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1996–97 ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Cambridge, Massachusetts, March 22–25, 1997

The Annual Meeting of the Association was held at the Massachusetts Institute of Technology, Cambridge, Massachusetts, March 22–25, 1997. The program included thirteen invited hour lectures. Solomon Feferman gave the eighth annual Gödel lecture, entitled *Occupations and Preoccupations with Gödel: His* Works *and the Work*. In addition there were thirty-five contributed talks and four papers presented by title. The Program Committee consisted of Peter Cholak, Sy Friedman (Chair), Warren Goldfarb, Grigori Mints, and Anand Pillay.

The following invited addresses were presented at the meeting:

John Baldwin (Illinois, Chicago), *Amalgamation, stability, and random structures*.

Timothy Carlson (Ohio State), *Can a machine know it is a machine?*

James Cummings (Carnegie-Mellon), *Combinatorics at successors of singular cardinals*.

Rod Downey (Victoria, Wellington), *Automorphisms of $\mathcal{E}$ after thirty years*.

Gregory Hjorth (UCLA), *When is an equivalence relation classifiable?*

Philip Kremer (Yale), *Propositional quantification in Kripke and topological semantics*.

Manuel Lerman (Connecticut), *Embedding parital lattices into the computably enumerable degrees*.

Richard Lipton (Princeton), *Using DNA to compute*.

Menachem Magidor (Hebrew University), *Mutually stationary sets*.

André Nies (Chicago), *Undecidability of the exponential time degrees and effective algebra*.

William Sieg (Carnegie-Mellon), *Hilbert's programs: 1917–1922*.

Sergei Starchenko (Vanderbilt), *On groups definable on O-minimal structures*.

Lou van den Dries (Illinois, Urbana), *On O-minimal expansions of the real field*.

Abstracts of contributed talks given in person or by title by members of the Association for Symbolic Logic follow.

For the Program Committee
SY D. FRIEDMAN

Abstracts of contributed talks

▶ KLAUS AMBOS-SPIES, DENIS R. HIRSHFELDT, and RICHARD A. SHORE, *Undecidability and* 1*-types in an interval of the computably enumerable degrees*.

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Early results in the study of the structure $\langle \mathbf{R}, \leq, \cup \rangle$ of the computably enumerable Turing degrees led to Sacks' conjecture that this structure is decidable. This conjecture turned out to be false. The undecidability of the theory of the partial ordering $\langle \mathbf{R}, \leq \rangle$ was first proved by Harrington and Shelah, using a very complicated $0'''$ argument. Ambos-Spies and Shore [1] gave a simpler infinite injury proof of the same result, as well as a proof that this theory has uncountably many 1-types. These proofs combine with the permitting technique to yield similar results for any ideal of the c.e. degrees. In this paper these results are further extended to any interval of the c.e. degrees by combining the methods of [1] with those of Slaman's proof [2] of the density of branching degrees and Fejer's proof [3] of the density of non-branching degrees.

[1] K. AMBOS-SPIES and R. A. SHORE, *Undecidability and* 1*-types in the recursively enumerable degrees*, ***Annals of Pure and Applied Logic***, vol. 63 (1993), pp. 3–37.

[2] P. A. FEJER, *The density of the nonbranching degrees*, ***Annals of Pure and Applied Logic***, vol. 24 (1983), pp. 113–130.

[3] T. A. SLAMAN, *The density of infima in the r.e. degrees*, ***Annals of Pure and Applied Logic***, vol. 52 (1991), pp. 155–179.

► ALEXANDRU BALTAG, *Interpolation and preservation for pebble logics*.

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It is well-known that the standard (obvious) analogues of the classical interpolation and preservation theorems fail for the infinitary logic $L_{\infty,\omega}$. In their paper *Interpolations, preservation and pebble games* (submitted for publication to the ***Journal for Symbol Logic***), J. Barwise and J. van Benthem have recently shown that there exist non-standard analogues of theorems for $L_{\infty,\omega}$ and for various subfragments, namely those for which there is a co-inductive pebble game characterising equivalence in the logic. While most of their method is general, there is one part that requires a tedious case-by-case analysis of each particular logic subjected to this method.

In our paper, we replace this analysis by a general theorem (the *Pebble Interpolation Theorem*), applicable to a wide class of fragments of $L_{\infty,\omega}$ (*pebble logics*). All the examples analysed by the previously-mentioned authors are covered by our theorem, but we also obtain new interpolation and preservation results for some other natural logics (e.g., the logics with only k variables and with counting quantifiers $L^k_{\infty,\omega}(C)$).

► PAUL BANKSTON, *A topological view of elementary chains*.

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For each ordinal α and pair X, Y of compacta (i.e., compact Hausdorff spaces), we define what it means for a function $f : X \rightarrow Y$ to be a *map of level* $\geq \alpha$, in such a way that a continuous map between Boolean spaces is of finite level $\geq \alpha$ if and only if it is the Stone dual of a Boolean lattice embedding that preserves all prenex first-order formulas of quantifier rank α. Maps of level ≥ 0 are just the continuous surjections, and the maps of level ≥ 1 (also called *co-existential maps*), preserve interesting topological properties; principally those of being of dimension $\leq n$, and of being an indecomposable continuum. Maps of level ≥ 2 preserve, in addition, being of dimension n, and of being a decomposable continuum.

Co-elementary maps are of level $\geq \alpha$ for all ordinals α; of course in the Boolean context, the co-elementary maps coincide with the maps of level $\geq \omega$. The results we wish to highlight in this talk are: (i) every map of level $\geq \omega$ is co-elementary; and (ii) the limit maps of an ω-

indexed inverse system of maps of level $\geq \alpha$ are also of level $\geq \alpha$.

Taken together, these results imply a perfect dual analogue to the elementary chains theorem of Tarski-Vaught; and can be used to give a new proof of Henson's Banach elementary chains theorem, in the context of commutative B^*-algebras.

▶ ANDREAS BLASS and XIAOKANG YU, *A basis theorem provable in* RCA_0.

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In the weak system of second-order arithmetic, we develop the theory of countable Boolean algebra and code the bounded Boolean power of $\mathbb{Z}(B)$, with respect to the countable algebra B, as a countable abelian group.

In RCA_0, we can prove the existence of a basis for the bounded Boolean power $\mathbb{Z}(B)$, with respect to any countable Boolean algebra B; and if B is not finite, $\mathbb{Z}(B)$ has an infinite basis.

▶ NATASHA BOZOVIC, *First-order theories without models of a given type*.

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For every property P from a large class C of algebraic properties (i.e., such that are preserved under isomorphisms) of finitely presented groups, it is proved here that there exists a finitely axiomatizable theory $T(P)$ having no models with a given property P. The class C of properties studied includes hereditary properties P such that both properties P and residually P are proper for finitely presented groups. This means that there exists a finitely axiomatizable theory of groups having no models which are:

solvable; nilpotent; torsion-free; one-relator; recursively presentable simple; etc.

While the main examples discussed here are group-theoretical (connected with some classes of recursively unrecognizable properties of groups), the results obtained are transferable to other algebras as well.

▶ JÖRG BRENDLE, *Mad families and large continuum*.

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This talk is about some results and questions on mad families of subsets of the natural numbers ω.

