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# 1997–98 ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

## Toronto, Ontario, Canada, May 22–25, 1998

The Annual Meeting of the Association was held at the University of Toronto, Canada, May 22–25, 1998. The program consisted of eight scheduled invited lectures and two special sessions with six talks each in model theory and set theory. The ninth annual Gödel lecture entitled *Current trends in descriptive set theory* was delivered by Alexander Kechris. Ehud Hrushovski was awarded the 1998 Carol Karp Prize; in his honor, Anand Pillay spoke on *Hrushovski's work on model theory and its applications*, Stephen Cook gave a three-lecture tutorial on *Proof complexity and bounded arithmetic*. In addition, there were seventeen contributed talks and three papers presented by title. The program committee consisted of Tim Carlson, Kevin Compton, Sy Friedman, Bradd Hart (Chair), Andre Nies, and Alisdair Urquhart.

The following invited talks were presented at the meeting:

Warren Goldfarb (Harvard), *On Dummett's "proof-theoretic justifications of logical laws"*.

Theodore A. Slaman (University of California, Berkeley), *On a question of Sierpiński*.

Richard Sommer (Stanford), *The strength of elementary computable infinitesimal analysis*.

Patrick Speissegger (University of Toronto/MSRI), *The Pfaffian closure of an o-minimal structure*.

Stevo Todorčevic (Université de Paris VII/CNRS), *Aspects of infinite dimensional Ramsey theory*.

Stanley Wainer (University of Leeds), *Accessible recursive functions*.

Hugh Woodin (University of California, Berkeley), *The axiom of determinacy, fine structure and the continuum hypothesis*.

Unfortunately, Alexei Myasnikov could not attend the meeting and presented only his title, *Equations over free groups*.

The following talks were presented in the Special Session on Model Theory:

Chris Miller (Ohio State), *Expansions of the real line by open sets: Some model completeness and quantifier elimination results*.

Ross Willard (University of Waterloo), *A non-progress report on decidable discriminator variety problem*.

Deidre Haskell (Holy Cross), *The model theory of valued fields*.

Carol Wood (Wesleyan), *Differential equations and model theory*.

José Iovino (Carnegie Mellon), *Types and $\ell_p$ subspaces of Banach spaces*.

Salma Kuhlmann (University of Saskatchewan), *The growth rate of exponentials in models of real exponentiation*.

The following talks were presented in the Special Session on Set Theory:

Douglas Burke (University of Nevada at Las Vegas), *Generic embeddings and combinatorial principles*.

Randall Dougherty (Ohio State University), *Congruences using open sets or property-of-Baire sets*.


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Steve Jackson (University of North Texas), *Combinatorics in $L(R)$*.
Matthew Foreman (University of California, Irvine), *A few remarks in Hungarian*.
Howard Becker (University of South Carolina), *Topics in invariant descriptive set theory*.
Ernest Schimmerling (University of California, Irvine), *Fine structure*.

Abstracts of talks presented in person or by title by members of the Association for Symbolic Logic follow.

For the Program Committee
BRADD HART

### Abstract of invited tutorial

► STEPHEN A. COOK, *Proof complexity and bounded arithmetic*.
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The central problem in propositional proof complexity is proving superpolynomial lower bounds on the lengths of proofs for various proof systems for tautologies. There has been some success for weak proof systems, but current methods appear hopeless for standard textbook systems (Frege systems). The question is important, since whichever way it is answered there will be interesting implications in complexity theory. There is an important three-way correspondence between propositional proof systems, complexity classes, and weak theories of arithmetic. The best known such triple consists of extended Frege systems, the class $P$, and Buss's system $S_2^1$. In general, lower bounds on propositional proof length, and computational complexity lower bounds, give rise to independence results in the corresponding arithmetic theory. Proofs of $\Pi_1$ statements in the arithmetic theory correspond to uniform families of propositional proofs. An intriguing open problem is to show a direct connection between propositional proof lower bounds and computational lower bounds.

The titles of the three talks are:

I. An overview of propositional proof complexity and bounded arithmetic
II. The second order theories $V^0$ and $V^1$
III. Translating proofs in arithmetic theories to propositional proofs.

### Abstracts of invited plenary lectures

► WARREN GOLDFARB, *On Dummett's "proof-theoretic justifications of logical laws"*.
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In Chapters 9–11 of *The logical basis of metaphysics*, Michael Dummett claims to formulate a proof-theoretic method for providing justifications of logical laws of inference. According to this method, a law is justified if any deduction of a certain standard type for the premises of the law can be transformed into a deduction of the standard type for the conclusion of the law. The laws justified by this method, Dummett claims, are just the laws of intuitionistic logic.

An analysis of Dummett's method is presented that makes its scope transparent. This uncovers counterexamples: clearly incorrect laws that are "justified" according to the method. A simple modification avoids such counterexamples, but is at odds with the philosophical

underpinnings of Dummett's enterprise. Even with the modification, some non-intuitionistic laws are "justified." The discrepancy reveals some fundamental difficulties in Dummett's view of the nature of logic and its relation to meaning.

► THEODORE A. SLAMAN, *On a question of Sierpiński*.
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Sierpiński (1936) raised the following question:

> Does there exist a set $U$ such that every analytic set is a bijective continuous image of $U$?

We observe that no analytic set can have the above property of $U$, and then define such a $U$ from a well-ordering of the reals.

