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1995–1996 ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Madison, Wisconsin, March 9–12, 1996

The Annual Meeting of the Association was held at the University of Wisconsin, Madison, March 9–12, 1996. The program consisted of seven invited hour lectures and special sessions of half hour talks in set theory and in philosophical logic. Yiannis Moschovakis gave the Retiring Presidential Address, entitled *Fixpoint Recursion*. Saharon Shelah gave the seventh annual Gödel lecture, entitled *Categoricity Without Compactness*. In addition, there were thirty-one contributed talks and six papers presented by title. The Program Committee consisted of Samuel Buss, Peter Cholak, H. Jerome Keisler (chair), Donald Martin, and Michael Resnik. The Local Organizing Committee consisted of Mirna Dzamonja, Kenneth Kunen, Steffen Lempp (chair), Terrence Miller, and Arnold Miller.

The following invited addresses were presented at the meeting:

Tomek Bartoszynski (Boise State), *Small sets related to cardinal invariants*.

Harvey Friedman (Ohio State), *Finite functions and the necessary use of large cardinals*.

Geoffrey Hellman (Minnesota), *Indispensability arguments in the philosophy of mathematics*.

Bakhadyr Khoussainov (Cornell), *Effective structures: presentations, isomorphisms, and models of theories*.

Phokion G. Kolaitis (UC Santa Cruz), *Almost everywhere equivalence of logics*.

Richard Shore (Cornell), *Standard models and definability in the computably enumerable degrees*.

Charles Steinhorn (Vassar), *Topological algebra in o-minimal structures*.

The following talks were presented in the special session on set theory:

Mirna Dzamonja (Wisconsin), *On ♣-principle and Suslin trees*.

Itay Neeman (UCLA), *Determinacy of long games*.

Slawomir Solecki (UCLA), *Analytic ideals*.

Jindrich Zapletal (Caltech), *Canonical models for several classical statements about* $\aleph_1$.

The following talks were presented in the special session on philosophical logic.

Jody Azzouni (Tufts), *Thick posits: Distinguishing the mathematical from the empirical*.

Mark Balaguer (Cal State LA), *Non-uniqueness as a non-problem*.

Paul Beame (Washington), *New directions in propositional proof complexity*.

Colin McLarty (Case-Western), *Geometry and logic*.

Keith Simmons (North Carolina), *Deflationary truth and the liar*.

Abstracts of invited talks and of contributed talks given in person or by title by members of the Association for Symbolic Logic follow.

For the Program Committee
H. Jerome Keisler

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Abstracts of invited talks

▶ MARK BALAGUER, *Non-uniqueness as a non-problem*.
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A response is given here to Benacerraf's [1] non-uniqueness (or multiple-reductions) objection to mathematical platonism. It is argued that non-uniqueness is simply *not* a problem for platonism. More specifically, it is argued that platonists can simply *embrace* non-uniqueness—i.e., that one can endorse the thesis that our mathematical theories truly describe collections of abstract mathematical objects while rejecting the thesis that such theories truly describe *unique* collections of such objects. I also argue that part of the motivation for this view is that it dovetails with the correct response to Benacerraf's *other* objection to platonism, i.e., his [2] epistemological objection.

[1] PAUL BENACERRAF, *What numbers could not be*, ***Philosophical Review***, vol. 74 (1965), pp. 47–73.

[2] ———, *Mathematical truth*, ***Journal of Philosophy***, vol. 70 (1973), pp. 661–680.

▶ PAUL BEAME, *New directions in propositional proof complexity*.
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It is a fundamental open question, equivalent to one of the main outstanding questions of computational complexity, whether or not there is a method of proof that permits efficient proofs of all propositional tautologies. With this question as a lens, propositional proof complexity developed as a pleasing theory, settling questions of the relative efficiency of many traditional proof methods and, by focusing on the limitations of existing methods, raising the question of whether or not better proof methods could be found by escaping the confines of traditional approaches.

Over the last decade, powerful techniques from computational complexity have been combined with ideas from the method of forcing and from commutative algebra to produce remarkable advances in our understanding of traditional propositional proof methods, such as resolution and its generalizations.

One product of these new techniques has been a better understanding of the range of proof methods possible. As a result, several new methods of propositional proofs have been developed. Of particular note are new algebraic methods based on Hilbert's Nullstellensatz which are in some cases provably more efficient than traditional methods and which have the promising feature that their associated search procedures are provably efficient relative to the size of the proof they seek.

This talk will survey these new developments, particularly concentrating on applications of circuit complexity techniques and on the new algebraic proof methods.

▶ PHOKION G. KOLAITIS, *Almost everywhere equivalence of logics*.
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We introduce a new framework for classifying logics on finite structures and studying their expressive power. This framework is based on the concept of *almost everywhere equivalence of logics*, that is to say, two logics having the same expressive power on a class of measure 1.

More precisely, if $\mathcal{L}$, $\mathcal{L}'$ are two logics and μ is an asymptotic measure, then $\mathcal{L} \equiv^{\mu}_{a.e.} \mathcal{L}'(\mu)$ means that there is a class $\mathcal{C}$ of structures such that $\mu(\mathcal{C}) = 1$ and $\mathcal{L}$ defines on $\mathcal{C}$ the same queries as $\mathcal{L}'$. Although this concept underlies much of the work on logic and random structures, so far it has not been singled out and studied in its own right.

