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GÖDEL'S PATH FROM THE INCOMPLETENESS THEOREMS (1931) TO PHENOMENOLOGY (1961)

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In a lecture manuscript written around 1961 (Gödel *1961/?), Gödel describes a philosophical path from the incompleteness theorems to Husserl's phenomenology. It is known that Gödel began to study Husserl's work in 1959 and that he continued to do so for many years. During the 1960s, for example, he recommended the sixth investigation of Husserl's *Logical Investigations* to several logicians for its treatment of categorial intuition (Wang 1997, p. 164). While Gödel may not have been satisfied with what he was able to obtain from philosophy and Husserl's phenomenology, he nonetheless continued to recommend Husserl's work to logicians as late as the 1970s. In this paper I present and discuss the kinds of arguments that led Gödel to the work of Husserl. Among other things, this should help to shed additional light on Gödel's philosophical and scientific ideas and to show to what extent these ideas can be viewed as part of a unified philosophical outlook. Some of the arguments that led Gödel to Husserl's work are only hinted at in Gödel's 1961 paper, but they are developed in much more detail in Gödel's earlier philosophical papers (see especially 1934, *193?, 1944, 1947, *1951, *1953/59). In particular, I focus on arguments concerning Hilbert's program and an early version of Carnap's program.

§1. Some ideas from phenomenology. Since Husserl's work is not generally known to mathematical logicians, it may be helpful to mention briefly a few details about his background. Husserl received his doctorate in mathematics in 1883 with a thesis on the calculus of variations. He then served for a brief period as an assistant to Weierstrass in Berlin. His interests soon turned to the philosophy of logic and mathematics. Eventually his philosophical interests widened considerably, although he continued to write on philosophical issues about logic. He was personally acquainted with Cantor, Zermelo,

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Hilbert and a number of other major figures in logic. He also corresponded with Frege, Schröder and others. Hilbert was evidently personally involved in helping Husserl to obtain a position in Göttingen in 1901. Husserl's philosophical work was deeply influenced by his mathematical background. This will become apparent below. Husserl exerted some influence on Hermann Weyl, on Arend Heyting by way of Oskar Becker, and on some of the Polish logicians who were active in the early part of this century. Heyting and Becker, for example, established an interesting connection with Husserl's work by identifying mathematical constructions (in the sense of constructive mathematics) with fulfilled mathematical intentions, where the notion of 'intention' is understood in terms of the theory of intentionality.

I begin with a description of some of Husserl's ideas that are directly relevant to Gödel's comments in 1961 (see also Tieszen 1992, 1995, 1997b, 1998; Føllesdal 1995; Wang 1996). The most important ideas are related to the fact that human cognition, including mathematical cognition, exhibits intentionality. This means that cognition is always about something. Consciousness is always consciousness *of*. One can see this clearly in acts of believing, knowing, willing, desiring, remembering, hoping, imagining, and so on. For example, an act of belief is always directed toward a particular object or state of affairs, even if this directedness is indeterminate in some respects. Try to imagine for a moment what it would be like for a belief to not be about anything in particular.

For an act to be directed in a particular way means that it is not directed in other ways. Cognitive acts are perspectival and we cannot take all possible perspectives on an object or domain. We do not experience everything all at once. Perhaps only a god could do that, but we cannot. Our beliefs and other cognitive acts at any given time are, in other words, always about certain *categories* of objects. The mind always categorizes in this way. It cannot help doing so, for our experience would have to be very different otherwise. Thus, many types or categories are always at work in our experience, even in everyday sense experience. I shall also refer to these categories as 'concepts'.

In order to introduce some specific texts of Husserl that clearly influenced Gödel, I now introduce a new term for what I have been calling 'categories' or 'concepts'. I will call these categories 'essences'. It is important not to let your mind wander to other associations you may have with the term 'essence'. I only mean for it to be another way of speaking of these categories in our experience. There are several reasons why it makes sense to use this term. Thus, at least the following associations are permissible at this point. First, we are concerned with *what* the experience is about at a given stage, and 'whatness' has been associated with the notion of essence at least since the time of Aristotle. Second, we can think of an essence in this sense as a universal. For example, I can see different instances of the category 'chair'. I can believe that there are different instances of the category 'natural number' or of the category 'function' or 'set'. I cannot believe that just anything is an instance of the category 'natural number'. The essences are not arbitrary. There are constraints on them and they are not subject to being changed at will. They are not freely variable.

It is safe to say we *know* that certain things are not instances of the category 'natural number' and that other things are instances of this category. Thus, we must have some grasp of this category. Of course our grasp of this essence has become more precise over time as a result of various refinements, including informal and even formal axiomatization. Similar remarks can be made about the categories 'function' and 'set'. Different sets of axioms are set up to capture and clarify different mathematical essences. Nothing about this claim to knowledge precludes the possibility of indeterminateness in the grasp of the essence or of a subsequent splitting or refining of the concepts involved. Indeed, history shows that this has happened with the categories 'function' and 'set'. We can be more specific and distinguish different types of functions and sets. For example, we can distinguish and investigate the essence 'mechanically computable function'. In this case it is possible to reach such a degree of clarification in understanding that we can say with great precision which things are instances of this essence and which things are not. No matter what ambiguities lurked in the early conceptions of functions or sets, however, it was still possible to know that certain things were not instances of these types. It should be noted that Husserl also calls for the genetic investigation of mathematical essences in order to reacquaint ourselves with the different meanings and refinements that have been sedimented across different times, places and persons.

Husserl thinks these essences always play a role in our experience even if we do not usually attend to them or we do not reflect on them in a conscious and systematic way. Essences are presupposed in any cognition of an object. We can reflect on essences, however, especially if we are interested in clarifying our grasp of them. We might, for example, try to arrive at a better understanding of their bounds, or of their relations to other essences. (Such relations could include, e.g., species/genus relations, consistency, implication, and so on.) Husserl distinguishes the study of mathematical and logical essences from the study of 'material' essences. In logic and mathematics, broadly construed, the study of essences could be and is regimented in various ways. One must turn to mathematical practice to see what has actually been established with respect to mathematical essences.