► RICHARD SOMMER, *The strength of elementary computable infinitesimal analysis*.
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Elementary Computable Infinitesimal Analysis (ECIA) is a very weak formal system of nonstandard analysis that is strong enough to formalize a significant fragment of real analysis. ECIA is weak in the sense that each ECIA-provably computable function on the natural numbers is bounded by an exponential term. The power of ECIA to carry out real analysis will be demonstrated, in part, by showing that a natural proof of the existence theorem for ordinary differential equations, in the language of infinitesimal analysis, can be directly carried out in ECIA. We go much further by showing that ECIA is bi-interpretable with a fragment of second-order arithmetic obtained by adding weak Koenig's Lemma to a base theory of elementary recursive strength; we will discuss some of the implications of this bi-interpretability result. This includes joint work with Patrick Suppes.

► PATRICK SPEISSEGGER, *The Pfaffian closure of an o-minimal structure*.
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Any o-minimal expansion $R$ of the field of real numbers can be expanded to an o-minimal structure $P(R)$, called the Pfaffian closure of $R$, in which solutions to Pfaffian equations with definable coefficients are definable. A special case of such solutions are antiderivatives of definable functions of one variable.

I will define $P(R)$ and give the main ideas of the proof of its o-minimality. More recent work suggests that its o-minimality in turn can be used to deduce further properties relative to $R$, like model completeness (in a language more suitable for applications than the one used to define $P(R)$), analytic cell decomposition and exponential boundedness.

► STANLEY WAINER, *Accessible recursive functions*.
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The class of all recursive functions fails to possess a natural hierarchical structure, generated predicatively "from within". On the other hand, many (proof-theoretically significant) sub-recursive classes do. This talk attempts to measure the limit of predicative generation in this context, by classifying and characterizing those (predictably terminating) recursive functions which can be successively defined according to an autonomy condition of the form: allow recursions only over well-orderings which have already been "coded" at previous levels. The question is: how can a recursion code a well-ordering? The answer lies in Girard's theory of dilators, but reworked here in a quite different and simplified framework specific to our purpose. The "accessible" recursive functions thus generated turn out to be those provably recursive in $(\Pi^1_1 CA)_0$.

### Abstracts of invited talks in the Special Session in Model Theory

► JOSÉ IOVINO, *Types and $\ell_p$ subspaces of Banach spaces*.
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The problem of what Banach spaces contain the classical sequence spaces $\ell_p$ (for $1 \leq p < \infty$) and $c_0$ has played a central role in the history of Banach space theory. (For a survey, see [2].) The first example of a Banach space not containing $\ell_p$ or $c_0$ was given by B. S. Tsirel'son in [7]. From that point on, it has become an important question to find a simple condition such that all spaces satisfying that condition contain $\ell_p$ or $c_0$. In the famous paper [5], J.-L. Krivine and B. Maurey introduced the class of *stable Banach spaces* and showed that every stable Banach space contains $\ell_p$ or $c_0$ almost isometrically. A fundamental concept in [5] is that of *type* on a Banach space. If $X$ is a Banach space, a type on $X$ is a function $\tau\colon X \to \mathbb{R}$ such that for some sequence $(a_n)$ in $X$,

$$\tau(x) = \lim_n \|a_n + x\|.$$

It was noticed that the space of types on a stable Banach space is *strongly separable*, i.e., separable with respect to the topology of uniform convergence on bounded sets. A question, then, was whether these two conditions (stability and strong separability of the space of types) are equivalent. The question was answered in the negative by E. Odell [6] by showing that Tsirel'son space of [1] has a strongly separable space of types. Then R. Haydon and B. Maurey showed in [3] that every space with a strongly separable space of types contains a reflexive space or a subspace isomorphic to $\ell_1$.

The problem then remained to find topological conditions on the space of types which characterize stability of the base space. In this talk we present such a condition. The condition is related to strong separability, and is inspired on the notion of type definability from model theory.

The result will appear in [4].

[1] T. Figiel and W. B. Johnson, *A uniformly convex Banach space which contains no $l_p$*, ***Compositio Mathematica***, vol. 29 (1974), pp. 179–190.

[2] W. T. Gowers, *Recent results in the theory of infinite-dimensional Banach spaces*, in ***Proceedings of the International Congress of Mathematicians***, vol. 1 and 2 (Zürich, 1994), Birkhäuser, Basel, pp. 933–942.

[3] R. Haydon and B. Maurey, *On Banach spaces with strongly separable types*, ***Journal of the London Mathematical Society***, vol. 33 (1986), pp. 484–498.

[4] J. Iovino, *Types on stable Banach spaces*, to appear in ***Fundamenta Mathematicae***.

[5] J.-L. Krivine and B. Maurey, *Espaces de Banach stables*, ***Israel Journal of Mathematics***, vol. 39 (1981), no. 4, pp. 273–295.

[6] E. Odell, *On the types in Tsirelson's space*, in ***Texas functional analysis seminar 1982–1983*** (Austin, Texas), Longhorn Notes, University of Texas Press, Austin, 1983, pp. 49–59.

[7] B. S. Tsirel'son, *It is impossible to imbed $1_p$ of $c_0$ into an arbitrary Banach space*, *Funktsional'nyĭ Analiz i ego Prilozheniya*, vol. 8 (1974), no. 2, pp. 57–60.

► SALMA KUHLMANN, *The growth rate of exponentials in models of real exponentiation* [Joint work with Franz-Viktor Kuhlmann].
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We give an explicit construction of non-archimedean models of real exponentiation which are contained in power series fields. Recent developments in the model theory of exponential fields have shown that in certain respects, the logarithm plays a more basic role than the exponential. And in fact, every power series field $\mathbb{R}((G))$ carries a non-surjective logarithm, as is shown in [4]. In the paper [1], van den Dries, Macintyre and Marker employ a first limit process to obtain a field with non-surjective exponential, and then a second (inverse) limit process renders the exponential surjective. Reversing the approach, we start with our non-surjective logarithm on a power series field and get it surjective by taking the union over an ascending chain of power series fields.