Given $A, B \subseteq \omega$ infinite, we say A and B are *almost disjoint* iff $A \cap B$ is finite. $\mathcal{A} \subseteq [\omega]^\omega$ is an *almost disjoint family* iff its members are pairwise almost disjoint; it is a *maximal almost disjoint* (*mad*) *family* iff it's a.d. and maximal with respect to inclusion. The *almost disjointness number* $\mathfrak{a}$ is the size of the least mad family. For $f, g \in \omega^\omega$ we say f *eventually dominates* g iff $f(n) \geq g(n)$ for all but finitely many $n \in \omega$. $\mathcal{F} \subseteq \omega^\omega$ is an *unbounded family* iff no $g \in \omega^\omega$ eventually dominates all members of $\mathcal{F}$. The *unbounding number* $\mathfrak{b}$ is the size of the smallest unbounded family. It is well-known that $\mathfrak{b} \leq \mathfrak{a}$.

The consistency of $\mathfrak{b} < \mathfrak{a}$, proved by Shelah [2], is rather hard. This is no accident because the "standard" ccc forcing for killing a mad family $\mathcal{A}$ may add a dominating real [1]. However, one can—under certain circumstances—find a ccc forcing which destroys a mad family without adding a dominating real [1]. In our talk we shall sketch proofs of these two results. Using the latter one can prove the consistency of $\mathfrak{b} < \mathfrak{a}$ with arbitrarily large continuum. This is an improvement on Shelah's original result which showed only the consistency of $\mathfrak{b} < \mathfrak{a}$ with $\mathfrak{c} = \omega_2$. We conclude our talk with some questions.

[1] J. BRENDLE, *Mob families and mad families*, preprint.

[2] S. SHELAH, *On cardinal invariants of the continuum*, ***Contemporary Mathematics***, vol. 31 (1984), pp. 183–207.

► JOHN BYRNES, *Normal forms resulting from proof search strategies*.
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Sieg developed intercalation calculi (IC) for the purpose of searching directly for natural deduction (ND) proofs; these are presented in [2, 3]. It is shown that by restricting strategies for indirect proof, only *normal* (in the sense of Prawitz [1]) ND proofs are found. A semantic argument for the completeness of the search yields the theorem that every provable formula has a normal proof, thus establishing normal form theorems without syntactic reduction rules.

I explore alternative restriction on IC search. The first result is that by applying only E-rules directly above major premises (and modifying the rules for vE), the class of ND proofs obtained is *exactly* the class of normal ND proofs for full classical logic. This is a wider class than treated by Prawitz, and most treatments since have made restrictions on certain rules (such as only allowing atomic conclusions); these restrictions are not needed in this framework, as we do not deal with proof transformations.

A natural further restriction on search is that elimination strategies are always pursued before introduction or indirect strategies. I define a class of IC derivations motivated by this restriction, and show that derivations in this class correspond to proofs in which no formula occurs twice on the same path under the same set of uncanceled assumptions. This yields a strict subclass of normal derivations; for the implicational fragment one can use the Curry-Howard isomorphism to see that the proofs in the more restricted class correspond to $\beta\eta$-normal terms of the typed λ-calculus, whereas normal proofs correspond to β-normal terms.

[1] D. PRAWITZ, ***Natural deduction: A proof-theoretical study***, Almqvist & Wiskell, Stockhold, 1965.

[2] SIEG and J. BYRNES, *Normal natural deduction proofs* (*in classical logic*), to appear in ***Studia Logica***, 1997.

[3] SIEG and SCHEINES, *Searching for proofs* (*in sentential logic*), ***Philosophy and the computer*** (editor, L. Burkholder), Westview Press, Boulder, 1992, pp. 137–159.

► PETER A. CHOLAK, CARL G. JOCKUSCH, JR., and THEODORE A. SLAMAN, *More on the strength of Ramsey's theorem for pairs*.
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Following is the vein of independent work by Seetapun, Slaman and Jockusch we show:

THEOREM 1. *$\mathbb{N} \to [\mathbb{N}]^2_2$ is Π^1_1-conservative over* $\mathrm{RCA}_0 + I\Sigma_2$.

Thus any Π^1_1 statement provable from $\mathrm{RCA}_0 + I\Sigma_2 + \mathbb{N} \to [\mathbb{N}]^2_2$ is already provable from just $\mathrm{RCA}_0 + I\Sigma_2$. In particular, $\mathrm{RCA}_0 + I\Sigma_2 + \mathbb{N} \to [\mathbb{N}]^2_2$ does not prove PA.

We discovered this result by first considering the associated recursion theoretic problem. Using a result of Jockusch and Stephan, we improved a theorem of Seetapun by showing that for any computable (recursive) partition of pairs into two pieces there is an infinite low_2 homogeneous set $X(X'' \equiv_T 0'')$. This refutes an old conjecture of Jockusch. Then, as suggested by a conjecture of Slaman, we adapted the argument to produce a notion of forcing for models of arithmetic and showed that adding a generic homogeneous set to a model of arithmetic preserves $I\Sigma_2$.

▶ TODD EISWORTH, *Forcing and stable ordered-union ultrafilters*.
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We give a characterization of those P-points in the ground model that continue to generate P-points in a generic extension by Matet forcing with respect to a given stable ordered-union ultrafilter. (These ultrafilters have been studied previously by Matet and Blass; they are a reasonable analog of Ramsey ultrafilters in the context of Hindman's partition theorem for finite unions.) We will discuss a few applications to questions about cardinal characteristics of the continuum.

▶ ILIJAS FARAH, *Embeddings of analytic quotients*.
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We investigate some questions associated to quotients $P(N)/I$ by analytic ideals I on the set N of integers, in particular the structure of homomorphisms with "definable" liftings. We prove that in some cases the relation of Borel-embeddability of quotients reduces to the Rudin-Keisler ordering between the ideals. Arbitrary homomorphisms are also considered working in the context of the Open Coloring Axiom.

▶ LEONA F. FASS, *Semi-active learning*.
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A system that employs an *active* learning process can influence its environment and so, the informational examples it may receive, rather than just *passively* observing available information [1]. In those learning processes we have examined, a system seeking to acquire a specific behavior acts within its environment to construct or alternately, to confirm, a "correct" behavioral model. In a constructive process the system actively experiments with selected information available within its environment, while in the alternate, confirmatory process the system actively questions its environment to determine if a possible behavioral model is, indeed, correct. Neither approach would be considered fully active or fully passive, but both involve substantial system- environment *interaction* and an environment that "cooperates" so that a learning system will succeed. Hence we classify such processes as *semi- active learning*.

We initially employed semi-active learning techniques in the area of linguistic modeling, where we found that by applying domain constraints we could achieve complete success [3]. By exploiting the structure of an observed language, we determined the existence of a constrained "perfect" model. We also defined complete/exhaustive information from which the model could be determined and techniques to determine it monotonically. A system might interact infinitely with its environment, wit a learner experimenting "forever". Hence, one of the most *active* aspects of a successful semi-active learning process is *knowing when to stop*.

