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1997–1998 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Baltimore, Maryland, January 9–10, 1998

The Winter Meeting of the Association was held in Baltimore, Maryland, on January 9–10, 1998, in conjunction with the annual meeting of the American Mathematical Society. The Program Committee consisted of A. Carbone, M. Detlefsen, Y. Gurevich, A. Kanamori, L. Moss, and R. Parikh (Chair). The program included eight invited talks, eight contributed talks, and two papers presented by title only.

The following invited talks were presented:

Juliet Floyd (Boston University), *Wittgenstein on Gödel and mathematics*.

Harvey Friedman (Ohio State University), *Finite trees and the necessary use of large cardinals*.

Marcia Groszek (Dartmouth), *A solution to Prikry's problem* (with Theodore A. Slaman).

Petr Hájek (Czech Academy of Sciences), *The liar's paradox and fuzzy logic* (joint work with J. Paris and J. Shepherdson).

Joel Hamkins (College of Staten Island, CUNY), *How tall is the automorphism tower of a group*?

Dexter Kozen (Cornell), *Kleene algebra with tests*.

Isaac Levi (Columbia), *Ramsey revision, recovery, and demands for information*.

Leonid Libkin (Bell Labs), *First-order logic, counting, and locality*: *Proving expressivity bounds over finite models*.

Abstracts of invited talks and of contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
ROHIT PARIKH

Abstracts of invited talks

JULIET FLOYD, *Wittgenstein on Gödel and mathematics*.
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Wittgenstein's remarks on Gödel's first incompleteness theorem often have been denounced: Kreisel calls them "wild," Dummett has insisted that they "are of poor quality and contain definite errors," Wright discreetly fails to mention them in his book on Wittgenstein's philosophy of mathematics. Gödel himself claimed that Wittgenstein had not understood the theorem, or at least had "pretended not to." On the contrary, I argue that Wittgenstein did understand the theorem, that he had no interest in trying to refute it, and that in fact he appropriated it for his own philosophical purposes. The result is a spare interpretation

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of the significance of the theorem, that is, a denial that it has any special relevance to our understanding of such general notions as proof or truth. For Wittgenstein, the Gödel result is just one more impossibility proof, perfectly analogous to classical impossibility proofs in geometry, such as the proof that it is impossible to trisect an angle with ruler and compass alone. No one has objected that Wittgenstein failed to understand these proofs. Russell and Frege's forms of logicism are the real targets of Wittgenstein's criticisms.

HARVEY FRIEDMAN, *Finite trees and the necessary use of large cardinals*.
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We combine a basic form of the greedy tree construction with classical Ramsey theory in order to obtain simple basic discrete and finite theorems which can only be proved by going well beyond the usual axioms of mathematics, as represented by the so-called large cardinal axioms. The results suggest a new area of research which we call "greedy Ramsey theory."

MARCIA J. GROSZEK AND THEODORE A. SLAMAN, *A solution to Prikry's problem*.
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We show that if there is a nonconstructible real, then every perfect set of reals contains a nonconstructible real. This answers a question of Prikry (cited in [1]).

[1] HARVEY FRIEDMAN, *102 problems in mathematical logic*, ***Journal of Symbolic Logic***, vol. 40 (1975), pp. 113–129.

PETR HÁJEK, *The liar's paradox and fuzzy logic* [Joint work with J. Paris and J. Shepherdson].
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Can one extend crisp Peano arithmetic PA by a possibly many-valued predicate $\mathrm{Tr}(x)$ saying "x is true" and satisfying the "dequotation schema" $\varphi \equiv \mathrm{Tr}(\bar{\varphi})$ for all sentences φ? This problem is investigated in the frame of Łukasiewicz infinitely valued logic.

JOEL HAMKINS, *How tall is the automorphism tower of a group*?.
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The automorphism tower of a group G is obtained by computing the automorphism group of G, the automorphism group of *that* group, and so on, iterating transfinitely. Each group maps canonically into the next using inner automorphisms; at limit stages one takes a direct limit. Does one eventually reach a fixed point, a group which is isomorphic to its automorphism group by the natural map? The classical result of Wielandt (1939) is that the automorphism tower of any finite centerless group reaches a fixed point at a finite stage. Extended by various authors to larger collections of groups, the analysis culminated in Thomas's elegant theorem (1985) that the automorphism tower of any centerless group eventually reaches a fixed point. In this talk, I will remove the 'centerless' assumption, and prove that the automorphism tower of *any* group eventually reaches a fixed point. How long does it take to reach the fixed point? The answer, using the set-theoretic technique of forcing, is that it depends on the model of set theory in which you live. One can construct a group whose automorphism tower various tremendously in height from one set-theoretic universe to another. Many simple-to-state problems remain open.