A few additional words of warning are in order about this idea of essences. First, it is clear that we need not have a complete or perfect grasp of essences. Typically we would not have such a grasp. That is why we speak of clarification in the first place. The fact that we do not have a complete grasp of them, however, does not render them useless in our experience. On the contrary, it only accurately reflects the fact that we are finite, limited beings and that we are not omniscient. At the same time, it cannot be denied that we do think in terms of categories. Moreover, a mathematical essence has an intrinsic unity that could not be replaced with some notion of family resemblance or similarity. It is not the case that the number nine I use on one occasion *resembles* the number nine I use on another occasion. It is *identical* to it. A definition or lemma I use on one occasion cannot merely resemble the definition or lemma I use on another occasion. It cannot differ essentially from it. If mathematical definitions, objects or theorems only resembled one another from occasion to occasion or from person to person then mathematical practice would be very different from the way in which it is actually given.

I now introduce another term. Instead of saying we have a partial 'grasp' of these essences I will say we *intuit* them. Thus, 'grasping the categories' involved in our experience means 'intuiting essences'. It is 'categorial intuition'. Once again, please do not let your mind wander to all sorts of other meanings that 'intuiting essences' could have. The expression, as it is being used here, is quite innocuous. The term 'intuition' is used because an essence is immediately given as a datum in reflection on our experience. It is given before we can begin to analyze it or to consider its relations to other essences. Another important point that should be clear from our description is that essences can be grasped prior to knowing whether or not there are individual instances of those essences. Some essences may be instanced in the physical world and some may not. Given what they are about, mathematical essences (e.g., the essence 'natural number') could not have individual physical instances. Generally, it does not follow that if there is intuition of an essence then there is intuition of the existence of an individual instance of the essence. We could also put this point another way by saying that it does not follow that if the intention (essence) associated with a mathematical act is grasped then that intention (essence) is fulfilled or fulfillable. It could happen that in unfolding the intuition of an essence we in fact meet a contradiction and thus discover that there could not in principle be individual instances of the essence.

With all of this said, I would now like to quote and briefly comment on a few passages from an essay by Husserl that was one of Gödel's favorites: *Philosophy as Rigorous Science* (1911). It should now be possible to read these passages with some understanding. Gödel was evidently quite taken with Husserl's ideal of philosophy as a rigorous science, perhaps because he saw in it a continuation of the spirit of Leibniz's work. The themes in these passages are repeated and developed extensively in other works by Husserl that were among Gödel's favorites (e.g., *Logical Investigations, Ideas, Cartesian Meditations*). First, Husserl (Husserl 1965, pp. 90–91) says that

> To study any kind of objectivity whatever according to its general essence...means to concern oneself with objectivity's modes of giveness and to exhaust its essential content in the process of "clarification" proper to it . . . . With this we meet a science of whose extraordinary extent our contemporaries have as yet no concept: ... a phenomenology of consciousness as opposed to a natural science about consciousness.

Husserl distinguishes sciences of fact from sciences of essence. Phenomenology itself is not supposed to be a natural science since it is concerned with essences. Essences cannot be adequately understood in terms of natural science. Indeed,

> The spell of the naturalistic point of view...has blocked the road to a great science unparalleled in its fecundity... The spell of inborn naturalism also consists in the fact that it makes it so difficult for all of us to see "essences", or "ideas"—or rather, since in fact we do, so to speak, constantly see them, for us to let them have the peculiar value which is theirs instead of absurdly naturalizing them. Intuiting essences conceals no more difficulties or "mystical" secrets than does perception. (Husserl 1965, p. 110)

Intuiting essences conceals no more difficulties than perception because, as I have been saying, it only refers to grasping categories in our experience.

> But one must in no instance abandon one's radical lack of prejudice, prematurely identifying, so to speak, "things" with empirical "facts". To do this is to stand like a blind man before ideas, which are, after all, to such a great extent absolutely given in immediate intuition. (Husserl 1965, p. 146)

And, for just this reason,

> It is important today to engage in a radical criticism of naturalistic philosophy. (Husserl 1965, p. 78)

By Husserl's sights, there are many ways in which essences have been 'absurdly naturalized'. It is important to note that much of what has gone by the name 'phenomenology' since Husserl has itself naturalized essences and shunned the ideal of philosophy as rigorous science. This development began immediately with Husserl's most famous and influential student, Heidegger. It goes without saying that Husserl was not pleased with this development.

Interestingly, Husserl laments many of the same reductionistic attitudes about essences that are mentioned in Gödel's discussions of mathematical content and abstract concepts: empiricism, naturalism, psychologism, nominalism, and forms of conventionalism and formalism that are coupled with these views. Aristotelian realism may also be added to this list. It is precisely this worry about reductionist attitudes that is a central theme in Gödel's 1961 paper. I now turn to Gödel's remarks.

§2. 'Leftward' and 'rightward' viewpoints in philosophy. Gödel (Gödel *1961/?) seeks to describe the development of foundational research in mathematics since the turn of the century in terms of a general schema of possible worldviews. He divides these worldviews according to the degree and manner of their affinity to, or renunciation of, metaphysics (or religion). He thinks we then obtain a division into two groups, consisting of skepticism, materialism and positivism on one ('leftward') side, and spiritualism, idealism and theology on the other ('rightward') side. There are also mixed cases. Gödel places 'apriorism' on the right side of this schema, and empiricism on the left. He also places optimism on the right, and pessimism on the left, describing skepticism as a form of pessimism. Idealists tend to see meaning, reason and purpose in everything in the universe, while strict materialists do not see meaning, reason or purpose in anything.