In other problem domains we have considered, constraints may not be possible, the structure of information within an environment may not be easily observable, or an all-encompassing structural definition may not exist. In such cases we consider it acceptable to *impose* some structure that *explains* observed environmental behavior. This can *force* at least some of a phenomenon into a constrained, semi-actively learnable "black box" [2]. Results will not be "perfect", but a subset of specified information will be represented correctly. Such an approach is the "ontological engineering" that we have generally observed in actual scientific reasoning [4].

[1] D. A. COHN and D. D. LEWIS, *Active learning symposium*, ***AI Magazine***, vol. 17, no. 1 (Spring 1996), p. 83.

[2] L. F. FASS, *Black box science*, in [4], pp. 116–117.

[3] ———, *Discovering a 'minimal' language model*, ***Proceedings of the Ninth Florida Artificial Intelligence Research Symposium***, Key West, May 1996, pp. 349–353.

[4] R. VALDES-PEREZ and L. DARDEN, ***Notes of the AAAI Spring symposium on systematic methods of scientific discovery*** (J. Lederberg, H. Simon, and D. Sleeman, editors), Stanford, May 1995, released as AAAI Press SS 95-03.

► SY D. FRIEDMAN and PETER KOEPKE, *The covering theorem for the constructible hierarchy with restricted Skolem functions*.
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The standard fine structure theory for the Gödel model L enriches the usual L_α-hierarchy by hull-operations which capture set formation in the Levy-hierarchy of formulae. The fundamental theorems of fine structure theory express that the enriched structures are respected by condensation arguments. In this talk the hierarchy is refined even further by introducing suitable restrictions on the domains of the hull operators.

Our approach circumvents the usual arguments involving iterated projects, standard parameters, and mastercodes and proceeds by simple-model theoretic arguments. We have earlier obtained Global Square and Morasses through the method. I am going to give a rough outline of the theory and apply it to a proof of Jensen's Covering Theorem for L.

► SU GAO, *The Glimm-Effros dichotomy for countable linear orderings*.
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The general Glimm-Effros dichotomy for an equivalence relation E states that either E is Borel reducible to the identity relation of a Polish space or E reduces the canonical Vitali equivalence relation. This property plays a prominent role in the study of the Topological Vaught Conjecture.

Building of Steel's proof of the Vaught Conjecture for trees we prove the Glimm-Effros dichotomy for countable linear orderings, that is, if an infinitary sentence in the language of a single binary relation only has linear orderings as its models, then the isomorphism relation of the countable models of this sentence has the Glimm-Effros dichotomy. Our theorem can be used to answer an open question of Friedman and Stanley regarding certain interpretability notions.

► VALENTIN GORANKO, *A metric space of closed theories*.
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Given a first-order language L we introduce a distance in the set $\mathcal{C}(L)$ of deductively closed theories in L as follows: for any $T_1, T_2 \in \mathcal{C}(L)$,

$$\mathbf{d}(T_1, T_2) = \begin{cases} 0 & \text{if } T_1 = T_2, \\ \frac{1}{n+1} & \text{if } n \text{ is the least integer such that a sentence of} \\ & \text{quantifier depth } n \text{ distinguishes } T_1 \text{ and } T_2. \end{cases}$$

$\mathcal{C}(L)$ endowed with that distance becomes a bounded and complete metric space.

We show that for every (non-trivial) first-order language L the space $\mathcal{C}(L)$ is disconnected, and give necessary and sufficient conditions for compactness and separability.

We further study important subspaces of $\mathcal{C}(L)$ including the subspace $\mathcal{CC}(L)$ of complete theories, as well as the subspaces of closed extensions of a given theory.

The metric space structure of $\mathcal{CC}(L)$ naturally transfers onto the set of all types of L-structures (classes of elementarily equivalent L-structures). The identity maps the resulting space $\Theta(L)$ continuously (but not homeomorphically) onto the model space of L.

An L-structure A is said to be *elementarily approximated* by a family of L-structures $\mathcal{S}$ if for every natural number n there is a structure $A_n \in \mathcal{S}$ such that $A_n \equiv_n A$.

We study elementary approximation in the framework of the space $\Theta(L)$.

Finally we discuss some problems and potential applications of the topic.

▶ RAYMOND D. GUMB and KAREL LAMBERT, *Definitions in nonstrict positive free logic*.

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Every "practical" programming language supplies the programmer with at least one nonstrict construct, such as the ALGOL60 arithmetic 'if-then-else' and the LISP 'cond'. Many programming languages also enable the user to define nonstrict functions. In some languages, this is accomplished through the lazy evaluation of procedure parameters, as realized, for example, by the call-by-name devices of ALGOL60 and SIMULA67 and the call-by-need mechanism of haskell. In other languages, such as Common LISP, a macro definition facility can serve a similar purpose. Programming languages that provide a mechanism for the user to define nonstrict functions are *nonstrict languages*, and we call the natural underlying logic of these languages *nonstrict positive free logic*. In this paper, we present the definition theory of nonstrict positive free logic. Suitable transformations of sentences in standard logic into sentences in nonstrict positive free logic preserve many properties of definitions in standard logic.

The full paper is scheduled to appear in a forthcoming issue of ***Modern Logic***.

▶ JOEL DAVID HAMKINS and ANDY LEWIS, *Infinite time Turing machines*.

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To provide a theory of infinitely long computations, we extend in a natural way the operation of Turing machines to infinite ordinal time. Our concept leads to a notion of computation on the reals and decidability for reals and sets of reals, with respect to which every Π^1_1 and Σ^1_1 set is decidable. The two kinds of jump operator and a notion of relative computability give a rich degree structure on the collection of reals and the collection of sets of reals.

▶ BERNARD HERWIG, STEFFEN LEMPP, and MARTIN ZIEGLER, *Constructive models of uncountably categorial theories*.

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A well-known theorem of Baldwin and Lachlan states that the countable models of an uncountably but not totally categorical theory form an elementary chain of order type $\omega + 1$. Computability theorists have studied the question of which combination of these models can be constructive or nonconstructive, and most conceivable combinations are known to be possible. All these, however, use infinite languages.

In joint work with Herwig and Ziegler, we show that there is an uncountably but not totally categorical theory in three binary relations such that the prime model is strongly minimal and the only constructive model of its theory. The proof codes an unsolvable word problem of a finitely generated group into the theory in such a way that it can only be recovered from the nonprime models.

▶ DARRYL JUNG, *A formalization of the logical theory of* Principia Mathematica.

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Both modern quantification theory and type theory in large measure have their origins in the logical theory that Bertrand Russell developed and presented in *Principia Mathematica*. Notwithstanding, *Principia*'s logical theory is not well known and, in important respects, not well understood today. Although several commentaries about the theory have appeared since *Principia*'s publication, most of these mischaracterize several of its gross structural features—for instance, those by Copi, Kleene, Kneale, and Ramsey—and even some of the better commentaries remain unclear about its details–for instance, those by Chihara, Hylton, and Quine. There are several reasons for these deficiencies. First, the conception of logic that underlies *Principia*'s technical development is, in many respects, at variance with more contemporary conceptions. Secondly, *Principia*'s discursive explanations and its formal work at times fall short of contemporary standards of rigour and clarity. Thus, for instance, these do not explicitly distinguish the sundry roles played by its axioms, rules of inference, and even rules of formation, usually simply calling them all "principles." *Principia* contrasts notably with Frege's *Grundgesetze der Arithmetik* in this respect. Thirdly, *Principia*'s discursive explanations and formal work contain several peculiarities which may even strike one as incongruous at first blush. Thus its Introduction and Chapter *12 present mutually incompatible definitions of the notion of predicative function. Chapter *9 specifies a very fine-grained notion of type for propositions and propositional functions, while *Principia*'s actual deductions may be taken to appeal to a rather coarse-grained of its notion of order have caused scholars to be confused about the nature of its ramified theory of types.

