THE BULLETIN OF SYMBOLIC LOGIC
Volume 3, Number 3, Sept. 1997

1996–1997 WINTER MEETING
OF THE ASSOCIATION FOR SYMBOLIC LOGIC

San Diego, California, January 10–11, 1997

The Winter Meeting of the Association was held in San Diego, California, on January 10–11, 1997, in conjuction with the annual meeting of the American Mathematical Society. The Program Committee consisted of Alexander Kechris (Chair), Sam Buss, and David Marker. The program included eight invited talks, eighteen contributed talks, and three papers presented by title only.

The following invited talks were presented:

Miklos Ajtai (IBM Almaden Research Center), *Induction, counting, and approximate counting in finite stuctures*.

Gregory Cherlin (Rutgers University), *Tame groups and finite simple groups*.

Daniel Mauldin (University of North Texas), *Some topics on the interface between set theory and analysis*.

Jeremy Avigad (Carnegie Mellon University), *The combinatorics of propositional provability*.

Russell Impagliazzo (University of California, San Diego), *The Groebner basis algorithm and propositional proof systems*.

Claude Laflamme (University of Calgary), *Gaps: Some structure and some applications*.

Monica McArthur (UCLA), *Nonconvergence for sparse random graphs*.

The scheduled invited address by Patrick Speissegger, *The field of reals with the gamma function is o-minimal*, was not given because the speaker was unable to attend the meeting due to severe weather.

Abstracts of invited and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
ALEXANDER S. KECHRIS

Abstracts of contributed talks

► KENSUKE BABA AND SACHIO HIROKAWA, *Normalization of proofs in classical logic.*
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According to Curry-Howard isomorphism, formulas correspond to types and proofs correspond to λ-terms. There have been many research to extend this correspondence to classical logic. One formulation is to use a new combinator $C : ((A \to\perp) \to\perp) \to A$ whose

1079-8986/97/0303-0006/$2.10

type corresponds to the elimination of double negation (DN). Here $\perp$ stands for falsehood. The reduction rule $CMN \Longrightarrow C(\lambda k.M(\lambda f.k(fN)))$ is one of the essential rules in [1]. This reduction rule corresponds to the simplification of the application of (DN) with respect to $(((A \to B) \to\perp) \to\perp) \to A \to B$ to (DN) with respect to $((B \to\perp) \to\perp) \to B$. In [2], another formulation was shown by introducing a combinator $P\colon (((A \to B) \to A) \to A)$ which corresponds to the Peirce formula. A simplification of "A" in the Peirce formula was defined by $P \Longrightarrow \lambda xy.P(\lambda z.x(\lambda y.z(uy))y)$.

In this talk, we define a translation between λ_C calculus and λ_P calculus and show that these two calculus simulates the normalization each other. We need to introduce a combinator $J\colon \perp\to A$ and its reduction rules. J_0 was introduced in [1]. We introduce J_1 and J_2, but these two properties are used implicitly in [3] concerning to boolean categories.

$$(J_0) \qquad JM \Longrightarrow M \quad \text{for } M\colon \perp$$

$$(J_1) \qquad M(JN) \Longrightarrow N \quad \text{for } N\colon \perp,\ M : A \to\perp$$

$$(J_2) \qquad J(MN) \Longrightarrow N \quad \text{for } M : A \to\perp,\ N : A$$

[1] P. de Groote, *On the relation between the λμ-calculus and the syntactic theory of sequential control*, Lecture Notes in Computer Science, no. 822, Springer-Verlag, 1994, pp. 31–43.

[2] S. Hirokawa, Y. Komori, and I. Takeuti, *A reduction rule for Peirce formula*, ***Studia Logica***, vol. 56, 1996, pp. 419–426.

[3] J. Lambek and P. J. Scott, ***Introduction to higher order categorial logic***, Cambridge University Press, 1986.

► JÖRG BRENDLE AND BENEDIKT LÖWE, *Δ^1_2 and Σ^1_2-measurability with respect to well-known forcing σ-algebras*.