The development of philosophy since the Renaissance has, on the whole, gone from right to left, and Gödel says it would be a miracle if this development had not also begun to prevail in the conception of mathematics. One sees this development, for example, in the work of Mill in the nineteenth century. At the turn of this century the antinomies of set theory were discovered. The significance of the antinomies was exaggerated by skeptics and empiricists as a pretext for a leftward upheaval. Mathematicians began to deny that mathematics depicts a system of truths. Instead, they acknowledged this for some part of mathematics and retained the remainder in, at best, a hypothetical sense.

§3. The incompleteness theorems and Hilbert's program. Hilbert's formalism is an example of this development. It seeks to do justice both to the leftward spirit of the time and to the nature of mathematics. In accordance with the spirit of the time, it is acknowledged that the truth of axioms cannot be justified or recognized in any way and that the deduction of consequences from them therefore has meaning only in a hypothetical sense. This deduction itself is construed as a mere game with symbols according to certain rules. On the other hand, in accordance with the rightward character of mathematics, it is held that a proof of a proposition must provide a secure grounding for the proposition, and that every precisely formulated yes-or-no question in mathematics must have a clear-cut answer. A certain part of mathematics must be acknowledged to be true in the older rightward sense if we are to have mathematical certainty, but this part is much less opposed to the leftward spirit of the time. It is the part that refers to concrete and finite objects in space in the form of combinations of symbols.

Hilbert hoped to show, in response to the foundational crisis brought on by the discovery of the paradoxes, that the Peano axioms for arithmetic, and indeed the formalization of all of higher mathematics, could be proved consistent using only concrete, finitist means. The hope was that 'Cantor's paradise' itself could be secured in this way. A concrete, finitist consistency proof would depend only on a finite number of discrete objects—sign-configurations—that would be immediately intuitable in space-time, and on the combinatorial properties of these objects. (The notion of intuition here is clearly much narrower than the Husserlian notion described above.) It was part of Hilbert's formalism that there should be no need to consider the meaning of the sign-configurations involved in the formalizations. Only syntactical properties and relations should figure into the consistency proof. Hilbert (Hilbert 1926, p. 376 of English translation) puts it the following way:

> ... as a condition for the use of logical inferences ... something must already be given to our faculty of representation, certain extralogical concrete objects that are intuitively present as immediate experience prior to all thought. If logical inference is to be reliable it must be possible to survey these objects completely in all their parts, and the fact that they occur, that they differ from one another, and that they follow each other, or are concatenated, is immediately intuitively given, together with the objects, as something that neither can be reduced to anything else nor requires reduction ... And in mathematics, in particular, what we consider is the concrete signs themselves, whose shape, according to the conception we have adopted, is immediately clear and recognizable.

The concern for reliability in Hilbert's conception of proof theory precluded any appeal to the thoughts or meanings associated with the concrete signs. The concrete signs should be given to our faculty of representation prior to all thought.

A finitist consistency proof could presumably be carried out in something like primitive recursive arithmetic (PRA), which may be viewed as a subsystem of PA. This constructive, finitist part of mathematics would, in Hilbert's view, contain only 'real' or 'contentual' propositions. The aim would be to formalize mathematics and to show that the formalism for higher parts of mathematics could be proved consistent using only the real, contentual part of mathematics. All the parts of mathematics in which purportedly abstract elements (like abstract 'meanings' of semantics) were found would thus be shown to be secure. Hilbert's program is thus, in effect, a conservation program: the formalizations that included ideal elements in mathematics would be shown to be conservative extensions of the real part of mathematics, thereby effectively eliminating the dependence on the ideal elements. The reference to ideal or 'abstract' elements would merely serve to shorten proofs, or to simplify a system of reasoning or to make it more perspicuous.

Now consider the meaning of Gödel's incompleteness theorems against this background. Gödel says they show, against Hilbert, that it is impossible to rescue the old rightward aspects of mathematics in such a manner as to be in accord with the spirit of our time. This is because it is impossible to even find an axiomatic system of number theory from which, for every number-theoretic proposition A, either A or ¬A would always be derivable. Furthermore, the second theorem tells us that for reasonably comprehensive systems of mathematics it is impossible to carry out a consistency proof merely by reflecting on the concrete combinations of symbols. It is necessary to introduce abstract elements.

Suppose the objects or concepts that can be represented in PRA (or even in PA) are completely representable in space-time as finite, discrete, sign-configurations that are in principle amenable to concrete intuition in Hilbert's sense. Let us further suppose that any objects or concepts that can be represented in PRA (or even PA) can, as Hilbert evidently thought, be considered 'concrete' by virtue of their representability in space-time as finite, discrete sign-configurations. There are some arguments in the literature about whether the exact bounds of the 'concrete' in finitist mathematics should be drawn more narrowly or more widely than PRA, but this need not detain us now. It seems unlikely that finitists would have extended the bounds beyond PA, given their concern for reliability in the face of the paradoxes. (If the bounds were extended beyond PA then there would still have to be some limit. We could then bring the form of our argument to bear at that point.)

Given these suppositions, it follows, by the second incompleteness theorem, that the objects or concepts needed for the proof of CON(PA) must *not* be completely representable in space-time as meaningless, finite, discrete sign-configurations which are amenable to concrete intuition. In other words, starting from Hilbert's original philosophical viewpoint, it appears that a proof of CON(PA) requires appeals to the *meaning* of the sign-configurations and to *abstract* objects or concepts that are in some sense *non-finite*. It will also involve a form of intuition that is not restricted to Hilbert's concrete intuition.

These kinds of remarks on the incompleteness theorems are expressed in many places in Gödel's writings. For example, Gödel (Gödel 1972, pp. 271–72) says that

> P. Bernays has pointed out on several occasions that, in view of the fact that the consistency of a formal system cannot be proved by any deduction procedures available in the system itself, it is necessary to go beyond the framework of finitary mathematics in Hilbert's sense in order to prove the consistency of classical mathematics or even of classical number theory. Since finitary mathematics is defined as the mathematics of *concrete intuition*, this seems to imply that *abstract concepts* are needed for the proof of consistency of number theory ... [What Hilbert means by "Anschauung" is substantially Kant's space-time intuition confined, however, to configurations of a finite

> number of discrete objects.] By abstract concepts, in this context, are meant concepts which are essentially of the second or higher level, i.e., which do not have as their content properties or relations of *concrete objects* (such as combinations of symbols), but rather of *thought structures* or *thought contents* (e.g., proofs, meaningful propositions, and so on), where in the proofs of propositions about these mental objects insights are needed which are not derived from a reflection upon the combinatorial (space-time) properties of the symbols representing them, but rather from a reflection upon the meanings involved.