DEXTER KOZEN, *Kleene algebra with tests*.
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Kleene algebra is the algebra of regular expressions. It arises in many contexts: relational algebra, semantics and logics of programs, automata and formal language theory, and the design and analysis of algorithms.

A Kleene algebra with tests is a Kleene algebra with an embedded Boolean subalgebra. The presence of tests allows us to model conventional programming constructs such as conditionals and while loops and to reason equationally with these constructs. Many simple program manipulations, such as loop unwinding and basic safety analysis, can be carried out purely equationally using the axioms of Kleene algebra with tests. Cohen has used a similar system to give an equational proof of correctness of a lazy caching protocol.

We illustrate the use of Kleene algebra with tests by giving a purely equational proof of the following classical results: every while program can be simulated by a while program with at most one while loop. This is interesting because of the widely held belief that there is no purely propositional proof of this result, and that it cannot be achieved without introducing extra variables. We show that this is incorrect.

We also discuss completeness, complexity, and expressiveness results.

ISAAC LEVI, *Ramsey revision, recovery, and demands for information*.
Columbia University.

Revision of a belief state, according to the approach developed by Alchourrón, Gärdenfors and Makinson is understood to have the following properties:

(1) When a theory K representing a belief state entails neither h nor $\sim h$, the revision K^*_h by adding h is the expansion $\mathrm{Cn}(K \cup \{h\}) = K^+_h$ by adding h.
(2) When a theory K entails h, $K^*_h = K$.
(3) When $K \models \sim h$, K^*_h is the result of contracting K by removing $\sim h$ to form $K^-_{\sim h}$ and expanding the result by adding h.

Revision so conceived in insensitive to whether the disputed Recovery Condition asserting that $[K^-_h]^+_h \supseteq K$ should hold or not. So doubts about the Recovery Condition have been ignored in applications of the operation of revision.

However, in cases where revision is used to clarify suppositional or conditional reasoning, AGM revision should be replaced by Ramsey Revision that replaces property (2) by the following:

(2′) When $K \models h$, $K^{*r}_h = [K^-_h]^+_h$.

Ramsey Revision is sensitive as to whether the Recovery Condition holds. The case for abandoning Recovery in the setting of suppositional reasoning is then elaborated by reference to the way losses of information are evaluated in the context of contraction.

LEONID LIBKIN, *First-order logic, counting, and locality*: *Proving expressivity bounds over finite models*.
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Proving expressivity bounds over finite models, one encounters difficulties, because many tools from model theory become unavailable: for example, compactness fails over finite models. The main tool that is available is Ehrenfeucht-Fraisse games, which are notoriously hard to play on complex structures. At the same time, expressivity bounds for first-order logic and its extensions have immediate applications in computer science, in particular, in the study of complexity classes and database query languages. In this talk, I shall survey tools for proving such bounds based on the idea of locality; intuitively, it means that the truth value of a formula $f(x)$ depends on a small portion of the structure—a neighborhood of a fixed radius around x. This idea goes back to Gaifman and Hanf. There are several ways of formulating the notions of locality. It turns out that they are closely related to each other, and that they apply to various counting extensions of first-order logic. I shall describe those notions, derive some recently obtained expressivity bounds for first-order logic with various forms of counting, and briefly explain applications in complexity and databases.

Abstracts of contributed talks

ANTHONY BONATO, *A pigeonhole principle for relational structures*.
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A relational structure A has the pigeonhole property $(\mathcal{P})$ if every finite partition of A has some block isomorphic to A. In this talk, which is based on joint work with Dejan Delić, I will present a survey of results about relational structures with $(\mathcal{P})$. In particular, I will present a generalization of a result of Cameron, characterizing the generic structure of a parametric class over a binary language via $(\mathcal{P})$ and a weak form of existential closedness.