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There are well-known classification results of Solovay, and Judah-Shelah [2] for the notions of Σ^1_2-Lebesgue measurability and Σ^1_2-Baire property. Most of the forcing partial orders from descriptive set theory are connected to some natural σ-algebra in the same way as Cohen forcing is connected to the σ-algebra of sets have the Baire property.

We give characterisations for Δ^1_2- and Σ^1_2-measurability for these σ-algebras using results from [3], [1], and [4].

Forcing	$\Delta^1_2(\mathbb{P})$	$\Sigma^1_2(\mathbb{P})$
$\mathbb{B}$	$\forall a \in \omega^\omega\colon\ \mathrm{Ran}(\mathbf{L}[a]) \neq \emptyset$	$\forall a \in \omega^\omega : \lambda(\mathrm{Ran}(\mathbf{L}[a])) = 1$
$\mathbb{C}$	$\forall a \in \omega^\omega\colon\ \mathrm{Coh}(\mathbf{L}[a]) \neq \emptyset$	$\forall a \in \omega^\omega\colon\ \mathrm{Coh}(\mathbf{L}[a]) \in (c^1)$
$\mathbb{D}$	$\forall a \in \omega^\omega\colon\ \mathrm{Hech}(\mathbf{L}[a]) \neq \emptyset \iff \Sigma^1_2(\mathbb{C})$	$\forall a \in \omega^\omega\colon\ \mathrm{Hech}(\mathbf{L}[a]) \in (d^1) \iff \forall a \in \omega^\omega : \aleph_1^{\mathbf{L}[a]} < \aleph_1$
$\mathbb{L}$	$\forall a \in \omega^\omega : \omega^\omega \cap \mathbf{L}[a]$ is σ-bounded	
$\mathbb{M}$	$\forall a \in \omega^\omega : \omega^\omega \cap \mathbf{L}[a]$ is not dominating	
$\mathbb{S}$	$\forall a \in \omega^\omega : \omega^\omega \cap \mathbf{L}[a] \neq \omega^\omega$	

[1] Jörg Brendle, Greg Hjorth, and Otmar Spinas, *Regularity properties for dominating projective sets*, ***Annals of Pure and Applied Logic***, vol. 72, 1995, pp. 291–307.

[2] Haim Judah (Jaime Ihoda) and Saharon Shelah, *Δ^1_2-sets of reals*, ***Annals of Pure and Applied Logic***, vol. 42, 1989, pp. 207–223.

[3] Alexander S. Kechris, *On a notion of smallness for subsets of the Baire space*, ***Transactions of the American Mathematical Society***, vol. 229, 1977, pp. 191–207.

[4] Grzegorz Łabędzki and Miroslav Repický, *Hechler reals*, ***Journal of Symbolic Logic***, vol. 60, 1995, pp. 444–458.

▶ WILLIAM C. CALHOUN, *Growth rate degrees*.
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Asymptotic growth rates of functions find applications in many branches of mathematics. For this talk, the most important example is the classification of an algorithm according to the growth rate of its time complexity.

We will consider functions from the positive integers to the positive reals, so we may define $f \in \Theta(g) \leftrightarrow f/g$ is bounded away from both 0 and ∞. We will call $\Theta(f)$ the Θ-*degree* of f and consider the set of Θ-degrees with the partial ordering $\leq$ induced by the O relation.

The idea is to explore the structure of the Θ-degrees the way other degree structures are explored in Computability Theory. I started this line of inquiry as a research project for one of my (undergraduate) students. It worked well as a student project since many properties of the Θ-degrees are easily observed (e.g., density and nonrigidity). But we also found some questions that seem harder and more interesting. For instance, are all nontrivial intervals in the Θ-degrees isomorphic?

Finally, we will consider ways to define growth rate other than Θ, and discuss an application of growth rate degrees in generalizing the Kolmogorov-style definition of *random sequence*.

▶ TOM COSTELLO AND ANNA PATTERSON, *Guilt-free exponentials*.
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Girard [1] introduces an operator $!_l$ called an exponential, to add structural rules to linear logic. We use subscript l to denote operators from linear logic. A formula $!_l A$ can be replicated or discarded. Thus the exponential removes the "linear" property of linear logic.