I attempt to explain the logical theory that Russell endeavored to put forward in *Principia Mathematics* by offering a clean formalization of its theory of apparent variables (i.e., quantification theory) and its theory of types.

► IRAJ KALANTARI and LAWRENCE WELCH, *Filters as a foundation for recursive analysis: Expelling points from domains of recursive functions*.
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We review our approach to recursive topology and analysis via filters with a focus on *points* and *functions*. Our interest is in investigation of the interplay between points and domains of functions. In traditional approaches to recursive analysis, all recursive functions are continuous and their domains are the entire set of reals. In our approach, we discover that, in addition to capturing the traditionally recursive functions, we have *recursive quantum functions* which do not have the entire set of reals as their domains, while they seem continuous and entire to the recursive observer.

We then note that there are three kinds of points: *recursive points* (computably describable), *avoidable points* (points which can be avoided from domains of recursive quantum functions), and *shadow points* (points that are impossible to avoid recursively). The partitioning of the reals into these types of points leads to interesting results on functions and their domains.

Some results are:

THEOREM 1 (Excision Theorem). *If f is a recursive quantum function, then there is a recursive quantum function g which agrees with f but its domain is smaller than the domain of f by uncountably many points.*

THEOREM 2 (Density Theory). *If f and g are two recursive quantum functions with f a proper subfunction of g and with domain of g containing at least two points not in the domain of f, then there is a quantum recursive function h which is a proper superfunction of f and a proper subfunction of g.*

The talk was given by Iraj Kalantari.

► IRAJ KALANTARI and LAWRENCE WELCH, *Filters as a foundation for recursive analysis: Codomains, avoidable sets of points, and uniformity*.
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We continue our study of recursive topology and analysis and direct our findings to *sets* of points avoidable from domains of recursive quantum functions. In particular, we are interested in recursion-theoretic properties of such sets, especially when they are complements of domains of recursive quantum functions.

A set of points S is *avoidable* if there is a countable open cover of the recursive points that is disjoint from S, and S is *recursively avoidable* if there is such a cover which is recursively enumerable. Also, S is *recursively piecemeal avoidable* if S is a countable union of recursively avoidable sets whose recursive avoidability is witnessed in a *uniform* fashion. We consider the interplay of these three properties and prove the following results, where by *codomain* we mean the *uncountable* complement of the domain of a recursive quantum function:

(1) There is a recursively avoidable codomain.
(2) There is a recursively piecemeal avoidable codomain which is not avoidable.
(3) There is a recursively piecemeal avoidable codomain which is avoidable but not recursively avoidable.

The talk was given by Lawrence Welch.

► BYUNGHAN KIM, *Stability and simplicity*.
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For the last two or three decades, one of the main subjects of model theory is the so-called "stability theory". One of the reasons that model theories are interested in stable theories is that there is a notion of "nonforking" which supplies uniform concept of independence to any stable theories. Although Shelah's definition of "nonforking" is sensible for any first-order theories, as it behaves nicely in stable theories, the dichotomy of stable/unstable structures are meaningful. Recently in Kim's dissertation, it is shown that Shelah's nonforking also produces a concept of independence to simple theories as well as stable theories. A family of simple theories properly contains that of stable theories. In other words, stable theories can be viewed as a particularly nice class of simple theories. Accordingly many questions naturally follow in trying to generalize results from stable theories.

► ULRICH KOHLENBACH, *General logical principles of uniform boundedness and their mathematical strength*.
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In [1] we have introduced a principle of uniform Σ^0_1-boundedness Σ^0_1-UB$^-$ which allows us to give short proofs (relatively to extremely weak subsystems $\mathcal{A}\restriction$ of arithmetic $\mathcal{A}$ in all finite types) of many analytical theorems, e.g., Dini's theorem, the sequential Heine-Borel property of $[0,1]^d$, the existence of a continuous inverse of a strictly monotone function $f \in C[0,1]$ and others, using higher types and avoiding tedious coding of analytical objects like functions $f \in C[0,1]$ (see [2]).

Σ^0_1-UB$^-$ follows (again relatively to $\mathcal{A}\restriction$) from quantifier-free choice AC- qf and a 'non-standard' axiom F^- which (like Σ^0_1-UB$^-$) is not true in the full set-theoretic type structure but is consistent with (analytical extensions) of $\mathcal{A}\restriction + \text{AC-qf}$. Moreover, F^-, which can be seen not to contribute to the growth of extractable uniform bounds or the provably recursive function(al)s, can be eliminated from the verification of the extracted bound.

Whereas the straightforward generalization of Σ^0_1-UB$^-$ to more complex formulas is inconsistent with $\mathcal{A}\restriction$ (even for Π^0_1-formulas instead of Σ^0_1 ones), this changes as soon as certain structural restrictions on the matrix of these formulas are imposed which are satisfied in many applications ([3]).

In this talk we introduce quite general rules of uniform boundedness (also in higher types) and investigate their mathematical strength and their impact on extractable bounds as well as their relation to comprehension principles.

[1] U. KOHLENBACH, *Mathematically strong subsystems of analysis with low rate of growth of provably recursive functionals*, to appear in ***Archive for Mathematical Logic***.

[2] ———, *The use of a logical principle of uniform boundedness in analysis*, submitted to ***Proceedings 'Logic in Florence 1995'***.

[3] ———, *Arithmetizing proofs in analysis*, submitted to ***Proceedings Logic Colloquium 96, San Sebastian***.

► OLIVIER LESSMANN, *Ranks and pregeometries in finite diagrams*.
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Shelah in 1969 initiated the study of classes of models of a finite diagram $D \subseteq D(T)$, namely classes of models of T omitting all the types in $D(T) - D$.

In this work, we introduce a rank for these classes, which is bounded for the $\aleph_0$-stable case. We focus our study on totally transcendental diagrams (those for which the rank is bounded), making no assumption on the cardinality of T. This allows us to define a natural dependence relation on elements, sharing many of the formal properties of forking. Moreover this dependence relation induces a pregeometry on the set of realizations of a type of minimal rank. Finally, these concepts are used to generalize many of the classical results for models of a totally transcendental theory. In fact, the picture is similar (but the proofs differ as we don't have compactness): models are determined by their pregeometries or their relationship with their pregeometries. This is illustrated with positive results (categoricity, tree decomposition) and negative results (construction of nonisomorphic models).

▶ BENEDIKT LÖWE, *Analytic measurability for forcing-σ-algebras*.
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It is well known that the σ-algebras of LEBESGUE measurable sets and of the sets having the BAIRE property encompass the analytic sets.

