oplus f, \Sigma^{f/e} \cup \Theta^{e/f} \rangle$ gives a pushout.

[1] J. L. Fiadeiro and A. Sernadas, *Structuring theories on consequence*, in ***Recent trends in data type specification*** (D. Sannella and A. Tarlecki, editors), Springer-Verlag, Berlin, 1988, pp. 44–72.

► ZHOU XUNWEI, *Inversism*: *The 4th school of mathematical logic*.
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Inversims is constructed by the author. It is different from logicism, intuitionism and formalism. In inversistic logic, there are four compounders and seven connectives. The four compounders are $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), $\oplus$ (exclusive or). The seven connectives are $\overset{-1}{\wedge}$ (inverse conjunction), $\overset{-1}{\vee}$ (inverse disjunction), $\overset{-1}{\leq}$ (inverse implication), $\overset{-1}{=}$ (inverse equivalence), $\overset{-1}{<}$ (inverse proper implication), $\overset{-1}{\overset{\triangledown}{\vartriangle}}$ (inverse quartic conjunction), $\overset{-1}{\oplus}$ (inverse exclusive or).

Each connective has an inductive composition truth table and two decomposition truth tables. Take $\overset{-1}{<}$ as an example. The inductive composition truth table of $\overset{-1}{<}$ is shown in Table 1, the decomposition truth tables are shown in Table 2 and Table 3.

Table 1

A	B	$A \overset{-1}{<} B$
F	F	T
F	T	T
T	F	F
T	T	T

Table 2

$A \overset{-1}{<} B$	A	B
F	F	F/T
F	T	F/T
T	F	F/T
T	T	T

Table 3

$A \overset{-1}{<} B$	B	A
F	F	F/T
F	T	F/T
T	F	F
T	T	F/T

"F/T" in Table 2 and Table 3 means "unable to determine whether it is true or false". Table 1 is the same with the truth table of $\rightarrow$ in classical logic. Table 2 tells us that if $A \overset{-1}{<} B$ is true and A is true then B is true, otherwise we can't determine the truth value of B. So, Table 2 describes affirmative expression of hypothetical inference. Likewise, Table 3 describes negative expression of hypothetical inference. Table 1 is from the truth values of A and B to that of $A \overset{-1}{<} B$. Table 2 and Table 3 are from the truth value of $A \overset{-1}{<} B$ to those of A or B. They are mutually inverse truth tables.

In inversistic set theory, there are four compounders and seven connectives. The four compounders are $\sim$ (complement), $\cap$ (intersection), $\cup$ (union), $\oplus$ (circular sum). The seven connectives are $\overset{-1}{\cap}$ (inverse intersection), $\overset{-1}{\cup}$ (inverse union), $\overset{-1}{\subseteq}$ (inverse inclusion), $\overset{-1}{=}$ (inverse equivalence), $\overset{-1}{\subset}$ (inverse proper inclusion), $\overset{-1}{\overset{\triangledown}{\vartriangle}}$ (inverse quartic intersection), $\overset{-1}{\oplus}$ (inverse circular sum).