The exponential is also used to embed intuitionistic logic in linear logic. Girard suggests that intuitionistic implication $A \supset B$ can be represented as $!_l A \multimap_l B$.

We suggest that these two uses can be separated. We look at how the second transformation can be carried out in a logic which already contains all the structural rules of intuitionistic logic.

The logic of constructible falsity, N^- [2], adds an involutory negation $\neg$ to intuitionistic logic. This negation asserts the existence of a counterexample to a formula, not merely that the truth of a formula implies contradiction. N^- contains the intuitionistic connectives $\wedge$, $\vee$, $\supset$ and $\sim$, as well as the new involutory negation, $\neg$. Strong implication, $A \multimap B = A \supset B \wedge \neg B \supset \neg A$, (written $\Longrightarrow$ in [3, 4]) and tensor, $A \otimes B = \neg(A \multimap \neg B)$ are definable.

We add a new constant, $\mathbf{I}$, to N^- which is the unit for tensor and is distinct from the other constants, falsity, $\mathbf{0}$, and truth, $\mathbf{1}$. $\mathbf{I}$ is not definable in N^-. The new exponential, !, is defined as $!A = A \wedge \mathbf{I}$.

This exponential ! obeys $!\mathbf{1} = \mathbf{I}$, $!(A \wedge B) = !A \otimes !B$, and $!!A = !A$, the characteristic equations of Girard's exponential. Further, intuitionistic implication is definable as $!A \multimap B$ using this new exponential.

Linear logic contains two exponentials, $!_l$ and $?_l$, which are DeMorgan duals. Adding $\mathbf{I}$ to N^-, gives us two dual exponentials, ! and ?, related by $?A = \neg!\neg A$.

More exponentials can be added to N^-. A weak exponential ¡ is definable as $¡A = A \vee \bot$, where $\perp$ is another constant independent from $\mathbf{0}$, $\mathbf{1}$, and $\mathbf{I}$. $\perp$ was first introduced by Vakarelov [4]. ¡ also obeys $¡\mathbf{I} = \mathbf{1}$, $¡(A \otimes B) = ¡A \wedge ¡B$ and $¡¡A = ¡A$. The dual exponential ¿ is definable as, $¿A = \neg¡\neg A$. The weak exponential also extracts intuitionistic implication from strong implication $\multimap$. However it is applied to the second argument via the rule $A \multimap ¡B$. All four exponentials are distinct.

These two new units complete N^-, in the sense that there is a representation theorem for N^- with these additional units with respect to $H \times H^{\mathrm{op}}$ where H is a Heyting algebra.

[1] J.-Y. Girard, *Linear logic*, ***Theoretical Computer Science***, vol. 50, 1987, pp. 1–102.

[2] David Nelson and Ahmad Almukdad, *Constructible falsity and inexact predicates*, ***Journal of Symbolic Logic***, vol. 49, no. 1, 1984, pp. 231–233.

[3] H. Rasiowa, ***An algebraic approach to nonclassical logics***, North-Holland, 1974.

[4] D. Vakarelov, *Notes on N-lattices and constructive logic with strong negation*, ***Studia Logica***, vol. 36, 1977, pp. 109–126.

▶ BILL GASARCH, JACOB LURIE, AND TIM MCNICHOLL, *Order of queries*.

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Let $Q(A_1, \ldots, A_n)$ consist of all sets that can be computed by first making a query to A_1, then making a query to $A_2, \ldots$, and finally a query to A_n. We say that a pair of sets (A, B) commutes if $Q(A, B) = Q(B, A)$. We show the following:

(1) The pair (K, Tot) commutes. More generally, if $(A_1, \ldots, A_n)$ is a sequence such that $A_i = K, \mathrm{Tot}$ for $i = 1, \ldots, n$, then $Q(A_1, \ldots, A_n) = Q(B_1, \ldots, B_n)$ where $(B_1, \ldots, B_n)$ is any permutation of $(A_1, \ldots, A_n)$.

(2) The only sets that commute with every set are the recursive sets.

(3) There are pairs of intermediate sets that do not commute.

▶ JOEL DAVID HAMKINS, *Between indestructibility and superdestructibility*.