Our approach exhibits the relation between order automorphisms of the value groups and the growth rates of the constructed exponentials. In particular, our construction can be used to provide models of arbitrary exponential rank, in the sense of [2]. It also allows to obtain on one and the same real closed field $K$ several exponentials of distinct exponential ranks, all of them making $K$ into a model of real exponentiation (cf. [3]). This result contrasts the fact that the order and hence also the rank of a real closed field is uniquely determined; it answers a question of B. Zilber to the negative.

[1] L. van den Dries, A. Macintyre, and D. Marker, *Logarithmic-exponential power series*, to appear in the *Journal of the London Mathematical Society*.

[2] F.-V. Kuhlmann and S. Kuhlmann, *The exponential rank of non-archimedean exponential fields*, to appear in the *Proceedings of the special semester in real algebraic geometry and ordered structures* (Baton Rouge, 1996).

[3] ———, *Explicit construction of exponential-logarithmic power series*, in ***Structures Algébriques Ordonnées***, Séminaire Paris VII, 1997.

[4] F.-V. Kuhlmann, S. Kuhlmann, and S. Shelah, *Exponentiation in power series fields*, ***Proceedings of the American Mathematical Society***, vol. 125 (1997), pp. 3177–3183.

► CHRIS MILLER, *Expansions of the real line by open sets: Some model completeness and quantifier elimination results*.

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Let $\mathfrak{R}$ be an expansion of the real line $(\mathbb{R}, <)$ by open sets $U_\gamma \subseteq \mathbb{R}^{n(\gamma)}$, $\gamma$ in some index set $\Gamma$. Expanding as necessary, suppose that every $\mathfrak{R}$-definable open subset of $\mathbb{R}^n$ $(n \in \mathbb{N})$ is equal to some $U_\gamma$. I'll discuss some results regarding quantifier elimination and model completeness for such structures.

► ROSS WILLARD, *A non-progress report on the decidable discriminator variety problem*.

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In 1989, McKenzie and Valeriote reduced the problem of determining which locally finite equational classes have a decidable first-order theory to two special cases: modules over finite rings, and discriminator varieties.

Discriminator varieties generalize Boolean algebras, and can be thought of as the classes $D(T)$ of certain covers of Boolean spaces by models of $T$, where $T$ ranges over universal theories.

In case the models of $T$ are locally finite, my previous work connected the decidability of $D(T)$ to homogeneity of the models of $T$ modulo finite sets of parameters.

In this lecture I will discuss the connection and outline some questions that remain open.

► CAROL WOOD, *Differential equations and model theory*.
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Some examples of differential equations whose solution sets exhibit model theoretically interesting behavior will be given, with details in one or two cases of recent interest.

## Abstracts of invited talks in the Special Session in Set Theory

► HOWARD BECKER, *Topics in invariant descriptive set theory*.
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Consider a continuous action a by a Polish group $G$ on a Polish space $X$. An important special case of this is the logic action for the language $L$: the group of permutations of N acts on codes for countable $L$-structures and the orbit equivalence relation is isomorphism. The subject of "invariant descriptive set theory", which originates with Vaught in 1974, is the study of definable $a$-invariant subsets of $X$. It is a generalization of ordinary descriptive set theory—the case of $G$ being the trivial group. It is also a generalization of (very elementary) model theory—the case of the logic actions. In this talk we discuss some recent results and some open questions in invariant descriptive set theory.

► DOUGLAS BURKE, *Generic embeddings and combinatorial principles*.
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It is a classical result, due to Solovay, that if $\kappa$ is a supercompact cardinal, then Jensen's square principle fails at every $\lambda \geq \kappa$, the singular cardinal hypothesis holds above $\kappa$, and stationary reflection holds at regular $\lambda \geq \kappa$ (on cofinality $< \kappa$). We investigate to what extent similar results hold assuming various generic embeddings rather than supercompact embeddings. We also discuss the relationship between various combinatorial principles (including weaker versions of square, scales, stationary reflection, and stationary reflection in $\mathcal{P}_\kappa\lambda$), and the influence of generic embeddings on these principles.

► MATTHEW FOREMAN, *A few remarks in Hungarian*.
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We give some new results for partition properties at successors of large cardinals (such as measurable cardinals and $\omega_1$).

► STEVE JACKSON, *Some combinatorial principles in $L(R)$*.
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Assuming $AD + V = L(R)$, we establish some combinatorial principles which seem important for understanding this model. In particular, these principles imply that all of the projective ordinals (and more) are supercompact through the largest Suslin cardinal.

## Abstracts of contributed talks

► ANDREW ABERDEIN, *How to be a deviant logician: A taxonomy of logical difference*.
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It is increasingly respectable to change a logical system in response to philosophical or scientific problems. This change may be effected either as a conservative extension or a revision of the prior system. Since the latter strategy raises greater philosophical concern, a taxonomy which could distinguish between logics advanced by the different strategies would be very useful. Classifications should be exhaustive of all logics likely to be of interest and mutually exclusive.