We give a uniform way of showing that for a large class of σ-algebras constructed from so-called arboreal forcing notions. This class includes all of the forcing notions from the systematics of [1].

[1] JÖRG BRENDLE, *Strolling through paradise*, ***Fundamenta Mathematicae***, vol. 148 (1995), pp. 1–25.

▶ ALEX M. MC ALLISTER, *A notion of forcing and computability of sets and enumerations*.
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We consider a "new" notion of forcing and two results that we can obtain using this notion of forcing. If $\mathcal{T}$ is a theory, we let $\mathcal{T}_n = \mathcal{T} \cap \Sigma_n$ for all $n \in \omega$. Now, given a complete theory $\mathcal{T}$, a Scott set $\mathcal{S}$ with $\mathcal{T}_n \in \mathcal{S}$ for all $n \in \omega$ and a set of new constants A, the forcing conditions consist of all $p = \Gamma(\overline{a})$ satisfying the following conditions:

- $\Gamma(\overline{x}) \in \mathcal{S}$.
- $\Gamma(\overline{x})$ is a complete B_n-type in variables $\overline{x}$.
- $\overline{a} \subset A$ is finite.
- $\Gamma(\overline{a}) \cup \mathcal{T}_n$ is consistent.

Using this notion of forcing we have produced two results of particular interest. The first characterizes the sets computable in all models of a countable complete theory. The second result generalizes a result of Lachlan and Soare on True Arithmetic (see [1]) to a much broader setting. In particular, we have the following:

THEOREM 1. *If $\mathcal{S}$ is a Scott set and $\mathcal{T}$ is a complete theory with $\mathcal{T}_n \in \mathcal{S}$ for all $n \in \omega$, then for all $X \subseteq \omega$ the following are equivalent*:

(1) *for all models $\mathcal{A}$ of $\mathcal{T}$ representing $\mathcal{S}$, $X \leq_T \mathcal{A}$*
(2) *there is a complete existential type $\Gamma \in \mathcal{S}$ consistent with $\mathcal{T}$ such that $\chi_X \leq_e \Gamma$.*

THEOREM 2. *If $\mathcal{S}, \mathcal{J} \subseteq \mathcal{P}(\omega)$ such that $\mathcal{S}$ is a Scott set, $\mathcal{J}$ is a jump ideal, $\mathcal{S} \subset \mathcal{J}$ and for all $X \in \mathcal{J}$, there exists a $C \in \mathcal{S}$ such that C is a "coding" set for the family of subtrees of $2^{<\omega}$ computable in X, and if $\mathcal{T}$ is a complete theory with $\mathcal{T}_n \in \mathcal{J}$ for all $n \in \omega$, then there exists a model $\mathcal{A}$ of $\mathcal{T}$ representing $\mathcal{J}$ such that no enumeration of $\mathcal{S}$ is computable in $\mathcal{A}$.*

We note that Theorem 2 is of particular interest when we also assume that $\mathcal{T}$ represents $\mathcal{S}$. If we add this additional assumption to the hypotheses of Theorem 2, then there exist completions of $\mathcal{PA}$ and $\mathcal{ZF}$ which satisfy these conditions. In particular, there exist models of certain completions of these theories from which one cannot enumerate the family of sets represented by the theory.

[1] A. H. LACHLAN and R. I. SOARE, *Models of arithmetic and upper bounds of arithmetic sets*, ***Journal of Symbolic Logic***, vol. 59, no. 3 (1994), pp. 997–983.

► G. MINTS, *Existential instantiation for intuitionistic logic*.
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We propose a manageable formulation NJi of intuitionistic logic with existential instantiation [2]. Extension to classical or higher-order case is straightforward. System $M\varepsilon$ of [1] using ε-symbol is probably the closest to NJi in the literature.

Formulas of first-order predicate logic are constructed in a familiar way from atomic formulas (including propositional constant $\perp$) by the connectives &, $\vee$, $\rightarrow$, $\forall$, $\exists$. Free and bound variables $\mathrm{FV}(A)$, $\mathrm{BV}(A)$ of a formula A are defined as usual. There are no redundant quantifiers: $\exists x\, A$, $\forall x\, A$ are formulas only if $x \in \mathrm{FV}(A)$.

Assumptions of the system NJi are formulas as well as expressions of the form $\langle\Gamma\rangle_{\exists x\, A,a}$ and $\langle\Gamma\rangle_{A\vee B,C}$ where Γ is a multiset of assumptions, A, B are formulas, C is A or B and a is an individual variable. By the definition, $\mathrm{FV}(\langle\Gamma\rangle_{\exists x\, A,b}) = \mathrm{FV}(A[x/b])$, $\mathrm{FV}(\langle\Gamma\rangle_{A\vee B,C} = \mathrm{FV}(C)$. Note that assumptions with $\langle,\rangle$ cannot be subformulas of (real) formulas. *Sequent* of NJi is any expression $\Gamma \Rightarrow A$ where Γ is a multiset of assumptions and A is a formula.

Axioms: $B \Rightarrow B$, $\perp\Rightarrow A$, where A is atomic.

Inference rules. Standard introduction and elimination rules for &, $\rightarrow$, $\forall$, standard $\vee$ introduction.

$$\exists I\,\frac{\Gamma \Rightarrow A[x/t]}{\Gamma \Rightarrow \exists x\, A} \quad \exists i\,\frac{\Gamma \Rightarrow \exists x\, A}{\langle\Gamma\rangle_{\exists x\, A,b} \Rightarrow A[x/b]} \quad \langle\rangle\exists\,\frac{\langle\Gamma\rangle_{A,b}, \Delta \Rightarrow C}{\Gamma, \Delta \Rightarrow C}$$

where $b \notin \mathrm{FV}(\Gamma, \exists x\, A)$ in $\exists i$, $b \notin \mathrm{FV}(\Gamma, \Delta, C)$ in $\langle\rangle\exists$.

$$\langle\rangle\vee\,\frac{\langle\Gamma\rangle_{A_0\vee A_1,A_0}, \Delta \Rightarrow C \quad \langle\Gamma\rangle_{A_0\vee A_1,A_1}, \Sigma \Rightarrow C}{\Gamma, \Delta, \Sigma \Rightarrow C}.$$

THEOREM. *$\langle\rangle$-free sequent is derivable in NJi iff it is derivable in intuitionistic logic.*

A deduction is *uniform* if it has pure variable property, all $\exists i$-inferences with the same eigenvariable have the same deduction of the premise (up to a suitable renaming), and similar property holds for $\vee$.

THEOREM. *Every deduction can be uniformized, and uniform deductions are strongly normalizable.*

[1] D. LEIVANT, *Existential instantiation in a system of natural deduction*, ***Math. Centrum-ZW 13-73***, 1973.

[2] W. V. QUINE, *On natural deduction*, ***Journal of Symbolic Logic***, vol. 15 (1950), pp. 93–102.

► LAWRENCE S. MOSS and GLEN T. WHITNEY, *Iterated corecursion*.
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In [1], Barwise and Moss define an operation of "corecursion," which is dual to ordinary notions of recursion. Any monotone operator on the anti-founded universe of sets has a greatest fixed class denoted Γ^*. If Γ is also *smooth*, i.e., has some additional regularity properties, then there is a natural way to take any function $\pi : B \longrightarrow \Gamma(B)$ and produce a new map $\phi : B \longrightarrow \Gamma^*$. This map ϕ satisfies appropriate fixed-point or "corecursion" conditions.