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Laver has obtained models of a supercompact cardinal k where any $< k$-directed closed forcing notion preserves the supercompactness of k. I have found models where any such forcing destroys the supercompactness of k. The fact which I would now like to discuss is that between these two extremes we can exactly control the class of $< k$-directed closed forcing notions which preserve the supercompactness of k.

▶ JEFFRY L. HIRST, *Reverse mathematics, induction and set comprehension*.

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When working in subsystems of second order arithmetic, we generally carefully distinguish between applications of set comprehension axioms and applications of induction schemes. Sometimes this is an artificial distinction, as shown by the following theorem.

Theorem ($\mathbf{RCA}_0$). *The following are equivalent*:

(1) $\mathbf{ACA}_0$ (*The arithmetical comprehension scheme*).

(2) **ATI** (*The arithmetical transfinite induction scheme*). *If $\psi(j)$ is an arithmetical formula and X is a well-ordered set, then*

$$\forall j\, [\forall i\, (i <_X j \rightarrow \psi(i)) \rightarrow \psi(j)] \rightarrow \forall j\, \psi(j).$$

The proof of this theorem uses some of the author's earlier results on reverse mathematics of ordinal arithmetic.

► M. RANDALL HOLMES, *Systems of first-order and higher-order logic without bound variables.*.

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The emphasis of this paper is on systems without bound variables equivalent to or slightly stronger than first-order logic, though some systems of higher-order logic will be considered. A historical survey of such systems is included, including a presentation of a complete set of axioms and rules for Quine's "predicate functor logic". A new system *EFT* ("external function theory") is described: it is a synthetic combinatory logic with the strength of first-order logic plus the existence of at least n objects for each concrete natural number n. The implementation of *EFT* as a subset of the higher-order logic of the Mark2 theorem prover is described.

This work has been supported by the U. S. Army Research Office, grant number DAAH04-94-G-0247.

► LUCIA R. JUNQUEIRA AND FRANKLIN D. TALL, *The topology of elementary submodels*.

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Given a topological space $\langle X, T\rangle$, we take an elementary submodel M of a sufficiently large initial fragment of the universe containing $\langle X, T\rangle$ and naturally define a space X_M. We consider under what conditions X_M is a (nice image of a) subspace of X, and what properties of X are retained by X_M.

► PHILIP KREMER AND GRIGORI MINTS, *Dynamic topological logic*.

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Dynamic topological logic concerns the confluence of three research areas: topological semantics, temporal logic, and topological dynamics. A standard propositional modal language can be interpreted in a topological space X by assigning subsets of X to the atoms, and interpreting &, $\vee$, $\neg$, and $\Box$ as intersection, union, complementation, and *topological interior*. S4, the set of formulas valid on all interpretations, is the resulting *topological* logic.

Topological dynamics studies the asymptotic properties of continuous maps. Let a *dynamic topological system* be a topological space X with a continuous function T. Extend S4 to a *dynamic* topological logic by adding two temporal modalities: $\circ$ (next), for formalizing T's immediate behavior; and $*$ (henceforth), for formalizing T's asymptotic behavior. If the formula A is assigned $|A| \subseteq X$, then $|\circ A| = T^{-1}(A)$ and $|*A| = \bigcap_{n\geq 0} T^{-n}(|A|)$. $\mathrm{DTL}_0 =_{\mathrm{df}} \{ A : A$ is valid on all interpretations$\}$ is the most general dynamic topological logic. Other DTLs result by focusing on a subclass of topological spaces, or of continuous functions, or both. For example, we define $\mathrm{DTL}_{\mathcal{H}}$ by requiring T to be a homeomorphism.

The paper is programmatic, containing more definitions and conjectures than theorems. We announce four results: (1) For most DTLs, the $\circ*$-free fragment is S4. (2) For most DTLs, the $\Box$-free fragment is the ω-time temporal logic W0 axiomatized by Segerberg [3] and others. (3) We give axiomatizations of the $*$-free fragment of $\mathrm{DTL}_{\mathcal{H}}$ and (4) of the fragment of DTL_0 where we do not allow occurrences of $\circ$ or $*$ in the scope of occurrences of $\Box$. The proofs for (3) and (4) are in Kremer, Mints and Rybakov [2] and Kremer [1], respectively.