The idea that it is necessary to reflect upon meaning plays a central role in Gödel's 1961 paper, as does the idea that reflection on meaning (or intuition of essence) is of a 'higher level' than reflection on the combinatorial properties of concrete symbols.

There are proofs of CON(PA) and these proofs must therefore require objects or concepts of the sort that would be recognized by mathematical or phenomenological realists (see also Tieszen 1994). That is, the proofs must require abstract concepts and/or meanings that are not available to concrete, sensible intuition. In addition, these objects must in some sense be non-finite. The requisite sense in which the objects or concepts needed for the proof of CON(PA) must be abstract and non-finite is seen in Gentzen's consistency proof, since the proof requires induction on the transfinite ordinals $< \varepsilon_0$. It is also seen in Gödel's consistency proof, since the theory of primitive recursive functionals requires the abstract concept of a "computable function of type t". If we combine the consequence drawn about meaning with the consequence about abstract elements, it appears that the meaning associated with arithmetic expressions must be 'abstract', and that abstract elements cannot be eliminated from mathematics in the way that Hilbert had hoped. Another way of putting this would be to say that Hilbert's appeal to finite, concrete *particulars* will not suffice for consistency proofs for interesting parts of mathematics. In addition, the abstract elements involved could not be given by Hilbert's concrete intuition since concrete intuition is restricted to finite sign-configurations. There must be, by the second incompleteness theorem and the consistency proofs for PA, a less restricted kind of mathematical intuition or insight that accounts for our mathematical knowledge. In Husserlian language, there must be an intuition of mathematical essences. (This is no doubt what lies behind Gödel's discussions with Wang (Wang 1974, pp. 84–86; 1987, pp. 188–192, 301–304) about how we perceive or intuit abstract concepts, i.e., intuit categories or essences.) It need not be claimed, however, that this kind of intuition is infallible (see below), even if it is taken to be a basic source of evidence.

If the Gentzen or Gödel proof of CON(PA) is evident to us on the basis of the meaning of the terms involved then this suggests that there are such proofs elsewhere in mathematics. There must be proofs that are not fully formalizable at a given stage in our mathematical experience but that are evident to us at that stage on the basis of the *meanings* involved in the terms of the proofs. There must be, in other words, a kind of 'informal rigor' in mathematics. This suggests the possibility that human minds might surpass machines in solving problems or in obtaining proofs of statements based on an understanding of the abstract meaning of the statements involved. Gödel has noted this implication in many places in his writings and I will come back to it below. In some writings he discusses an 'absolute' concept of proof that is abstract and non-formal. This is not a concept of proof that is always relative to some particular formal system or machine. It is a concept according to which 'proof' is to be understood as 'that which provides evidence' (Gödel *193?, 1946). What is 'provable' in this sense is what is 'knowable to be true' (Gödel *1951, p. 318; *1953/59, p. 341).

Since the Hilbertian combination of materialism and aspects of classical mathematics proves to be impossible, Gödel (Gödel 1961/?) says that only two possibilities remain: one must either give up the rightward aspects of mathematics or one must attempt to uphold them in contradiction to the prevailing spirit of our time. Gödel says that nothing about the mathematical results we achieve requires us to give up the idea of the certainty of mathematical knowledge or the belief that for clear questions posed by reason, reason can also find clear answers. Only the desire to remain in agreement with the prevailing philosophy compels this.

Hilbert embraced a particular philosophy about the infinite, about what has meaning, about what objects can be recognized as existing, and about what objects can be intuited. This part of Hilbert's philosophy is refuted by the incompleteness theorems. In the Husserlian language quoted above, Hilbert's program does not give mathematical essences or abstract concepts their due. It exhibits a certain blindness or prejudice about them. It is an attempt, in effect, to show that directedness toward essences or abstract concepts can be reduced to directedness toward concrete, finite sign configurations and combinatorial operations on such objects. On the other hand, Hilbert's optimism about mathematical problem solving and his concern for achieving certainty in mathematical proofs were admirable ideals. In an interesting unpublished paper written already in the 1930s (Gödel *193?), Gödel says that the incompleteness theorems can be interpreted as having one of two consequences for Hilbert's optimism about mathematical problem solving: either they mean that (i) it is not the case that every clearly posed mathematical problem can be solved or they show that (ii) something was lost in translating the concept of proof as 'that which provides evidence' into a purely formal or mechanistic concept. Gödel says it is easy to see that (ii) is true, since number-theoretic questions that are not decidable in a given formalism are always decidable by evident inferences not expressible in the formalism. The new inferences turn out to be exactly as evident as those of the given formalism. Perhaps Gödel is overstating the case here. The claim that number-theoretic questions not decidable in a given formalism are always decidable by evident inferences not expressible in the formalism seems to be true of questions like that of the consistency of the formalism (for reasonable formalisms) but it is not so obvious for arbitrary number-theoretic questions. Nonetheless, the conclusion we should be able to draw is that Hilbert's optimism about mathematical problem solving remains untouched even though it is not possible to formalize or mechanize mathematical evidence in the domain of number theory. The reason Hilbert's optimism remains untouched is that we can hope to make these decisions on the basis of our directedness toward and intuition of the underlying abstract concepts or essences. Gödel does not say this in *193? but by *1961/? it is clear that this is what he has in mind (see Section 6 below). Some parts of mathematics might be completely formalized or mechanized but, on the whole, it is not possible to mechanize mathematical reasoning.