JOHN CORCORAN, *Mathematical induction and specific-case semantic omega properties*.
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Although the axiomatizations of second-order number theory preferred by mathematicians take as primitive entities (besides the universe of natural numbers beginning with zero or with one) the initial number and the successor function, there are axiomatizations whose languages do not involve numerals, i.e., terms composed of a symbol for the initial number preceded by zero or more symbols for the successor function. Taking the *property* of being initial (instead of the initial *number*) and taking the *relation* of [immediate] succession (instead of the successor *function*), the following property-universal proposition can serve as the principle of mathematical induction: every property that belongs to every initial number and to every number that succeeds a number to which it belongs also belongs to every number. In such contexts the usual (numeral-dependent) omega notions do not apply; instead a [semantic] theory, or set of propositions closed under consequence, is defined to be omega complete (respectively, consistent) if and only if it contains (respectively, lacks the negation of) every number-universal proposition all of whose "specific cases" it contains. By the specific cases of a number-universal proposition, "every number has [property] P," is meant the following: "every initial number has P," "every number succeeding an initial number has P," "every number succeeding a number succeeding an initial number has P," etc.

Presupposing a standard formalization of classical second-order logic, as found in the 1956 Church book, e.g., this paper shows that a theory is omega complete (respectively, consistent) if and only if it contains (respectively, lacks the negation of) the principle of mathematical induction. Strictly analogous results hold in the case of the language obtained by deleting the primitive for "initial" as long as the expression for "is initial" is replaced by an expression for "does not succeed any number" in the sentence for mathematical induction and in the definition of "specific case."

However, when second-order languages are replaced by corresponding first-order languages and the sentence expressing mathematical induction is replaced by the infinite set of its first-order substitution instances, i.e., the instances of the induction scheme, difficulties arise. Although it becomes clear that in this case every omega complete (respectively, consistent) theory contains (respectively, lacks the negation of) every particular instance of the induction scheme, as one would expect, the converse question seems open. In particular it is not known to the author, either when the language contains constants for "initial" and for "succeeds" or when it contains only the latter, whether every theory containing every instance of the appropriate induction scheme is omega complete, i.e., whether the theory axiomatized by the induction scheme is omega complete.

DENNIS F. CUDIA, *Information and entropy in Aristotelian logic*.
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Following E. Post a semi-Thue system G' is given that generates exactly the set of 96 analytic judgments over the propositions A, E, F, G, I, O, U, Z, where F and G are universal propositions defined by

$$\text{All } {\sim}\alpha \text{ are } \beta \quad \text{and} \quad \text{All } {\sim}\alpha \text{ are } {\sim}\beta,$$

respectively, and U and Z are particular propositions defined by

$$\text{Some } {\sim}\alpha \text{ are } \beta \quad \text{and} \quad \text{Some } {\sim}\alpha \text{ are } {\sim}\beta,$$

respectively. A, E, U, Z distribute subjects and E, O, G, Z distribute predicates.

Definition. An *analytic mood* is a mood such that there exists a figure such that the figure-mood combination is an analytic judgment.

G' is a context-sensitive grammar with

$$V_T = \{1, 2, 3, 4, A, E, F, G, I, O, U, Z\}$$

whose context free productions generate exactly the set of 54 analytic moods and whose context sensitive productions generate explicitly the set of figure(s) for each analytic mood. The context free productions are:

$S \to \langle\text{figure}\rangle\ \langle\text{mood}\rangle$,
$\langle\text{mood}\rangle \to \langle S \text{ or } P \text{ distributed}\rangle \mid \langle S \text{ and } P \text{ or neither distributed}\rangle$,
$\langle\text{one figure only}\rangle \to \langle\text{fig. 1}\rangle \mid \langle\text{fig. 2}\rangle \mid \langle\text{fig. 3}\rangle \mid \langle\text{fig. 4}\rangle$,
$\langle\text{fig. 1}\rangle \to AAA \mid GGG \mid OGO \mid GOO \mid UAU \mid AUU$,
$\langle\text{fig. 2}\rangle \to GAA \mid AGG \mid AOO \mid UGO \mid OAU \mid GUU$,
$\langle\text{fig. 3}\rangle \to AGA \mid GAG \mid OAO \mid GUO \mid AOU \mid UGU$,
$\langle\text{fig. 4}\rangle \to GGA \mid AAG \mid UAO \mid AUO \mid OGU \mid GOU$,
$\langle\text{all figs}\rangle \to FEA \mid EFG \mid EIO \mid ZFO \mid IEU \mid FZU$,
$\langle S \text{ and } P \text{ or neither distributed}\rangle \to \langle\text{fig. 1, 2}\rangle \mid \langle\text{fig. 3, 4}\rangle \mid \langle\text{fig. 2, 4}\rangle \mid \langle\text{fig. 1, 3}\rangle$,
$\langle\text{fig. 1, 2}\rangle \to EAE \mid ZAZ \mid EUZ \mid \langle\text{Neither 1, 2}\rangle$,
$\langle\text{fig. 3, 4}\rangle \to EGE \mid ZGZ \mid EOZ \mid \langle\text{Neither 3, 4}\rangle$,
$\langle\text{fig. 2, 4}\rangle \to AEE \mid AZZ \mid UEZ \mid \langle\text{Neither 2, 4}\rangle$,
$\langle\text{fig. 1, 3}\rangle \to GEE \mid GZZ \mid OEZ \mid \langle\text{Neither 1, 3}\rangle$,
$\langle\text{Neither 1, 2}\rangle \to FGF \mid IGI \mid FOI$,
$\langle\text{Neither 3, 4}\rangle \to FAF \mid IAI \mid FUI$,
$\langle\text{Neither 2, 4}\rangle \to GFF \mid GII \mid OFI$,
$\langle\text{Neither 1, 3}\rangle \to AFF \mid AII \mid UFI$.

The context sensitive productions are:

$\langle\text{figure}\rangle\ \langle\text{fig. } i\rangle \to i\langle\text{fig. } i\rangle$ for $i = 1, 2, 3, 4$,
$\langle\text{figure}\rangle\ \langle\text{all figs}\rangle \to 1\langle\text{all figs}\rangle \mid 2\langle\text{all figs}\rangle \mid 3\langle\text{all figs}\rangle \mid 4\langle\text{all figs}\rangle$,
$\langle\text{figure}\rangle\ \langle\text{fig. } i, j\rangle \to i\langle\text{fig. } i, j\rangle \mid j\langle\text{fig. } i, j\rangle$ for $(i, j) = (1, 2), (3, 4), (2, 4), (1, 3)$.

It is explained why G' is not a violation of the second law of thermodynamics.

DOUGLAS E. ENSLEY AND RAMI P. GROSSBERG, *Ramsey's theorem in finite structures*.
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Frank Ramsey in his fundamental paper was interested in "a problem of formal logic." Ramsey proved his theorem in order to construct a finite model for a given finite universal theory so that the universe of the model is canonical with respect to the relations in the language. This paper seeks the improvement in the size of this construction that intuitively must follow from stability-like assumptions on the theory. It turns out that for theories without the independence property, we can derive better upper bounds than for the general Ramsey numbers.

SU GAO, *Another dichotomy for simple trees*.
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This is a continuation of our study of the isomorphism relation of countable models. In this paper we focus on simple trees, or countable rooted trees of height $\leq \omega$. We show that if an $L_{\omega_1\omega}$ sentence describes an invariant Borel class of simple trees, then either the isomorphism relation on this class is Borel or else the class is Borel complete. This dichotomy theorem implies the author's earlier theorem of the Glimm-Effros dichotomy for simple trees and partially answers a question of Friedman and Stanley.

STEPHEN HARNISH, *GST: A theory of hypersets*.
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This paper gives an overview of the properties of GST-graph isomorphism set theory, a new theory of non-well-founded (NWF) sets. The axioms, properties, and relative consistency results concerning GST were developed in [2].

GST does not satisfy the foundation axiom. Peter Aczel has presented several non-well-founded set theories within a unified framework [1]. For each bisimulation $\sim$, he defined the axiom $\mathrm{AFA}^{\sim}$: *ann accessible pointed graph* (*apg*) *is an exact picture if and only if it is* $\sim$*-extensional*. When $\sim$ is a regular bisimulation whose definition is absolute for full systems, then the resulting set theory $\mathrm{AFA}^{\sim}+\mathrm{ZFC}^{-}$ (i.e., ZFC$-$foundation) is known to be consistent relative to ZFC. Regularity causes such an $\mathrm{AFA}^{\sim}$ set theory to satisfy extensionality.