This paper is available at `http://www.stanford.edu/~kremer`.

[1] P. Kremer, *Temporal logic over S4: an axiomatizable fragment of dynamic topological logic*, manuscript available at `http://www.stanford.edu/~kremer`.

[2] P. Kremer, G. Mints, and V. Rybakov, *Axiomatizing the next-interior fragment of dynamic topological logic*, manuscript available at `http://www.stanford.edu/~kremer`.

[3] K. Segerberg, *von Wright's tense logic*, in ***The philosophy of Georg Henrick von Wright*** (P. A. Schilpp and L. E. Hahn, editors), Open Court, La Salle, 1989, pp. 603–635.

▶ SILVIA B. LA FALCE, *Algebraizability versus strong algebraizability in fragments of Łukasiewicz's many-valued logics*.
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A deductive system $\mathcal{S}$ over a language $\mathcal{L}$ is called *algebraizable* if there exists a class $\mathcal{K}$ of $\mathcal{L}$-algebras such that $\mathcal{K}$ is an equivalent algebraic semantics for $\mathcal{S}$. The class $\mathcal{K}$ can always be assumed to be a quasivariety. When this is the case, the deductive system is called *strongly algebraizable*.

We will consider the deductive systems $\mathcal{S}_\omega$ ($\mathcal{S}_n$) of Łukasiewicz's infinite-valued (n-valued) logics and two important fragments of them, one consisting only of the biconditional connective $\leftrightarrow$, and another containing also a product $\cdot$ that can be interpreted as intentional conjunction. We know that $\mathcal{S}_n$ and its two fragments are strongly algebraizable. We will show that, while $\mathcal{S}_\omega$ is known to be strongly algebraizable, its $\langle \cdot, \leftrightarrow \rangle$-fragment is not. We do no know yet whether or not the $\langle \leftrightarrow \rangle$-fragment is strongly algebraizable.

We will also discuss some deduction theorems and results and open questions about finite axiomatizability for all these systems.

▶ THOMAS E. LEATHRUM, *Internet protocols and semi-automated formal logic*.
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In computer science, the field of Formal Methods Research has brought a new level of rigor to the task of writing specifications for hardware, software, and (the application presented here) communication protocols. To do this, researchers in the field use a "semi-automated" theorem-prover based on a high-order logic which is strongly typed (in the sense of computer programming languages like Pascal). The Prototype Verification System (PVS) from SRI is the software tool used in the current research to define and verify specifications for the new IPv6 Internet protocol. This presentation, designed to bring this research to the attention of other mathematical logicians, will begin with a standard mathematical application, algebraic group theory, and show how to state the axioms and prove a few lemmas in PVS. To establish the mathematical foundation for the Internet protocol specification, a logical network models is developed based on the combinatorics theory of an undirected graph. This provides the basis for the network topology on which mechanisms for addressing and packet transfer can be defined.

(Based upon *Formal Verification of Communication Protocols*, joint work with James F. Leathrum and Rasha Morsi, both of Old Dominion University, Department of Electrical Engineering, presented as IASTED Conference on Parallel and Distributed Computing Systems, October, 1996. The work also includes formal specifications for header structure, routing, and fragmentation, and will eventually include other standard protocols such as TCP to verify that the different protocols can function well together.)

► CYRUS F. NOURANI, *Functorial model theory and generic fragment consistency models*.
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Infinitary language categories are defined and their preliminary categorical properties were presented in our 1994–95 papers [1, 2]. In this paper a model theory is defined for functors starting with a countable fragment of an infinitary language. The infinite language category $\mathrm{L}\omega 1$, B, generic sets on $\mathrm{L}\omega 1$, B and a functor from $\mathrm{L}\omega 1$, B to Set, called the generic functor are defined. A new technique for generating generic models with categories is defined by inventing infinite language categories and functorial model theory. Specific functor models, the String Models [3], are defined by infinite chains on fragment models defined on an infinite language category functor. Techniques similar to Robinson's consistency theory are invented to define limit models. Functorial models are further defined on a generalized diagram by limit chains. Functorial models are further defined on a generalized diagram by limit chains. There are models defined by the generic diagrams, $D\langle A, G\rangle$, isomorphic to the string models, defined by a functor $F\colon \mathrm{L}\omega 1, \mathrm{B} \overset{\mathrm{Op}}{\to} \mathrm{Set}$. The models are limits defined on language strings. An example theorem is:

(1) There are functorial elementary chain models.