§4. The incompleteness theorems and Carnap's program (Gödel *1953/59). Another 'leftward' attempt to accommodate mathematics can be found in Carnap's early work. Gödel (Gödel *1953/9-III) says that around 1930 Carnap, Hahn and Schlick developed a conception of the nature of mathematics, under the influence of Wittgenstein, which was a combination of nominalism and conventionalism. Its main objective was to reconcile strict empiricism with the *a priori* certainty of mathematics. Gödel characterizes empiricism as the view that all knowledge is based on external or internal sense perception. It holds that we do not possess an intuition into some realm of abstract mathematical objects or concepts. In light of Gödel's 1961 paper, it holds that we do not have an intuition of mathematical essences. Mathematics is thought to have an *a priori* certainty, but because such a realm cannot be known empirically it must not be assumed to exist at all. The objective of the program of these logical empiricists (or positivists) can therefore be viewed as one of building up mathematics as a system of sentences which are valid independently of sense experience without using mathematical intuition or referring to any mathematical objects or facts. This is to be done by showing that mathematics can be completely reduced to and in fact is nothing but syntax of language. This would mean that the validity of mathematical theorems consists solely in their being consequences of certain syntactical conventions about the use of symbols. It would not consist in their describing states of affairs in some realm of objects. As Gödel puts it, mathematics is to be viewed as a system of auxiliary sentences without content or object. The syntactical conventions involved in the program are those by which the use of a symbol is defined by stating rules about the truth or assertibility of the sentences containing the symbol, where the rules refer only to the outward structure of expressions and not to their meaning or to anything outside of the expressions. Assertions that conflict with the rules are excluded on account of their structure, exactly as are assertions that conflict with rules of grammar.

Gödel lays out some basic conditions that would need to be met for the program to succeed. Among these conditions are the following: 'mathematics' should mean classical mathematics, since the syntactical program aims to dispense with mathematical intuition but without impairing the usefulness of the full extent to which mathematics can be used in the empirical sciences. Gödel does say, however, that his argument against the syntactical viewpoint goes through even if the term 'mathematics' is restricted to intuitionistic mathematics. Another condition is that 'language' must mean some symbolism that can actually be exhibited and used in the empirical world. Sentences will have to consist of a finite number of symbols, for sentences of infinite length would evidently not exist and could not be produced in the empirical world. For the same reason, the rules of syntax must also be finitary.

Another important condition is that a rule about the truth of sentences can be called 'syntactical' only if it can be known that it does not imply the truth or falsehood of any factual or empirical sentence. This follows from the concept of a convention about the use of symbols, but also from the fact that mathematics must lack content if it is supposed to be admissible in spite of strict empiricism. This condition implies that the rules of syntax must be demonstrably consistent since from an inconsistency every proposition follows, including all factual propositions. In the proof of consistency, as well as in the rules of syntax, only syntactical concepts may be used. If mathematical intuition and the assumption of mathematical objects or facts is to be dispensed with by means of syntax, then the use of 'abstract' and 'transfinite' concepts of mathematics will have to be based on considerations about finite combinations of symbols. Gödel notes that this same condition is involved in Hilbert's program. He says that nominalism and conventionalism could be used against realism only if mathematics could be interpreted as syntax in accordance with these conditions. By 1961 the reference to realism here can be understood in terms of a phenomenological realism that recognizes the intuition of essences.

Gödel's central argument against Carnap's syntactical program is developed by way of a philosophically interesting application of the second incompleteness theorem. Suppose that mathematics is syntax of language, as understood according to the conditions mentioned above. In order to know that this was true we would need to know that the rules of this syntactical system are consistent. To know this, we will, by the second incompleteness theorem, need to use mathematics that is not captured by the rules in question. We will need to use mathematical concepts or principles that are formally independent of and stronger than those captured by the rules in question. The supposition that mathematics is syntax of language is thereby contradicted.

The argument suggests that mathematical sentences are not void of content. On the contrary, they must have their own kind of meaning and reference. They are not 'true by definition' nor are they empty tautologies (see Gödel *1951, *1953/59). The mathematical essences we intuit could not be linguistic conventions. There are constraints on them that we do not freely invent or create. One might also say that this content or meaning will be 'abstract' relative to the rules of syntax. Mathematical intuition will therefore not be eliminable. In Husserl's language, categorial intuition will not be eliminable. Thus, instead of clarifying the meaning of abstract and non-finitary mathematical concepts by explaining them in terms of syntactical rules, abstract and non-finitary concepts are used to formulate the syntactical rules. Instead of justifying mathematical axioms by reducing them to syntactical rules, some of these axioms are required to justify the syntactical rules as consistent.

§5. Against the elimination of rational intuition. In papers written prior to 1961, Gödel makes various comments about the nature of mathematical content and the 'rational perception' by which we come to have mathematical knowledge (see also Parsons 1995). It is remarkable how closely these comments parallel Husserl's view of categorial intuition. In some passages, Gödel likens the conceptual content of sentences to Frege's notion of sense (Sinn) (Gödel *1953/59, p. 350). He says the conceptual content of sentences, or their 'sense', is objective and non-psychological. This meaning is not man-made and does not consist merely of syntactical conventions. He says these concepts form an objective reality of their own, which we cannot create or change, but only perceive and describe (Gödel *1951, p. 320). It is clear that he means to reject psychologism, nominalism, empiricism, and Aristotelian realism about concepts. He says we know particular objects and their properties and relations through ordinary sense perception. With mathematical reason, however, we perceive the most general concepts and their relations (Gödel *1953/59, p. 354). This rational perception is analogous in certain respects to ordinary sensory intuition. Gödel suggests there is an analogy in several respects. First, our perceptions in both cases are constrained or 'forced' in certain respects (see especially Gödel *1951 and 1947/64). Second, we can be under illusions in each case (see Wang 1974, pp. 85–86). Third, there is a kind of inexhaustibility in each case (Gödel *1951, *1953/59). Gödel says the 'inexhaustibility' of mathematics makes the similarity between rational perception and sense perception even closer because it shows that there is a practically unlimited number of perceptions also in the case of rational perception (Gödel *1953/59, p. 353). This inexhaustibility appears not only through foundational investigations, like the incompleteness theorems, but also in the actual development of mathematics. It appears, for example, in the unlimited series of axioms of infinity in set theory which Gödel says are analytic and evident in the sense that they only explicate the general content of the concept of set.