One attempt at such a taxonomy is developed in Susan Haack's ***Deviant Logic*** (Cambridge, 1974 and Chicago, 1996). She draws a syntactic distinction between extending a logic (conservatively of inferences), quasi-deviating (a non-conservative extension) and deviating (disturbing the validity of inferences without adding new material). I argue that the extensive criticism this has received can be answered if the taxonomy is rigorized and augmented with an account of equivalence between notational variants of a logical system. The development of such an account is complicated by unexpected flaws in the most obvious candidates. I argue that equivalence consists in a pair of surjections between the classes of formulae of the systems which preserve the partition of the classes of inferences into valid and invalid subclasses.

The Haack taxonomy is wide ranging, but at the expense of content. Another approach, utilizing sequent calculus methods, has been developed by Kosta Došen (*Logical Constants as Punctuation Marks*, ***Notre Dame Journal of Formal Logic***, 1989). He distinguishes alternative logics, which differ in structural inferences, and supplementary logics, which differ in the analysis of their constants. (Logics may be both.) I explore the relation between the two taxonomies, and show that on certain auxiliary assumptions Došen's categories are proper sub-classes of Haack's. Thus I offer the means for attempting a general account of logical plurality.

► EDWARD BELAGA, *Post-Hilbertian programme and its post-Gödelian stumbling-block*.
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> Had I been present at the creation, I would have given some useful hints for the better ordering of the universe.
>
> (Alfonso X the Wise, King of Castile, 1252–84)

Before Georg Cantor has entered the scene of mathematical infinity, two types of infinite totalities were already known "experimentally" to the mathematical community: the denumerable and the continuum. It is only after the advent of Cantor's discoveries that mathematical infinity has become both an attractive and perilous mathematical "Klondike" of sorts. Many mathematical "fortunes" were made there, and even more hopes were dashed, provoking from time to time sudden and powerful mathematical and philosophical "quakes" which could be felt far away from the "infinite epicentre". Fratricidal wars were waged, won and lost. And yet, the "infinite dust" is still far from settled!

A leading researcher into infinite wonders why so "many of my colleagues, including the best minds in the field of set theory, feel apologetic about their subject" [4]. Meanwhile, his well-known contemporary flatly dismisses his and his colleagues efforts of "setting up new axioms in the never-never land of large cardinals" [3]. To consummate the disunion, on the "infinite" side, dramatic announcements abound [2], whereas the other side remains unconvinced, indifferent, unaware, if not outright hostile.

This painful discord continues to be accompanied by no less painful conflicts of foundational philosophies of mathematics, of educational methodologies, etc. In fact, the integrity of mathematics [5], if not its very existence [1], are at stake. The question is now: Why? What is the meaning of this "foundational crisis of nearly unprecedented magnitude" (paraphrasing [2, p. 93])?

[1] VLADIMIR I. ARNOLD, *Will mathematics survive? Report on the Zurich congress*, ***Mathematical Intelligencer***, vol. 17 (1993), no. 3, pp. 6–10.
[2] HARVEY FRIEDMAN, *Necessary uses of abstract set theory in finite mathematics*, ***Advances in Mathematics***, vol. 60 (1986), pp. 92–122.
[3] SAUNDERS MAC LANE, *The health of mathematics*, *Mathematical Intelligencer*, vol. 4, (1983), no. 4, pp. 53–55.
[4] SAHARON SHELAH, *The future of set theory*, in ***Set theory of the reals*** (H. Judah, editor) Bar-Ilan University, Bar-Ilan, 1993, pp. 1–12.
[5] STEPHEN G. SIMPSON, *Partial realizations of Hilbert's program*, ***Journal of Symbolic Logic***, vol. 53 (1988), pp. 349–363.

► ANTHONY BONATO, *The model companion of colour classes*.
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Colour classes are $\forall_1$ classes over a finite relational language that generalize the classes of $n$-colourable graphs. Uniquely colourable structures play a special role in the theory of colour classes, leading to a proof of the existence of the model companion of a colour class. In this talk, we describe the connections between uniquely colourables and the $\aleph_0$-categorical generic whose theory axiomatizes the model companion of the colour class. Furthermore, we will sketch the proof that the model companion is non-finitely axiomatizable even modulo a strong form of the axiom of infinity.

► NORMAN DANNER, *Ordinals and functional abstraction*.
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In this talk, I will discuss an alternate method for ordinal measurement of arithmetic theories. Traditionally, such a measure is really a measure of the amount of additional "strength" one must add to a weak metatheory in order to prove consistency of the object theory. For example, one might associate to an object theory $T$ the least ordinal $\alpha$ such that the consistency of $T$ is provable in Primitive Recursive Arithmetic with transfinite induction for all ordinals $< \alpha$.

The novelty of the approach I will discuss is to consider certain derivations in the object theory as representing ordinals themselves. Given a theory $T$, one views derivations in $T$ as computational processes by mapping them into an appropriate version of the polymorphic lambda calculus via the Curry-Howard correspondence. One then interprets the terms of the lambda calculus as functionals in the finite type structure over $\omega_1$. The terms that are interpreted as ordinals are those with the type of the polymorphic Church numerals: $\forall R.(R \to R) \to (R \to R)$. Thus, we consider ordinals as "transfinite iterators", extending the interpretation of numbers as finite iterators (an induction axiom in the object theory is mapped to a term that represents an $\omega$-iterator). In this talk, I will concentrate on this second stage of the method by introducing an appropriate polymorphic lambda-calculus, its interpretation, and discussing bounds on the ordinals represented in the calculus.