[1] Cyrus F. Nourani, *Functorial model theory and infinitary language categories*, Proceedings of ASL Conference, January 1995, San Francisco.

[2] Cyrus F. Nourani, *Functorial models and infinite language categories*, ASL, Wisconsin, Spring 1996.

[3] Cyrus F. Nourani, *The incredible string models*, versions accepted at Wollic, June 1996, and Kosice, July 1996.

► C. E. ORTIZ, *Approximate truth for infinitary formulas in metric spaces*.
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Many relevant problems in classical Fourier Analysis and Geometry of Banach spaces have the following form:

(∗) The approximations of an infinitary property P are true for every structure in a class $\mathcal{M}$. Is the property P true for every structure in $\mathcal{M}$?

Examples of such questions abound, from asymptotic infinite dimensional problems in Functional Analysis to the Distortion Problem for various classes of Banach spaces. We study this situation from a model theoretic point of view. The natural semantic notion of approximation for metric structures is the concept of approximate truth ($\models_{\mathrm{AP}}$) introduced by Henson for positive bounded formulas and extended by Fajardo and Keisler to a logic closed under infinite conjunction, finite disjunction and bounded existential and universal quantification.

We generalize $\models_{\mathrm{AP}}$ to a very expressive language L_A that allows countable conjunctions, negation, and bounded existential quantification over infinitely many variables. We introduce the notion of complete and uniform classes of models. We show that many of the usual collections of structures in Functional Analysis are complete and uniform collections of models. For these classes we prove a Compactness Theorem. This result generalizes the usual Compactness Theorem for first order logic and the Compactness Theorem for $\models_{\mathrm{AP}}$ obtained by Henson for positive bounded formulas. We use this theorem to give a partial positive answer to $(*)$. Finally, we give applications to Functional Analysis.

► ANDRZEJ ROSLANOWSKI, *More ccc ideals on Cantor space.*.
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We use the method of norms on possibilities to answer a question of Kunen and construct ccc σ-ideals on Cantor space with various closure properties and distinct from the ideal of null sets, the ideal of meager sets and their intersection.

Joint work with Saharon Shelah.

► ZORAN SPASOJEVIC, *Uniformization of colorings of ladder systems on trees*.
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We formulate the notion of uniformization of colorings of ladder systems on trees, prove that MA implies that every nice coloring of full ladder systems on special Aronszajn trees can be uniformized, and give an application to topological properties of special Aronszajn trees.

► GLEN WHITNEY, *Set-recursion in the anti-founded universe*.
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The Anti-Foundation Axiom explicitly allows the construction of sets x satisfying recursive equations like $x = \{x\}$ or $x = \langle 1, x\rangle$. Therefore, a rich class $\mathcal{C}$ of operators on sets obtain canonical fixed points in the anti-founded universe. The resulting fixed-point operation of "set-recursion" can be used to interpret a simple formal language FLR_0 of equational logic terms augmented with a recursion construct.

THEOREM 1. *The* FLR_0 *equational theory of set-recursion is identical to that of ordinary least-fixed-point recursion in complete partially ordered sets.*

This theorem leads to a complete proof system for valid equations between terms built from operators in $\mathcal{C}$ and set-recursion. The proof system fails to be complete for the full consequence relation (with hypotheses) between such equations. This incompleteness leads naturally to a new language $\mathcal{L}_{\mathrm{AFA}}$ for equations between "finitely presented" set-recursive terms. Augmenting the proof system from Theorem 1 with a small number of set-theoretic axioms yields a complete proof system for the consequence relation of $\mathcal{L}_{\mathrm{AFA}}$, as well as

THEOREM 2. *The consequence relation of* $\mathcal{L}_{\mathrm{AFA}}$ *is decidable.*

This second result generalizes the well-known theorem that bisimilarity between two finite pointed graphs is decidable.