Similar themes are found in the 1947 version of *What is Cantor's continuum problem*? and Gödel elaborates on the notion of mathematical intuition in the supplement to the 1964 version of the paper. Gödel is already at pains in the early part of the 1947 version of paper to show that the continuum hypothesis (CH) is meaningful and determinate enough to expect that an unambiguous answer should be forthcoming in a natural extension of set theory. He does say that the solution of this problem will require an analysis of the meanings of the terms occurring in the axioms of set theory that is more profound than mathematicians are accustomed to providing, but he rules out the meaning analyses proposed by Brouwer, Weyl and Poincaré on the grounds that they change the original meaning of the problem. The negative attitude toward Cantor's set theory that is found in these writers is the result of a philosophical conception of mathematics that admits objects only to the extent that they are interpretable as our own constructions or can be completely given in mathematical intuition. In effect, these mathematicians are considering categories or essences that are different from the essence of 'set' as it is given in Cantorian set theory. Gödel says that there is a satisfactory foundation of Cantor's set theory provided that one is willing to concede that mathematical objects exist independently of our constructions and of our having an intuition of them individually, and requires only that the general iterative concept of set be sufficiently clear to recognize the soundness and the truth of the axioms concerning it. Thus, instead of acquiescing in these reductionist views, we need to find new axioms that only analytically unfold the content of the given (abstract) iterative concept of set. At the time, Gödel suggested the use of large cardinal axioms, but he also said that other unknown axioms may play a role.

In the 1964 supplement to the second edition of the 1947 paper, Gödel says that despite their remoteness from sense experience we do have "something like a perception of the objects of set theory". This is seen in the fact that the axioms force themselves upon us as being true.

> I don't see any reason why we should have less confidence in this kind of perception, i.e., mathematical intuition, than in sense perception, which induces us to build up physical theories and to expect that future sense perceptions will agree with them, and, moreover, to believe that a question not decidable now has meaning and may be decided in the future. (Gödel 1964, p. 268.)

Gödel goes on to make some additional remarks about the nature of mathematical intuition.

It is worth noting Gödel's suggestion that in order to make progress in set theory we need not have an individual intuition of the sets of higher set theory. It is only required that our grasp of the 'general mathematical concepts' be sufficiently clear. In other words, there need not be an intuition of arbitrary individual instances of the essence under analysis in order to obtain some clarification of the essence. We are already forced in some ways by the essence or concept. It is not freely variable. This is why Gödel adds the qualification that we have *something like a perception* of the objects of Cantorian set theory. There arguably are intuitions of some individual instances of the Cantorian concept of set. There may be 'object-like' perceptions, however, in cases where our thinking is directed by our intentions (essences) even if we cannot intuit arbitrary individual instances of the essence.

§6. From the incompleteness theorems to phenomenology. We have seen how Gödel uses his incompleteness theorems to argue against two of the fundamental twentieth century schemes in the foundations of mathematics. Gödel says the correct attitude appears to be that the truth lies in the middle, or consists of a combination of the leftward and rightward viewpoints. Hilbert's effort to combine the two directions had been too primitive and too strongly oriented toward the leftward direction. Carnap's program also failed. We can perhaps hope to obtain a workable combination in a different way. In this case, the certainty of mathematics is not to be guaranteed through proving certain properties by projecting them onto material systems like physical sign configurations. Since the second incompleteness theorem suggests that we must reflect on meaning, or on abstract concepts, we can instead try to obtain a workable combination of the two directions through

> cultivating (deepening) knowledge of the abstract concepts which themselves lead to the setting up of those mechanical systems, and further, according to the same procedures [for clarifying meaning], seeking to gain insights about the solvability, and the actual methods for the solving of all meaningful mathematical propositions. (Gödel *1961/?, p. 383.)

These comments run directly parallel to Husserl's view. (The reader might find it helpful to read again the passages by Husserl quoted earlier in the paper.) Cultivating knowledge of abstract concepts is cultivating intuition of essences. How is it possible to extend our knowledge of these abstract concepts or essences? How is it possible to make these concepts precise, and to gain comprehensive and secure insight into the fundamental relations that hold among them (i.e., into the axioms that hold for them)? This cannot be done by trying to give explicit definitions for concepts, and proofs for axioms, since we would then need other undefinable abstract concepts with their own axioms. Gödel says that the procedure must consist, at least to a large extent, in a clarification of meaning that does not consist in defining.

It is at this point that Gödel explicitly mentions Husserl's philosophy. With its emphasis on the non-reductionistic, descriptive clarification of meaning, Husserl's view provides a combination of the two directions that can yield a workable foundation for mathematics. (It should be noted that he does not suggest some other philosophical view. Since he would have been aware of a wide range of views at this time, it is worth asking why he does not think that other views would provide a workable combination. I return to this question in Section 7 below.) Gödel says that there "exists today the beginnings of a science which claims to possess a systematic method for such clarification of meaning, and that is the phenomenology founded by Husserl". Continuing, he says

> Here clarification of meaning consists in concentrating more intensely on the concepts in question by directing our attention in a certain way, namely, onto our own acts in the use of those concepts, onto our own powers in carrying out those acts, etc. In so doing, one must keep clearly in mind that this phenomenology is not a science in the same sense as the other sciences. Rather it is [or in any case should be] a procedure or technique that should produce in us a new state of consciousness in which we describe in detail the basic concepts we use in our thought, or grasp other, hitherto unknown, basic concepts. (Gödel *1961/?, p. 383.)