It is also possible to parametrize this operation. Let $A + B$ denote a disjoint union or coproduct operation on sets A and B, and define $\Gamma_C(X) = \Gamma(C + X)$. Then one can start with a function $\pi : B \longrightarrow \Gamma(B + C)$ and produce $\phi : B \longrightarrow \Gamma_C^*$ by corecursion. However, the resulting map ϕ is not of the proper form to apply corecursion again. This contrasts with ordinary least-fixed-point recursion for monotone maps on complete partial orders: If $f : D \times E \longrightarrow D$ is monotone, then the map $g : E \longrightarrow D$ produced by taking least fixed points in D is again monotone. Thus, it is not possible to iterate the Barwise-Moss corecursion.

We generalize the notion of corecursion to take a map $\sigma : B \longrightarrow \Gamma_B^*$ and produce $\psi : B \longrightarrow \Gamma^*$. The resulting map ψ satisfies exactly the same corecursion conditions as in [1], which means that if the image of σ happens to lie in $\Gamma(B) \subseteq \Gamma_B^*$, then the generalized corecursion agrees with [1]. This construction eliminates the obstacle to iterating corecursion. We may then interpret the recursion construct of the Formal Language of Recursion (FLR_0) using corecursion.

THEOREM. *The valid equational identities of corecursion coincide with those for ordinary recursion.*

Finally, we consider some applications of these ideas to the semantics of programming languages.

[1] J. BARWISE and L. S. MOSS, ***Vicious circles***, CSLI Press, 1996.

► CYRUS F. NOURANI, *Functorial admissible models*.
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Infinite language categories were presented and their categorical properties are defined by our 1994–95 papers. Computable Functors are defined in a December 1996 paper [11]. By defining admissible generic diagrams we define functors to admissible structures. Starting with infinite language category $L\omega 1, B$ [4] we define generic sets on $L\omega 1, \omega$. Let $\mathcal{L}$ be the admissible language [11] defined on the Keisler $L\omega 1, \omega$ fragment [1, 4]. Generic diagrams, denoted by G-diagrams were defined since [3] to be diagrams for models defined by a specific function set, for Example Σ_1 Skolem functions.

At the summer colloquium [9] we had put forth descriptive computing principles, briefed by the following definition and theorem.

DEFINITION 1. A set is Descriptive Computable iff it is derivable by a G-diagram with computable functions.

THEOREM 1. *For A an admissible computable set, A is descriptive computable.*

Thus we can apply generic diagrams to define admissible computable sets and models. Let us define computable functors [11]. The functors define generic sets from language strings to form limits and models [4, 7, 8, 11].

DEFINITION 2. A functor $F : A \rightarrow B$ is computable iff there is an effective procedure for defining the arrow object mapping from A to B.

DEFINITION 3. A model is admissible iff its universe, functions, and relations are admissible sets.

THEOREM 2. *There is a generic functor defining an admissible model.*

Let $\mathcal{L}$ be an admissible language [10] defined on the Keisler fragment $L\omega 1, \omega$. The functor on the category $L\omega 1, B$ [4] might be defined with admissible horn sets.

THEOREM 3. *The generic functors define admissible models, provided the functions and relations in the language fragment are $\mathcal{L}$-admissible.*

[1] H. J. KEISLER, ***Model theory for infinitary logic***, North-Holland, Amsterdam, 1971.

[2] F. W. LAWVERE, *Functorial semantics of algebraic theories*, ***Proceedings of the National Academy of Sciences, USA***, 1963.

[3] C. F. NOURANI, *Forcing with universal sentences*, ***Proceedings of the Association for Symbolic Logic***, vol. 49, Boston, Massachusetts, 1983.

[4] ———, *Functorial model theory and infinitary language categories*, September 1994, ***Association for Symbolic Logic, San Francisco***, January 1995, see ***Association for Symbolic Quarterly***, Summer 1996, for recent abstract.

[5] S. MAC LANE, *Categories for the working mathematician*, ***GTM***, vol. 5, Springer-Verlag, Berlin, Heidelberg, New York, 1971.

[6] J. Barwise, editor, ***Handbook mathematical logic***, North-Holland, 1978.

[7] C. F. NOURANI, *Functorial string models*, ***WOLLIC***, 1995.

[8] ———, *Functorial model theory and generic fragment consistency models*, ***American Mathematical Society-Association for Symbolic Logic, San Diego***, January 1997.

[9] ———, *Descriptive computing*, ***Summer Logic Colloquium, San Sebastian, Spain***, July 1996.

[10] J. BARWISE, *Implicit Definability and compactness in infinitary languages*, ***Syntax and semantics of infinitary languages*** (J. Barwise, editor), Springer-Verlag Lecture Notes in Mathematics, vol. 72, Berlin, Heidelberg, New York.

[11] C. F. NOURANI, *Computable functors and generic models diagrams*, December 1996.

► ŽIKICA PEROVIĆ, *An example of η_1-ordered real closed field which is not hyper-real.*
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In [2] the model theory of reals with restricted analytic functions, called T_{an} was reexamined. The same paper also considered the theory $(T_{an}, \exp, \log)$ of reals with exponentiation and logarithms of positive elements. Following ideas of this paper we show that hyper-real fields could be expanded to models of $(T_{an}, \exp, \log)$.

THEOREM 1. *Let F be a hyper-real field. F could be expanded to a model of* $(T_{an}, \exp, \log)$.

COROLLARY 2. *Let F be a hyper-real field. We can define real powers in F so that they satisfy usual axioms for powers.*

There were many long standing questions about η_1-ordered fields. Some of them are already answered, some of them are still open. In [1] and [3], was asked to find an example of an η_1-ordered real closed field which is not a hyper-field. In [4] such an example under some additional set-theoretic assumptions was constructed. Here we provide such an example in ZFC.

THEOREM 3. *Let F be a hyper-real field, and G its value group. Then there is an R-module over G.*

This way we have proved that there is a specific property of hyper-real fields which distinguishes them among η_1-ordered real closed fields. We construct now an example of η_1-ordered real closed field, such that there is no R-module over it value group.

THEOREM 4. *There exists an η_1-ordered real closed field which is not isomorphic to any hyper-real field.*

[1] M. ANTONOVKIJ, D. CHUDNOVSKY, G. CHUDNOVSKY, and E. HEWITT, *Rings of real-valued continuous functions II*, ***Mathematische Zeitschrift***, vol. 176 (1981), pp. 151–186.

[2] L. VAN DEN DRIES, A. MACINTYRE, and D. MARKER, *The elementary theory of restricted analytic fields with exponentiation*, ***Annals of Mathematics***, vol. 140 (1995), pp. 183–205.

[3] M. HENRIKSEN, *Rings of continuous functions from an algebraic point of view*, ***Ordered algebraic structures*** (J. Martinex, editor), Kluwer Academic Publishers, 1989, pp. 143–174.

[4] Ž. PEROVIĆ, *At least one cannot prove that they are the same*, to appear.