► ILIJAS FARAH, *Gaps in analytic quotients*.
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One of the most striking early discoveries in Set Theory was Hausdorff's construction of a gap in the quotient $P(\mathbb{N})/\mathrm{Fin}$. Recently Mazur and Todorcevic constructed Hausdorff gaps in every quotient over an analytic ideal on $\mathbb{N}$. We investigate some new gap phenomena occuring in quotient algebras over analytic ideals. In particular, we construct a very simple

$(F_\sigma)$ gap in some quotients, an object which is known not to exist in the quotient $P(\mathbb{N})/\text{Fin}$. On the other hand, the gap-spectra of some quotients close to $P(\mathbb{N})/\text{Fin}$ are closely related with that of $P(\mathbb{N})/\text{Fin}$.

► IBRAHIM GARRO, *Resolving paradoxes of self reference using the theory of non-wellfounded sets*.
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Yahya bin Adi, a Christian Arab logician of the 10th century, formulated in a treatise defending the doctrine of the trinity, what I called the paradox of the empty relation.

In an unpublished manuscript whose results were sketched by Stachowiak (*Handbook of Pragmatics*, vol. V, pp. 291, 300), I demonstrated structural similarities between Yahya's paradox and the well-known paradoxes of the Liar and Russell.

I also show that Yahya's rudimentary treatment of a theory of relations could be developed into a mathematical theory based on the duality relation-object (fulfilling that relation). Thus I arrive at limitations on human knowledge (cf. Ibid, p. 291).

To resolve Yahya's paradox using the theory of non-wellfounded sets, an infinite general flat system is established recursively. The solution to this system resolves Yahya's paradox. In a similar fashion the Liar's and Russell's paradoxes are resolved thus reflecting the structural similarities between these paradoxes.

These results have applications in logic and computer science, such as reasoning with contradictions and non-monotonic reasoning.

► JAMES HIRSCHORN, *Towers of Borel functions*.
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We give a mathematical reformulation of the cardinals $\mathfrak{p}$ and $\mathfrak{t}$ in terms of families of Borel functions. As an application we show that $\mathfrak{t}$ is invariant under the addition of a single Cohen real.

[1] M. Bell, *On the combinatorial principle $P(\mathfrak{c})$*, ***Fundamenta Mathematicae***, vol. 114 (1981), no. 2, pp. 149–157.

[2] E. K. van Douwen, *The integers and topology*, in *Handbook of set-theoretic topology* (K. Kunen and J. E. Vaughan, editors), North-Holland, Amsterdam-New York, 1984, p. 116.

[3] I. Farah and S. Todorčević, ***Some applications of the method of forcing***, Yenisey Publishing Company, Moscow, 1995, pp. 11–12.

[4] T. Jech, ***Set theory***, Pure and Applied Mathematics, vol. 79, Academic Press, New York-London, 1978, p. 538.

[5] Z. Piotrowski and A. Szymański, *Some remarks on category in topological spaces*, ***Proceedings of the American Mathematical Society***, vol. 101 (1987), no. 1, pp. 156–160.

[6] J. Roitman, *Correction to*: "*Adding a random or a Cohen real*: *topological consequences and the effect on Martin's axiom*", ***Fundamenta Mathematicae***, vol. 129 (1988), no. 2, p. 141.

► DANIEL LEIVANT, *Peano's lambda calculus: The functional abstraction implicit in arithmetic*.
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We define a fragment of Girard's second order lambda-calculus, in which arguments of type application are required to be "Peano types", namely types generated, using arrow and substitution, from type variables and types of the form $(\forall R)\tau$, $\tau$ a quantifier free type of rank $\leq 2$. We show that the provably recursive functions of first order arithmetic are precisely the numeric functions that are lambda-definable in this calculus.

A consequence is that the consistency of intuitionistic second order arithmetic with prenex $\Pi^1_1$ comprehension is provable in a small subsystem of intuitionistic arithmetic. This gives a new illustration of the crucial role of implication in constructive mathematics.

► WENDY MACCAULL, *Generalized dependencies for information systems*.
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An information system, $S$, is a four-tuple $(X, A, \{ V(a) \mid a \in A \}, f)$, where $X$ is a set (of objects), $A$ is a set (of attributes) and, for each $x \in X$, and $a \in A$, $f(x, a) \subseteq V(a)$. (A database, where $f(x, a)$ can have more than one entry). Given such a system, one goal is to extract knowledge from the data. Among the problems considered are the determination of various constraints called data dependencies. Here we describe a deductive system based on frame semantics for information systems.

Information relations are (binary) relations among the objects in $X$ determined by the values of their attributes. The most well known are the indiscernability relations: for each $P \subseteq A$, and $x, y \in X$, $(x, y) \in R_P$ if and only if for all $p \in P$, $f(x, p) = f(y, p)$. Many data dependencies may be expressed using conditions on indiscernibility relations.

An information frame, $K$, is a two-tuple $(X, \{ R_P \mid P \subseteq A \})$ where the $R_P$ are binary relations indexed by subsets of a set $A$. To each information system $S$, and each kind of information relation, we may associate an information frame. If, for example, the information relations are indiscernibility relations, the $R_P$ must be equivalence relations satisfying the conditions that for all $P, Q \subseteq A$, $R_P \cap R_Q = R_{P \cup Q}$, and $R_\emptyset = X \times X$.

A tableaux deductive system allows us to determine dependencies from known dependencies for indiscernibility frames and generalizes to allow us to derive dependencies arising from known dependencies for other information relations, such as association relations and the many information relations arising in Rough Set Theory. In general, tableaux-completeness holds, so that if a branch of a tableau is open we may construct a frame countermodel. For some kinds of information relations, one can use this to construct a countermodel which is an information system; for other information relations, including association relations, the question remains open.