Gödel argues that the phenomenological approach cannot be dismissed on *a priori* grounds. Empiricists, in particular, should be the last to suppose there is an *a priori* argument against the phenomenological approach since *a priori* arguments about such matters are not available to them and would merely be dogmatic.

Thus, Gödel's path from the incompleteness theorems to Husserl's phenomenology is not surprising at all. Indeed, there is a sense in which the incompleteness theorems support and are supported by a phenomenological view. They cohere with a Husserlian view of meaning and the clarification of meaning, of what kinds of objects exist, and of categorial intuition. They support a phenomenological view in the sense that they suggest that an intuition of mathematical essences (or a grasp of abstract concepts) that cannot be understood reductionistically is required in order to solve certain mathematical problems, to obtain consistency proofs for formal systems, and to facilitate the development of mathematics. They are supported by a phenomenological view in the sense that phenomenology, unlike other recent viewpoints, does give mathematical essences their due. If it is held from the outset that we intuit mathematical essences and that reductionism about intuiting essences is a prejudice, then it is highly unlikely that one would believe that mathematical essences could be completely captured in formal or mechanical systems. There would not be the same kind of blindness to 'ideas' that one finds in the various leftward viewpoints. The 'radical lack of prejudice' needed to let essences have their own particular value might in fact be cultivated. From this perspective, however, logicians like Hilbert, Carnap, Skolem and others appear to display a certain kind of blindness or prejudice. Of course the incompleteness theorems are not derivable from this phenomenological attitude alone. One must also develop a very precise understanding of the essence of formal or mechanical systems in order to actually give a mathematically rigorous proof of incompleteness. The basic point about the irreducibility of mathematical essences comes into even sharper focus, however, with the subsequent clarification of the essence 'mechanically computable function' in Turing's work and with the fact that many different characterizations of this notion were shown to be equivalent.

It might even be suggested that the incompleteness theorems and related results on decidability and consistency are in fact examples of philosophy as rigorous science. They are, that is, supported by a particular philosophy and they show in a rigorous way the limits of a purely formal, syntactical or mechanical conception of mathematics. They show that the essence 'arithmetical truth' is not exhausted in a purely formal, syntactical, or mechanical (and hence relative) concept of provability. Indeed, the theorems show that virtually none of the mathematical concepts one might like to axiomatize can be exhausted in formal concepts and methods. This does not, however, undermine mathematical rigor. It does not preclude informal rigor or axiomatization. 'Rigorous science' need not be identified with purely formal science.

In the 1961 paper Gödel therefore says that not only is there no reason to reject phenomenology out of hand but one can even present reasons in its favor. Gödel argues that the incompleteness theorems show how a further development in the rationalistic direction takes place even without having applied a conscious and systematic phenomenological procedure. New axioms of mathematics, which do not follow by formal logic alone from those previously established, become evident to us again and again. One example of this can be found in the unlimited series of new arithmetic axioms, in the form of Gödel sentences, that one could add to the present axioms on the basis of the incompleteness theorems. These axioms become evident again and again and do not follow by formal logic alone from the previous axioms. We can use these axioms to solve problems that were previously undecidable. We are simply unfolding our intuition of an essence. Of course in this particular case the axioms are not mathematically very interesting. The incompleteness theorems for arithmetic have, however, led to interesting results like those of Paris and Harrington (1977). The Paris/Harrington theorem is a genuinely mathematical statement that refers only to natural numbers but is undecidable in PA. Its proof requires the use of infinite sets of natural numbers. It provides a good example of Gödel's idea of having to ascend to stronger, more abstract (in this case, set-theoretic) principles to solve lower level (number-theoretic) problems.

Another one of Gödel's favorite examples that is mathematically more substantial, but also more controversial, concerns extensions of the axioms of set theory. This example is mentioned in many of his other papers (e.g. Gödel *1951, *1953/9, 1964, 1972a) and is also related to his views on the inexhaustibility of mathematics. Gödel refers to the unlimited series of axioms of infinity in set theory, which are evident in the sense that they only explicate the meaning or content of the general concept of set. He says that such a series may involve a very great and perhaps even an infinite number of actually realizable independent rational perceptions. This is seen in the fact that the axioms concerned are not evident from the beginning, but only become so in the course of the development of mathematics. To understand the first transfinite axiom of infinity, for example, one must first have developed set theory to a considerable extent. One could then rise to a 'higher' state of consciousness at a later stage in which one understands the next axiom of infinity, and so on. This is also an example of meaning clarification, albeit one that did not result directly from an application of the kind of conscious and systematic procedure that Gödel seeks. Gödel hoped that in unfolding our intuition of the essence 'set' for which particular set-theoretic problems could be formulated, we could solve those problems, including the continuum problem. Perhaps we know enough today to say that adding strong axioms of infinity will not induce the kinds of constraints needed to settle the continuum problem in a convincing way. Gödel did, however, suggest that other unknown axioms might play a role.

One might argue that we only obtain the unfolding of the concept of set *if* we are willing to take this concept as given in the first place, and that the concept is very problematic. Perhaps it is even inherently ambiguous or vague. There are of course many more issues about this matter to be discussed than I can go into here. Nonetheless, I shall mention a few things that can be said on the basis of the comments made above. First, a general iterative concept of set is given in our experience even if it is not completely understood. There seems to be no *a priori* reason why it should not be explored. In unfolding our intuition of a general iterative concept of set we have not yet found a contradiction. At the same time, we do not have a precise grasp of the consistent extensions of it that may exist. But how could we be certain at this point that it is inherently ambiguous or vague? What would it mean to *know* this? The fact that the unfolding is even possible suggests that the concept gives some direction to research and has some coherence and stability. Indeed, consider all of the new results and methods to which researchers have been led by exploring aspects of this concept.