► GRAHAM PRIEST, *Inconsistent models of arithmetic. Part II: The general case*.
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The existence of models of paraconsistent theories that extend the set of sentences true in the standard model of arithmetic is demonstrated in [2]. In [1] a taxonomy of the finite models is given. This paper establishes the *general* structure of such models, and some of their further features.

- A *nucleus* is any set, N_x, of the form $\{y; x \leq y \leq x\}$.
- A nucleus, N, has *period* p iff for all $y \in N$, $y + p = y$.
- A nucleus is *proper* if it has period $p \geq 1$.
- A *chromosome* is any sequence of successors unbounded in both directions.
- A chromosome is *cyclical* if it is finite, otherwise it is *linear*.

The main results include the following.

THEOREM 1. *A model comprises a linear sequence of nuclei with periods inherited up the ordering. There are three segments (any of which may be empty). The first contains only proper nuclei. The second contains proper nuclei with linear chromosomes. The third contains proper nuclei with cyclical chromosomes.*

THEOREM 2. *For any ordinal, α, there are models where the order-type of the proper nuclei is α.*

THEOREM 3. *There are models where the order-type of the proper nuclei is that of the rational numbers.*

[1] R. K. MEYER and C. MORTENSEN, *Inconsistent models for relevant arithmetic*, ***Journal of Symbolic Logic***, vol. 49 (1984), pp. 917–29.

[2] G. PRIEST, *Inconsistent models of arithmetic*, ***Journal of Philosophical Logic***, forthcoming.

► CHAZ SCHLINDWEIN, *Hemi-proper forcing*.
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We say P is hemi-proper iff whenever λ is a large enough regular cardinal and M and N are countable elementary substructures of H_λ, and p is in $P \cap M$, and $P \in M \in N$, then p does not force that $\omega - 1 \cap M[G]$ is not a subset of N. Hemi-properness is more general than semi-properness while maintaining the key properties: it is preserved under iteration and it ensures $\omega - 1$ is not collapsed.

▶ KRISTER SEGERBERG, *Irrevocable revision*.
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We assume an underlying consequence relation Cn and use the term "theory" as in [1] except that we require theories to be closed under Cn. By a *theory complex*, or *complex* for short, we mean a pair (V, T) where V and T are theories such that $V \subseteq T$; one may refer to V as the *doxastic commitment* and to T as the *belief set* (of an agent). Only one operation on the set of complexes is considered, viz., *irrevocable revision*, denoted by $*$: if (V, T) is a complex and φ is a formula, then $(V, T) * \varphi$ is also a complex. We write $*_1$ and $*_2$ for the two projects of $*$: thus if $(V', T') = (V, T) * \varphi$, then $V' = V *_1 \varphi$ and $T' = T *_2 \varphi$. We study the metatheory defined by the following postulates, where (V, T) is any complex and φ and ψ are any formulæ:

($*0$) $V *_1 \varphi = \mathrm{Cn}(V \cup \{\varphi\})$.
($*1$) $V *_1 \varphi \subseteq T *_2 \varphi$.
($*2$) $\varphi \in T *_2 \varphi$.
($*3$) If $\neg\varphi \notin \mathrm{Cn}\, T$ then $T *_2 \varphi = \mathrm{Cn}(T \cup \{\varphi\})$.
($*4$) If $\neg\varphi \notin \mathrm{Cn}\, V$ then $\bot \notin T * \varphi$.
($*5$) If $\mathrm{Cn}\{\varphi\} = \mathrm{Cn}\{\psi\}$ then $T *_2 \varphi = T *_2 \psi$.
($*6$) $T *_2 \varphi = \mathrm{Cn}(((T *_2 \varphi) \cap T) \cup \{\varphi\})$.
($*9$) $(T *_2 \varphi) *_2 \psi = T *_2 (\varphi \wedge \psi)$.

The ≪one-shot≫version of this metatheory with $V = \emptyset$ is (virtually) a conservative extension of AGM (which explains the numbering of our postulates). Our metatheory, however, allows iteration of $*$.

The idea behind irrevocable revision arose in collaboration with Horacio Arló Costa; cf. [2].

[1] Carlos E. Alchourrón, Peter Gärderfors, and David Makinson, *On the logic of theory change: Partial meet contraction and revision functions*, ***Journal of Symbolic Logic***, vol. 50 (1985), pp. 510–530.

[2] Horacio Arló Costa and Krister Segerberg, *Conditionals and hypothetical revision*, abstract, to appear in ***Theoria***.

▶ DAVID REED SOLOMON, *Reverse mathematics and fully ordered groups*.
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Reverse mathematics has had considerable success in using a weak base system, RCA_0, to prove the equivalence of ordinary mathematical theorems to naturally occurring subsystems of second-order arithmetic. The focus here will be on results in reverse mathematics concerning fully ordered (f.o.) groups. An f.o. group is a group with a linear ordering such that order is preserved by multiplication on both the right and the left. This talk will include results about orderings of direct products, quotient groups, and free groups. It will also consider a theorem classifying the possible order types for a countable f.o. group. For the reverse mathematics, equivalence of WKL_0, ACA_0, and $\Pi^1_1 - \mathrm{CA}_0$ will be given. The work on free groups answers an open question of Downey and Kurtz from *Recursion theory and ordered groups*, ***Pure and Applied Logic***, vol. 32(1986).

▶ STEVE WALK, *ANR degrees and lattice embeddings*.
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Array nonrecursive (a.n.r.) degrees were introduced by Downey Jockusch, and Stob to capture certain natural constructions: specifically, constructions in which multiple permissions are necessary to fulfill each positive requirement. Such constructions include (i) the construction of an f-c.e. set, where f is a strictly increasing computable function; (ii) the construction of disjoint c.e. sets whose union is cofinite and which cannot be separated by any set of degree $\mathbf{0}'$; and (iii) the construction of a Martin Pour-El theory of a certain degree.

It was shown by Downey, Jockusch, and Stob that every nonlow$_2$ degree is a.n.r. Downey and Shore have demonstrated that the lattice M_5 can be embedded within the computably enumerable degrees below every nonlow$_2$ degree, via a technique they refer to as *multiply inductive* permitting. It is reasonable, then, to wonder whether the multiple permitting associated with a.n.r. degrees is enough to allow us to embed M_5 below every a.n.r. degree. However, using techniques of Cholak, Downey, and Shore, we have shown that

THEOREM. *There is a* (*nonlow*) *computably enumerable a.n.r. degree below which the lattice* M_5 *cannot be embedded in the computable enumerable degrees.*

This result is the best possible in terms of jump classes. Also, by this result, there is a definable difference between the a.n.r. degrees and the nonlow$_2$ degrees, namely that each of the latter bounds M_5 but some of the former do not.

The author thanks his advisor, Peter Cholak, for guidance and encouragement.

► DAN E. WILLARD, *The tangibility reflection principles and the Liar's Paradox*.
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Define a function $F(x_1, x_2, \ldots, x_k)$ to be *Non-Growth* iff $F(x_1, x_2, \ldots, x_k) \leq \mathrm{MAX}(x_1, x_2, \ldots, x_k)$. Consider a language $\mathcal{L}$ based around a finite set of non-growth functions. One can obviously rewrite the conventional statements of arithmetic by replacing Addition and Multiplication with Subtraction and Division (where we define $x - y = 0$ when $y \geq x$ and $\lfloor \frac{x}{y} \rfloor = x$ when $y = 0$). Define a sentence Ψ to be Π_1^- iff it is equivalent to a prenex sentence all of whose existential quantifiers are bounded.