This work is a continuation of works by Orlowska and others in relational deduction for non-classical logics.

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We use forcing techniques to examine questions in computable model theory. We review a theorem of Ash, Knight, Manasse, and Slaman which relativized a result of Goncharov. We then use this theorem together with a forcing construction to argue that finite computable dimension does not relativize. In addition, we relativize a result by Davey about computable separability.

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We will show how to construct, using Martin's Axiom, a c.c.c. Boolean algebra which is not $\sigma$-centered and which contains no uncountable independent family. Such an algebra can be thought of as having properties similar to the interval algebra of a Souslin line (a c.c.c. non-separable linear order).

► MARCUS SCHAEFER, *A guided tour of minimal indices and shortest descriptions*.
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The set of minimal indices of a Gödel numbering $\varphi$ is defined as $\mathrm{MIN}_\varphi = \{\, e : (\forall i < e)[\varphi_i \neq \varphi_e] \,\}$ [2]. It was proved by Meyer [3] in 1972 that $\mathrm{MIN}_\varphi \equiv_{\mathrm{T}} \emptyset''$, but beyond this $\mathrm{MIN}_\varphi$ has remained mostly uninvestigated. This paper collects the scarce results on $\mathrm{MIN}_\varphi$ from the literature and adds some new observations including that $\mathrm{MIN}_\varphi$ is autoreducible, but neither regressive nor $(1, 2)$-computable. We also study several variants of $\mathrm{MIN}_\varphi$ that have been defined in the literature like size-minimal indices [1], shortest descriptions, and minimal indices of decision tables [4]. Some challenging problems remain open.

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► FRANK TALL, *Elementary submodels and the real line*.
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Given a topological space $\langle X, \mathcal{T} \rangle$ in an elementary submodel $M$ of $H_\lambda$, $\lambda$ sufficiently large, we consider $X_M = X \cap M$ with topology generated by $\mathcal{T} \cap M$ [1]. A particularly interesting case is when $X_M$ is homeomorphic to a subspace of the real line. In case $X_M$ is locally compact uncountable separable metric, $X = X_M$, but for Polish spaces, this is open in ZFC, even for the irrationals. However, assuming $0^\sharp$ does not exist, the theory does extend. These results will appear in [2]

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► MAKOTO TATSUTA, *Uniform continuity of graph model and partial function model*.
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The uniform continuity property MCONT for a partial combinatory algebra with pairing and natural numbers is that there exists a functional which uniformly produces from a type 2 functional its associate in the finite type structure on the algebras [2]. It is important since the consistency of systems $\mathbf{IZF} + \mathbf{RDC}$ and $\mathbf{EM}_0 + \mathbf{J} + \mathbf{RDC}$ with continuity principles $\mathbf{BP}_0 + \mathbf{LC}^+(\mathbf{X}, \mathbf{Y}) + \mathbf{UC}(\mathbf{X}, \mathbf{Y})$ is proved by using models for which MCONT holds [1, 2]. Known models with MCONT are only the partial function model and the graph model $\mathbf{P}(\mathbf{N})$, and the former is used in [1] and the latter is used in [2] for this purpose.

This paper shows that actually either the partial function model or the graph model does not have MCONT. It means that the consistency proofs in [1, 2] are incorrect and the consistency of the systems with the continuity principles is left unproved.

THEOREM 1. *The property* MCONT *does not hold for the graph model* $\mathbf{P}(\mathbf{N})$.

It implies that Theorem 8.2.1 in Page 138 of [2] is false.

The theorem is proved by supposing the functional $M$ exists and deriving contradiction by using extensionality of $M$. Let $F_i$ be a type 2 functional such that $F_i \cdot x$ outputs $x \cdot \bar{0}$ after getting $x \cdot \bar{0}, x \cdot \bar{1}, \dots, x \cdot \bar{\imath}$ and choose $k$ such that $M \cdot F_0 \cdot \overline{\bar{\alpha}(k)} > \bar{0}$. Then $M \cdot F_k \cdot \overline{\bar{\alpha}(k)}$ produces contradiction.

THEOREM 2.

1. *There exists a non-extensional type* 2 *functional in the partial function model.*
2. *The property* MCONT *does not hold for the partial function model.*

It implies that the proof of Theorem 1.2.1 in [1] is incorrect.

Intensionality of type 2 functionals proves (1) and (2) follows immediately from (1).

The property MCONT does not hold for models with non-extensional type 2 functionals. Hence many intensional models such as the partial function model, Kleene's first recursive model $\mathbf{K}_1$ and Kleene's second recursive model $\mathbf{K}_2$ do not have MCONT. On the other hand, MCONT does not seem to hold for extensional models such as the graph model in which type 3 functionals are extensional. Therefore, to find a model with MCONT, we should search a model with extensional type 2 functionals and non-extensional type 3 functionals.

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► J. V. TUCKER AND J. I. ZUCKER, *Generalised computations and equational specifications of functions on many-sorted algebras*.
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There is a stable theory for the class of functions computable on arbitrary many-sorted algebras $A$ by means of models of computation that generalise those of classical recursion theory on the naturals (Tucker and Zucker [6]). We investigate definability of such computable functions on $A$ by algebraic specification methods. We prove adequacy and soundness theorems using initial algebra semantics for theories composed of (1) *conditional equations*, and (2) a new class of formulae called *conditional bounded universal equations*. Among many results about these methods, we show that for any signature $\Sigma$ and any type $\tau$ on $\Sigma$, there is a uniform bound $B_{\Sigma,\tau}$ such that for all $\Sigma$-algebras $A$ and computable functions $f$ of type $\tau$ on

$A$, there is a conditional equational specification of $f$ of size at most $B_{\Sigma,\tau}$.