► MICHAEL ZAKHARYASCHEV, *Modal formulas in one variable*.
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The paper presents two general methods for proving the finite modal property of modal logics with transitive frames represented by their canonical axioms [4]. In particular, they give positive solutions to two well-known open problems in modal logic concerning formulas in one variable. A modal formula is called a *modal reduction principle* if it is of the form $Mp \to Np$, where M and N are strings (possibly empty) of $\Box$ and $\Diamond$. Modal reduction principles were studied by van Benthem [3] who described their first order equivalents on Kripke frames.

THEOREM 1. *Every normal extension of* **K4** *by modal reduction principles is finitely axiomatizable, has the finite model property and so is decidable.*

Note by the way that it is still unclear whether extensions of **K** by modal reduction principles or even formulas of the form $\Box^n p \to \Box^m p$ have the finite model property.

Another theorem concerns normal extensions of **S4** by axioms in one variable. Sobolev [2] and Shehtman [1] showed that a formula in two variables or an infinite set of one-variable formulas can axiomatize a logic above **S4** without the finite model property. However we have the following:

THEOREM 2. *Every normal extension of* **S4** *with a finite number of formulas in one variable has the finite model property and is decidable.*

This result is rather unexpected because the structure of the 1-generated free topological Boolean algebra looks like the 2-generated Heyting algebra, and according to [2] there is an intuitionistic formula in two variables axiomatizing an extension of intuitionistic logic without the finite model property.

[1] V. B. SHEHTMAN, *Topological models of propositional logics*, ***Semiotics and Information Science***, vol. 15, 1980, pp. 74–98, in Russian.

[2] S. K. SOBOLEV, *On the finite approximability of superintuitionistic logics*, ***Mathematics of the USSR, Sbornik***, vol. 31, 1977, pp. 257–268.

[3] J. F. A. K. VAN BENTHEM, *Modal reduction principles*, ***Journal of Symbolic Logic***, vol. 41, 1976, pp. 301–312.

[4] M. V. ZAKHARYASCHEV, *Canonical formulas for K4, Part I: Basic reults*, ***Journal of Symbolic Logic***, vol. 57, 1992, pp. 1377–1402.

Abstracts presented by title

► PHILIP KREMER, *Temporal logic over S4: An axiomatizable fragment of dynamic topological logic*.
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Dynamic topological logic (Kremer and Mints [2]) concerns the confulence of three research areas: topological semantics for S4; temporal logic; and topological dynamics. Let a *dynamic topological system* (DTS) be a topological space X with a continuous function T. Given a DTS, interpret a propositional trimodal language with one topological modality $\Box$ (interior), and two temporal modalities, $\bigcirc$ (next) and $*$ (henceforth) as follows. Assign $|p| \subseteq X$ to each atom p; interpret &, $\vee$, $\neg$, and $\Box$ as intersection, union, complementation and topological interior; and let $|\mathsf{O}A| = T^{-1}(A)$ and $|{*}A| = \bigcap_{n\geq 0} T^{-n}(|A|)$. This extends the topological interpretation of S4. Define DTL_0 as the set of formulas validated by every DTS. The fragment of DTL_0 whose only modality is $\Box$ is simply S4. And the fragment whose only modalities are O and $*$ is the ω-time temporal logic W0, axiomatized by Segerberg [3] and others.

Define W0/S4 as the fragment

$$\{\, A \in \mathrm{DTL}_0 : A \text{ contains no occurrences of either } \mathsf{O} \text{ or } * \text{ in the scope of an occurrence of } \Box \,\}.$$

The main result: W0/S4 can be axiomatized as follows (building the scope restriction into the definition of a well-formed formula).

Axioms: Standard axioms for S4 (for $\Box$) and W0 (for O and $*$). Rules: *Modus Ponens*; Necessitation for each modality; Continuity: from $(\mathsf{O}^n\Box A \supset B)$ infer $(\mathsf{O}^n\Box A \supset \Box B)$; Antecedent Elimination: from $(A \supset \mathsf{O}B)$ infer $\mathsf{O}B$, where A contains no temporal modalities and $\neg A \notin$ S4.