Gödel goes on to say in *1961/? that it is not excluded by the incompleteness results that every clearly posed mathematical yes-or-no question is nevertheless solvable through cultivating our knowledge of abstract concepts (i.e., through developing our intuition of essences), for it is this activity in which more and more new axioms become evident on the basis of the meaning of the primitive concepts that a machine cannot emulate. Gödel suggests in other writing (Gödel 1934, from the postscript added in 1964; 1972a) that mental procedures may extend beyond mechanical procedures because there may be finite, non-mechanical procedures that make use of the meaning of terms. The intuition of mathematical essences would be just such a procedure (see also Tieszen 1996). Given the incompleteness theorems, there can for most mathematical essences be no consistent machine that solves all of the well defined yes-or-no questions that are left undecided by the original sets of axioms for those essences. Human reason, however, may be able to achieve such a development by virtue of its ability to reflect on essences. The mind can constantly develop without diagonalizing out of the mathematical essence it is intuiting. There might be a constant development of machines to capture more of the essence but only by diagonalizing out of each particular machine under consideration. Thus, it is through an adjusted philosophical viewpoint according to which we intuit essences that Gödel seeks to make a place for Hilbert's optimism about mathematical problem solving as well as Hilbert's idea that in mathematical proofs we should strive for certainty.

Gödel says that the intuitive grasp of ever newer axioms that are logically independent of earlier ones is necessary for the solvability of all problems even within a very limited domain. He says that this agrees in principle with the Kantian conception of mathematics. Gödel points out, however, that Kant was wrong to think that for the derivation of geometrical theorems we always need new intuitions, and that a logical derivation of these theorems from a finite number of axioms was therefore impossible. But in the case of mathematics in a more general sense, Kant's observation is correct. Gödel says many of Kant's assertions are false if literally understood, but that they contain deeper truths in a more general sense. It is Husserl's (transcendental) phenomenology that for the first time does justice to the core of Kantian thought. It avoids both the "death-defying leap of idealism into a new metaphysics as well as the positivistic rejection of every metaphysics" (Gödel *1961/?). It could be argued that the phenomenological approach is not prey to the excesses and lack of balance that characterized earlier 'rightward' viewpoints.

§7. Why phenomenology? Gödel worked mostly on philosophy after 1942. It is interesting to ask why he would have settled on Husserl's phenomenology and not some other viewpoint. He certainly knew about extended forms of Hilbert's program, intuitionism, predicativism, and other foundational views. Quine's ideas had become very influential. He could have appealed to earlier forms of platonism. There were even developments in post-Husserlian phenomenology.

The answer to the question is straightforward. First, most of post-Husserlian phenomenology had itself succumbed to leftward pressures. Second, consider the views about provability, about what has meaning, what objects can be recognized, and what can be intuited in (i) modified forms of Hilbert's program, (ii) traditional intuitionism and (iii) predicativism. To insist on restrictions like those found in (i), (ii), or (iii) is to virtually insure there will be certain clearly posed mathematical problems that will not be solved, including questions about the consistency of formal systems. It is to insure blindness or prejudice. In the case of the limitation to PRA, for example, it is to insure that the Gödel sentence for PA is undecidable, or that no consistency proof for PA would be forthcoming. Or consider whether the following is a clearly posed mathematical question: is ZF + CH consistent or not? Adherence to the views (ii) and (iii) would probably lead (or would have led) one to believe it is not a clearly posed mathematical question, much less that it has a clear-cut solution. On the basis of the meaning theories associated with these views, it would be difficult to see how one could give meaning to the problem. In any case, one would be blinded to the solution on views (i)-(iii) because the proof that CH is consistent with the axioms of ZF requires impredicatively specified sets. It also requires that we transcend the intuitionistic ordinals and take the classical ordinals as given. For some problems or consistency proofs there is a need to ascend to higher types, new axioms of infinity, classical ordinals, etc., where this emerges naturally in the course of unfolding the given essence.

Quine's views on meaning, on what kinds of objects can be recognized, etc., are beset with analogous problems (see Tieszen 199?). It will not help to be told that one can only recognize as legitimate the mathematics that is needed to round out our theories of nature, much less that open mathematical problems can or should be decided on the basis of whatever mathematics this happens to be.

Most modern philosophical conceptions of mathematics are more skewed in the leftward direction than ever before. Thus, the trend that Gödel saw has not abated but has become stronger than ever. To philosophers like Husserl and Gödel this signals a crisis. Indeed, it is one form of the crisis that Husserl writes about (see Husserl 1970a; also Tieszen 1997b).

A few words should also be said about platonism. Husserl criticizes earlier forms of platonism or realism. He considers them to be naive. A central reason to avoid earlier, naive forms of platonism is that they place essences outside of all possible experience. They treat essences as abstract things-in-themselves. On the phenomenological view, however, we are clearly directed toward and have access to essences. At least some essences are here in our experience, if only partially. They are not all completely outside of or beyond our experience. To deny that essences are here in our experience is simply to deny that we can grasp various categories in our experience without attempting to reduce them to something else. This, in turn, is to deny the undeniable: that consciousness exhibits intentionality.

I conclude by briefly considering a skeptical objection to the so-called 'phenomenological method'. The objection is this: What exactly is the method supposed to be? How does one learn it? What are some examples of the method? What are some of its fruits? My response is to argue that these questions misplace the proper emphasis. For students of Gödel's work, the substantive point about the phenomenological approach lies in its distinctive form of realism and its anti-reductionism about mathematical concepts and concept analysis. All questions about method should be viewed in this light. Suppose, for example, that the arguments of Frege and Husserl against psychologism had not been successful. Would thinking of logic and mathematics in terms of empirical psychology have advanced logic and mathematics? Would it be an advance to move logic and mathematics into the psychology department? It seems clear that this would hardly be a desirable development. How would the 'methods' of logic and mathematics be conceived in those circumstances? One can ask similar questions about strict formalism, mechanism, nominalism, and other reductionistic and naturalistic viewpoints. In each case, the resulting conception of the methods of logic and mathematics, I would argue, is at least inadequate and possibly even harmful. A correct method would be one that gives mathematical essences their due without necessarily denying the usefulness or importance of formalization. That would be the method of a phenomenological or critical realism in which the epistemological role of categorial intuition is not eliminable.

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