This is part of an ongoing research program on computability theory on many-sorted algebras (Tucker and Zucker [1, 2, 3, 4, 5, 6, 7]). It is also a generalisation of the theory of algebraic specifications for *computable* algebras developed by J. A. Bergstra and J. V. Tucker.

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► KRISTYO VAKARELOV, *Some generalizations of the notion "logical form."*

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In this paper is made an attempt to generalize the traditional logical forms—concept, statement and conclusion. As a basis is taken the conception of logic as a science studying ways for transformation of different types of information contained in the elements of a given system. Such an approach allows generalizations to be sought in the area of logic, based on the analysis of thought processes and, as well as in the functioning of artificial informational systems. Particular attention is paid to the analysis of *theory* as a logical form.

Formulated is the thesis that concept brings about statement when we consider a system of compatible concepts, whereas statement brings about theory when we consider a system of compatible statements. This hierarchy of logical forms may be extended. The different types of logical forms, and methods for their transformations in accordance to logical norms, forces a reconsideration of many logical notions, such as: *truth*, *entailment*, *conclusion*, *proof*, *logical law*, *theoretical transformation*, etc.

The apparatus of symbolic logic springs from a number of presuppositions in explication of the logical notions. Supported is the thesis for the linguistic foundation of this apparatus; something which raises problems for its adequacy and applicability. The basic notions are valid only for first order formal languages. As far as application is concerned, however, it is necessary first, to extend the formal decies—*signature*, *formula*, *formal language*, *formal theory*—and second, to adopt some content devices from the realm of application—*informal intuitive theory*, *universe of discourse* and *system of predicates*. For the purpose of logic and its applications it is useful to be considered *classes of languages* for description, *classes of theories*, and *classes of various universes* for interpretation. The characterization of these classes through the basic characteristics of the elements of the other classes allows an answer to the problem of the connection among the elements of the triple ⟨*language*, *theory*, *model*⟩ to be found. The generalization of *logical form* introduces the needed dynamics in the triple, which also may help to clarify the process of development of science.

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A permutation $g \in \text{Sym}(\omega)$ is cofinitary if and only if $g$ has only finitely many fixed points. A group $G \leq \text{Sym}(\omega)$ is cofinitary if and only if every non-identity element is cofinitary. It is easily seen that $G \leq \text{Sym}(\omega)$ is cofinitary if and only if

1. $G$ is an almost disjoint set of permutations, and
2. $G$ is a group.

Since the union of a chain of cofinitary permutation groups is cofinitary, Zorn's Lemma implies that maximal cofinitary groups exists; and indeed any cofinitary group is in a maximal one. We first state a theorem which was proved by Truss and Adeleke.

THEOREM 1. *If $G \leq \text{Sym}(\omega)$ is a maximal cofinitary group, then $G$ is not countable.*

P. Cameron pointed out the following result which was proved by P. Neumann.

THEOREM 2. *There exists a maximal cofinitary group of cardinality* $2^\omega$.

The following question was asked by P. Cameron.

QUESTION 3. *If Continuum Hypothesis* (CH) *fails, is it possible that there exists a maximal cofinitary group $G$ such that* $|G| < 2^\omega$?

I answered this question by proving the following results.

THEOREM 4 (MA). *If $G \leq \text{Sym}(\omega)$ is a maximal cofinitary group, then $G$ has cardinality* $2^\omega$.

THEOREM 5. *Let $M \models (\text{ZFC}+\neg\text{CH})$. Let $\kappa \in M$ be a cardinal such that $\omega_1 \leq \kappa < 2^\omega = \lambda$. Then there exists a c.c.c. notion of forcing $\mathbb{G}$ such that the following statements holds in $M^{\mathbb{G}}$.*

1. $2^\omega = \lambda$.
2. *There exists a maximal cofinitary group $G \leq \text{Sym}(\omega)$ of cardinality $\kappa$.*

Therefore, we can consider the following cardinal number: Let $\mathfrak{a}_\mathfrak{g}$ be the least $\lambda$ such that there exists a maximal cofinitary group $G$ of cardinality $\lambda$.

It would be interesting to know the relationship between $\mathfrak{a}_\mathfrak{g}$ and other well-known continuum invariants. Some results have already been proved in this area. For example,

1. $\mathfrak{a}_\mathfrak{g} < \mathfrak{d}$ is consistent;
2. $\mathfrak{d} < \mathfrak{a}_\mathfrak{g}$ is consistent;
3. $\mathfrak{a} < \mathfrak{a}_\mathfrak{g}$ is consistent,
4. $\mathfrak{a}_\mathfrak{g} = \text{cover(meager)} < \text{cover(null)}$ is consistent;
5. $\text{cover(meager)} < \mathfrak{a}_\mathfrak{g} = \text{cover(null)}$ is consistent.