For completeness, we construct a canonical model whose points are complete consistent ω-closed theories, where a theory u is ω-closed if and only if $(\forall n)(\mathsf{O}^n A \in u \Longrightarrow {*}A \in u)$. The bulk of the paper consists in proving that every consistent formula is in such a theory. For this we prove that if $(A\ \&\ \neg{*}\neg B)$ is consistent, then $(A\ \&\ \mathsf{O}^n B)$ is consistent for some n, adapting ideas from Goldblatt [1].

This paper is available at `http://www.stanford.edu/~kremer`.

[1] R. Goldblatt, ***Logics of time and computation***, Center for the Study of Language and Information Lecture Notes, no. 7, Stanford University Press, Stanford, second edition.

[2] P. Kremer and G. Mints, *Dynamic topological logic*, manuscript available at `http://www.stanford.edu/~kremer`.

[3] K. Segerberg, *von Wright's tense logic*, in ***The philosophy of Georg Henrick von Wright*** (P. A. Schilpp and L. E. Hahn, editors), Open Court, La Salle, 1989, pp. 603–635.

► PHILIP KREMER, GRIGORI MINTS, AND VLADIMIR RYBAKOV, *Axiomatizing the next-interior fragment of dynamic topological logic*.

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Dynamic topological logic (Kremer and Mints [1]) concerns the confulence of three research areas: topological semantics, temporal logic, and topological dynamics. Let a *dynamic topological system* (DTS) be a topological space X with a continuous function T. Given a DTS, interpret a propositional trimodal language with one topological modality $\Box$ (interior), and two temporal modalities, O (next) and $*$ (henceforth) as follows. Assign $|p| \subseteq X$ to each atom p; interpret &, $\vee$, $\neg$, and $\Box$ as intersection, union, complementation and topological interior; and let $|\mathsf{O}A| = T^{-1}(A)$ and $|{*}A| = \bigcap_{n\geq 0} T^{-n}(|A|)$. This extends the topological interpretation of S4. Define DTL_0 ($\mathrm{DTL}_{\mathcal{H}}$, $\mathrm{DTL}_{\mathrm{open}}$) as the set of sentences validated by every DTS (whose function is a homeomorphism, open). Define $\mathrm{DTL}_{\mathbb{R};\mathcal{H}}$, $\mathrm{DTL}_{\mathbb{R};\mathrm{open}}$, $\mathrm{DTL}_{[0,1];\mathcal{H}}$, $\mathrm{DTL}_{[0,1];\mathrm{open}}$ by requiring the topological space X to be the real line or the closed unit interval.

The main result: the next-interior fragment—i.e., the $*$-free fragment—of all of the above logics except DTL_0 can be axiomatized by the classical tautologies; S4 axioms for $\Box$; $(\mathsf{O}(A \vee B) \equiv (\mathsf{O}A \vee \mathsf{O}B))$; $(\mathsf{O}\neg A \equiv \neg \mathsf{O}A)$; $(\mathsf{O}\Box A \equiv \Box \mathsf{O}A)$; *modus ponens* and necessitation for $\Box$ and O.

The main points of the proof. (1) Every formula has a normal form resulting from pushing all the Os to the atoms, since O commutes with everything. (2) A formula in normal form is a theorem of the above axiomatization if and only if it is a theorem of S4, formulated as if $\mathsf{O}^n p$ were atomic (for p atomic). (3) Since S4 can be interpreted in the open unit interval $(0, 1)$, we get an interpretation of formulas in normal form in $(0, 1)$. (4) Make appropriate copies of this interpretation in the intervals $(n, n + 1)$, and use the homeomorphism $T(x) = x + 1$ to get the desired interpretation in $(\mathbb{R}, T)$. Note that step 1 fails for the next-interior fragment of DTL_0, since $(\Box\mathsf{O}p \supset \mathsf{O}\Box p) \notin \mathrm{DTL}_0$.

This paper is available at `http://www.stanford.edu/~kremer`.

[1] P. Kremer and G. Mints, *Dynamic topological logic*, manuscript available at `http://www.stanford.edu/~kremer`.