Isomorphism between apgs (denoted by $\cong$) is a bisimulation. However, this bisimulation is not regular. Now consider the axiom $\mathrm{AFA}^{\cong}$. It implies the following criterion for set equality: two sets are equal if and only if their canonical graphics are isomorphic. GST is the theory $\mathrm{AFA}^{\cong}+\mathrm{ZFC}^{--}$ where ZFC^{--} is a modification of ZFC that excludes foundation and the standard statement of extensionality. More specifically, ZFC^{--}'s axioms of separation, extensionality, infinity, and choice add requirements that certain sets are noncyclic (i.e., not in any $\in$-cycles). These modified axioms of GST are equivalent to the standard ones when working within ZFC.

It is known that the GST universe has an inner $\in$-model of ZFC [2]. However, since GST satisfies neither extensionality nor foundation, it has a much "richer" universe than does ZFC. For instance, we will consider two surprising cardinality results which show that in GST some finite sets have infinite power sets.

We will end by considering several topics from geometry and the theory of language which could utilize the richness of the GST universe of sets. Key tools for such applications are analogs of the solution lemma for anti-foundation axioms besides AFA [3]

[1] P. ACZEL, ***Non-well-founded sets***, Center for the Study of Language and Information, Stanford, 1988.

[2] S. HARNISH, *A non-well-founded set theory (GST)*, ***Ph.D. thesis***, University of Illinois, Urbana, 1995.

[3] ———, *Bisimulation collapses and analogs of the solution lemma*, Winter Meeting of the Association for Symbolic Logic, Orlando, Florida, January 13, 1996.

JEFFRY L. HIRST, *Sorting, spanning, and reverse mathematics*.
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Employing the techniques of reverse mathematics, we will analyze the logical strength of some theorems from algorithmic graph theory. In particular, we will consider infinite versions of some theorems about topological sortings and minimal spanning trees. These results are the product of joint work with P. Clote.

WENDY MACCAULL, *Relational tableaux for tree models, language models and information networks*.
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A tableaux-style proof calculus is given for formalisms for finite tree models, which rests on the Orlowska-style relational semantics for nonassociative Lambek calculus (see [1]). By studying such finite tree models, we work toward finding a proof calculus for Moortgat and Oehrle's system for categorial grammar which treats the linguistic phenomena of headedness and discontinuous constituency (see [2]). This proof system may be extended to deal with the full spectrum of substructural logics; modifications to provide a proof calculus for Barwise's logic for information, which formalizes the flow of information through an information network, are outlined. The relationship of the Orlowska-style relational semantics to other relational semantics (such as the arrow logics of van Benthem, Venema and Vakarelov, and the gaggle theory of Dunn) is clarified. Some comparisons to Gabbay's labelled deduction method are made. This work is part of the general program to find effective, modular proof systems for nonclassical logics, based on relational semantics.

[1] WENDY MACCAULL, *Relational tableaux for tree models, language models and information networks*, submitted.

[2] YDE VENEMA, *Tree models and (labeled) categorial grammar*, ***Journal of Logic, Language and Information***, vol. 5 (1996), pp. 253–277.

Abstracts presented by title

GREG HJORTH, *Isomorphism of countable abelian groups*.
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The isomorphism relation on countable abelian groups is *as complicated* as the isomorphism relation on arbitrary countable structures.

PAUL STRAUSS, *The Russell paradox in arithmetical set theory*.
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We assume familiarity with [1]. Let ℓ be a Gödel number of the representing function of the predicate $\sim\{\psi(x)\}(x) = 0$, then

$$(x).{\sim}x \in 3^{\psi(x)} \equiv {\sim}\{\psi(x)\{(x) = 0 \equiv \{\ell\}(x) = 0 \equiv x \in 3^{\ell}.$$

From this, we infer that although ℓ defines a general recursive function, this function is not primitive recursive. Of course, $\lambda x\ \{\psi(x)\{(x)$ is an even simpler function with this property. We easily see that there exists a numeral p such that $2 \cdot 3^{\psi(p)}$ is a set equivalent to 3^{ℓ}.

[1] P. STRAUSS, *Arithmetical set theory*, ***Studia Logica***, vol. 2 (1991), pp. 343–349.