Axiom systems employing non-growth do not necessarily recognize Successor as a total function. Nevertheless, Löb's Theorem can be generalized to show no consistent axiom system can prove all Peano Airthmetic's Π_1^- theorems and also verify Equation (1) for every Π_1^- sentence Ψ.

$$\{\exists\, y\, \mathrm{Prf}_\alpha(\ulcorner \Psi \urcorner, y)\} \supset \Psi. \tag{1}$$

The surprising facet is that close approximations to the impermissible Equation (1) are feasible. Let us use the following notation:

a. $\mathrm{TangPred}(x) = \{\exists v\, x = v - 1\}$
b. $\mathrm{TangDiv}(x) = \{\exists v\, x < \frac{v}{2}\}$
c. $\mathrm{TangRoot}(x) = \{\exists v\, x\ < v^{1/2}\}$.

If Ψ denotes a prenex sentence, let Ψ^i denote a sentence identical to Ψ except that each *unbounded* universally quantified variable in Ψ is bounded by i. Then for any consistent r.e. axiom system A, we can construct a system α which can verify all A's Π_1^- theorems and verify for any prenex Ψ:

$$\forall x \{[\exists\, y\, \mathrm{Prf}_\alpha(\ulcorner \Psi \urcorner, y) \wedge \mathrm{TangPred}(x)] \supset \Psi^x\}. \tag{2}$$

Moreover, such an α can recognize Bitwise-Or as a total function if TangDiv replaces TangPred in (2).

The preceding paragraph assumed "Prf" denoted Hilbert-style proofs. If "Prf" was weakened to designate a Herbrand or Semantic Tableaux type proof and TangRoot replaced TangPred, then the resulting α can be allowed to recognize Addition as a total function.

However, Robert Solovay (private communications) has generalized a theorem by Pudlàk to show that the preceding α are inconsistent if they recognize Successor as a total function and use a version of Equation (2) with "Prf" designating Hilbert proofs and *with any type* of Tangible(x) formula where Tangible(0) is simply true!

Abstracts presented by title

▶ JOHN CORCORAN, *Teaching on categoricity of arithmetic*.
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This paper presents a new, simple, and instructive proof of the categoricity of arithmetic together with a series of remarks useful to students and instructors in the classroom. Arithmetic is represented by the axiom system of the 1931 Gödel Incompleteness paper which uses three axioms (the zero axiom, the successor axiom, and the induction axiom) expressed in a higher-order language involving only two non-logical constants: '0', a symbol for zero; '*s*', a symbol for the successor function. An isomorphism is defined as a structure-preserving function, or homomorphism, which is one-one and onto. Every interpretation of this language determines its own unique denotation function mapping the numerals of the language to the universe of discourse of the interpretation, the interpretation's "numbers." Besides the intended model, the interpretation whose "numbers" are the natural numbers, whose "zero" is zero and whose "successor" is the successor function, this approach highlights the Hilbert interpretation, also called the algebra of constant terms, whose universe of "numbers" is actually the set of (synctactic) numerals, whose "zero" is the zero-symbol and whose "successor" is the (syntactic) operation of prefixing an occurrence of the successor symbol. It is easy to verify that the Hilbert interpretation is also a model. Likewise it is easy to verify that in any given interpretation the denotation function is a homomorphism from the Hilbert model to the given interpretation, that satisfaction of the first two axioms results in the denotation function's being one-one, and that satisfaction of the induction axiom results in the denotation function's being onto. Thus in any given model of all three Gödel axioms, the denotation function is an isomorphism from the Hilbert model to the given model. Thus, any two such models are isomorphic. The paper is written in ordinary scholarly English avoiding special mathematical formalisms so that the conceptual structure of the subject stands out and so that the paper will be compatible with any number of different courses using incompatible notations.

▶ VLADIMIR KANOVEI, *Two dichotomy theorems on colourability of non-analytic graphs*.
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THEOREM 1. *Let κ be an uncountable cardinal. Every κ-Souslin graph G on reals satisfies one of the following two requirements*: (I) *G admits a $\kappa + 1$-Borel colouring by ordinals below κ*; (II) *there exists a continuous homomorphism* (*in some cases embedding*) *of a certain locally countable Borel graph G_0 into G*.

THEOREM 2. *In the Solovay model, every OD graph G on reals satisfies one of the following two requirements*: (I) *G admits an OD colouring by countable ordinals*; (II) *as above.*

▶ Ž. MIJAJLOVIĆ, *On the effective solution of a mathematical programming problem with algebraic constraints*.

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Robinson's solution of Seventeenth Hilbert problem by methods of model theory represents an important contribution to the model-theoretic algebra. The solution is based on the method of elimination of quantifiers and notion of model completeness, the model-theoretic version of the elimination of quantifiers. Besides, this notion can be understood as a transfer principle, which is of significant importance for the applications in algebra.

DEFINITION. A theory T in the first-order predicate calculus admits elimination of quantifiers if for every formula φ of T there is a formula ψ in the language of T, without quantifiers such that: $T \perp \varphi \Leftrightarrow \psi$.

Now we consider a problem in mathematical programming. Programming problem with algebraic constraints in several variables $x_1, \dots, x_n$

$$f \longrightarrow \min, \quad p_1 = 0, \dots, p_k = 0, \quad q_1 > 0, \dots, q_m > 0$$

where $f\, p_1, \dots, p_k, q_1, \dots, q_m$ are polynomials in variables $x_1, \dots, x_m$ with rational coefficients, is easily stated in the first-order theory of ordered fields as follows:

$$\begin{aligned}
&\exists x_1 \dots x_n (y = f(x_1, \dots, x_n) \\
&\wedge p_1(x_1, \dots, x_n) = 0 \wedge \dots \wedge p_k(x_1, \dots, x_n) = 0 \\
&\wedge q_1(x_1, \dots, x_n) > 0 \wedge \dots \wedge q_m(x_1, \dots, x_n) > 0 \\
&\wedge \forall x_1 \dots x_n (p_1(x_1, \dots, x_n) = 0 \wedge \dots \wedge p_k(x_1, \dots, x_n) = 0 \\
&\wedge q_1(x_1, \dots, x_n) > 0 \wedge \dots \wedge q_m(x_1, \dots, x_n) > 0 \\
&\Rightarrow y \le f(x_1, \dots, x_n))).
\end{aligned}$$

Eliminating quantifiers from the above formula by use of Tarski's algorithm for the theory T_{RCF} of closed fields, we obtain a formula $\psi(y)$ of the theory of ordered fields in the single variable y. As T_{RCF} is o-minimal, this formula is a finite disjunction of the formulas of the form $y > r$, $y < r$, $y = r$, where r is a rational number. We take it as a starting step in the algorithm of finding a solution of the above stated mathematical programming problem. Observe that, in fact, this proves that the problem of finding solutions of mathematical programming problems with polynomial constraints is decidable.