## Abstracts presented by title

► JOHN CORCORAN, *Numerically qualified omega completeness*.
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In *GF*, the framework of the 1931 Gödel paper, there are two explicit non-logical constants, '$o$' and '$s$'; the concept "number" is absorbed into the intension of the individual variables by taking the domain $D$ of number theory as the universe of discourse. In the framework of the 1889 Peano booklet, $D$ is taken as a subset of the range of individual variables and, accordingly, "number" takes a distinctive non-logical constant. In a modern version, *PF*, of the Peano framework, the numbers are treated as individuals having a distinctive quality, say, "being numerical", indicated by a one-place predicate '$N$' supplementing '$o$' and '$s$'. In view of their higher-order character, in both frameworks versions of mathematical induction are expressible by single sentences; Gödel induction is 'every property belonging to zero and to the successor of every number to which it belongs belongs to every number', Peano induction is 'every property belonging to zero and to the successor of every individual to which it belongs belongs to every numerical individual.' In $GF$, a semantic theory (or set of sentences closed under consequence) is omega-complete if and only if it contains every unqualified universal sentence 'every number has $P$' whenever it contains every numeral instance, '$o$ has $P$', '$so$ has $P$', '$sso$ has $P$' and so on. One characteristic metatheorem of $GF$ is that a semantic theory is omega-complete if and only if it contains Gödel induction. This paper extends these concepts and results to the modern Peano framework. Here a semantic theory is defined to be numerically qualified omega-complete if and only if it contains every numerically qualified universal sentence 'every numerical individual has $P$' whenever if contains every unqualified instance, '$o$ has $P$', '$so$ has $P$', '$sso$ has $P$', and so on. Surprisingly, in $PF$ it turns out that a semantic theory is numerically qualified omega-complete if and only if it contains Peano induction.

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DEFINITION. A $G$-diagram for a structure $M$ is a diagram $D\langle A, G\rangle$, such that the diagram has a proper definition by a specific function sets.

THEOREM 6. *$G$-diagrams encode possible worlds.*

We define Deterministic Epistemology, D.E., to be logic and epistemics definable by a deterministic logic and model theory. We further define Probabilistic Epistemology, P.E., to be a logic and epistemics defined by probabilities, as in known probabilistic logics.

THEOREM 7. *There is no reduction from D.E. to P.E.*

DEFINITION. A situation consists of a nonempty set $D$, the domain of the situation, and two mappings: $g$, $h$. $g$ is a mapping of function letters into functions over the domain as in standard model theory. $h$ maps each predicate letter, $pn$, to a function from $Dn$ to a subset of $\{t, f\}$, to determine the truth value of atomic formulas as defined below. The logic has four truth values: the set of subsets of $\{t, f\}.\{\{t\}, \{f\}, \{t, f\}, 0\}$. The latter two corresponding to inconsistency, and lack of knowledge of whether it is true or false.

A compatible set of situations is a set of situations with the same domain and the same mapping of function letters to functions. In other words, the situations in a compatible set of situations differ only on the truth conditions they assign to predicate letters.

THEOREM 8. *Two situations are compatible if and only if their corresponding generalized diagrams are compatible with respect to the Boolean structure of the set to which formulas are mapped (by the function h above, defining situations).*

THEOREM 9. *There is a Boolean algebra $A'$ in the class $A$, $A'$ containing only $\{0, 1\}$, such that for the initial model $B$ defined by the $G$-diagram, $B \models$ is equivalent to $\models$ for $A'$.*

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Alfred North Whitehead published *Universal Algebra* (UA) in 1897 followed by Bertrand Russell's publication of *Principles of Mathematics* (PoM) in 1903. Each had planned a second volume, but instead decided to collaborate on a common work. The collaboration didn't produce the expected second volume, but, instead, *Principia Mathematica* (PM). Neither UA nor PoM looks like PM. This paper explores the collaboration between Russell and Whitehead that ultimately resulted in PM by tracing the historical and mathematical developments as well as some of the differences between UA, PoM, and PM.

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In this paper, we discuss various properties in saturated $B\Sigma_2$ model. We prove that any $\Sigma_3$ set is either below $0'$ or above $\omega$, the $\Sigma_2$ cut of the model. An conclusion is that $0'$ and $\omega$ are definable in the class of degrees below $0''$ in this model. The other main result is about the jump of degree. We show that $I \oplus 0'$ is not a jump of any degree. Some results in $B\Sigma_1$ model and $I\Sigma_2$ model are also included in this paper.

► ANDRES VILLAVECES AND AMIR LESHEM, *Destroying GCH at unfoldable cardinals.*
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I will present a proof of a joint result with Amir Leshem that if $\kappa$ is an unfoldable cardinal in $L$ then there is a class forcing extension in which $\kappa$ is strongly unfoldable and $2^\kappa = \kappa^{++}$. This answers a question of mine from [1].

I first defined Unfoldable cardinals in [1]. The basic motivation for their definition is the study of chains of elementary chains of models of ZFC.

Unfoldability relativises down to $L$, their consistency strength is less than the consistency strength of a subtle cardinal (if $\theta$ is subtle then there is a $\theta$-unfoldable cardinal, and one can use $V_\theta$ as the inner model for unfoldability). If a subtle cardinal $\theta$ exists then one can force that the GCH fails at every inaccessible below $\theta$ without destroying its subtlety. From these two results it is clear that the consistency strength of the failure of GCH at an unfoldable cardinal is less than the consistency strength of a subtle cardinal. I conjectured in [1] that the consistency strength of the failure of GCH at an unfoldable cardinal is that of a strongly unfoldable cardinal.

I will present some of the previous results, and will sketch a proof of

THEOREM. *If an unfoldable cardinal exists, then it is consistent that there exists a strongly unfoldable cardinal, at which the GCH fails.*

This shows that the above conjecture holds: even more, GCH actually fails at every $L$ unfoldable cardinal, while preserving its strong unfoldability.

[1] A. VILLAVECES, *Chains of elementary end extensions of models of set theory*, ***Journal of Symbolic Logic*** (1997), accepted.