We embark on a systematic investigation of $\equiv^{\mu}_{a.e}$ with respect to the uniform measure and analyze the $\equiv^{\mu}_{a.e.}$-equivalence classes of several logics that have been studied extensively in finite model theory. In particular, we show that first-order logic FO is almost everywhere equivalent to the finite-variable infinitary logic $\mathcal{L}^{\omega}_{\infty\omega}$. At the other end of the spectrum, we establish that the infinitary logic $\mathcal{L}_{\infty\omega}$, which can express all queries on finite structures, is almost everywhere equivalent to extensions of $\mathcal{L}^{\omega}_{\infty\omega}$ with generalized quantifiers that express *counting* properties, such as the *Even* quantifier and *Rescher's* quantifier. These results are obtained by unveiling a rather unexpected connection between the analysis of the $\equiv^{\mu}_{a.e.}$-equivalence class of $\mathcal{L}_{\infty\omega}$ and the average complexity of algorithms for the *Graph Isomorphism Problem*. As a byproduct, we show the PTIME is almost everywhere equivalent to the extensions of inflationary fixpoint logic IFP with the *Even* quantifier or with *Rescher's* quantifier. This is joint work with Lauri Hella and Kerkko Luosto of the University of Helsinki.

▶ S. SHELAH AND JINDRICH ZAPLETAL, *Canonical models for several classical statements about* $\aleph_1$.

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Using Woodin's $P_{\max}$ method, we build canonical models for several statements ϕ about the relationship of $\aleph_1$ and the continuum. We shall find an $L(\mathbb{R})$-definable forcing notion P_ϕ such that granted enough large cardinals, the generic extension $L(\mathbb{R})^{P_\phi}$ is a model of

$$\mathrm{ZFC} + \phi + \mathfrak{c} = \aleph_2 + \text{ the nonstationary ideal is saturated}$$

and the following holds.

THEOREM (Π_2-compactness criterion). *If in the original ZFC universe there are large enough cardinals and ϕ holds and $\langle H_{\aleph_2}, \in, NS_{\omega_1}\rangle \models \psi$ for some Π_2 statement ψ, then in $L(\mathbb{R})^{P_\phi}$ $\langle H_{\aleph_2}, \in, NS_{\omega_1}\rangle \models \psi$.*

Here, NS_{ω_1} is a predicate for the nonstationary ideal. If Π_2-compactness can be proved, we say that the statement ϕ is Π_2-*compact*. Note that if large cardinals exist, then the theory of $L(\mathbb{R})$ is invariant under forcing and thus so is the theory of $L(\mathbb{R})^{P_\phi}$. From the Theorem it follows that $L(\mathbb{R})^{P_\phi}$ must necessarily realize all Π_2-forcing effects ever achievable in conjunction with ϕ and large cardinals. Roughly, if ϕ is Π_2-compact and $\psi_i : i \in I$ are Π_2-facts one by one consistent with ϕ, then also their conjunction is consistent with ϕ.

We show that the following statements are Π_2-compact:

(1) the dominating number $= \aleph_1$,
(2) cofinality of the meager ideal $= \aleph_1$,
(3) density of the amoeba forcing $= \aleph_1$,
(4) existence of certain kinds of Souslin trees,
(5) certain variations on uniformity of meager and null ideal $= \aleph_1$,
(6) bounding number $= \aleph_1$,

and that "reals can be covered by $\aleph_1$-many meager sets" is an example of a noncompact statement. We give a user-friendly axiomatisation of the $P_{\max}$ method and provide general framework for proving Π_2-compactness type results.

Abstracts of contributed talks

► G. ALDO ANTONELLI, *Extensional quotients for type theory*.
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In this paper we explore the applicability of the method of extensional quotients, independently due to Hinnion and Aczel, for the constructions of models of the simple theory of types and its variants. An extensional quotient, or contraction, is an equivalence relation R that is a fixed point of the following operator $(\cdot)^{\forall\exists}$, where

$$R^{\forall\exists}(x,y) \iff (\forall u \in x)(\exists v \in y)R(u,v) \wedge (\forall v \in y)(\exists u \in x)R(u,v).$$

Clearly, the quotient of any model of type theory modulo a contraction is extensional. But what other properties does this operation preserve? It is easy to show that if R is an *internal* equivalence relation with the property that $R(x,y)$ implies $(\forall u \in x)(\exists v \in y)R(u,v)$ and $(\forall v \in y)(\exists u \in x)R(u,v)$ then the quotient satisfies a (type-theoretic) comprehension principle, although it might not be extensional. In this paper we combine and strengthen these results identifying conditions sufficient in order that the quotient of a model of Specker's Theory of Negative Types give rise to a model of Quine's New Foundations.

► JEREMY AVIGAD AND RICHARD SOMMER, *Combinatorial largeness properties and ordinal analysis*.
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We present a model-theoretic alternative to cut elimination for the ordinal analysis of subsystems of second-order arithmetic. Using the finitary combinatorial notion of an α-large set of natural numbers, we show how appropriately large intervals in a nonstandard model of arithmetic can be used to construct models of the theories in question. We apply these methods to obtain sharp upper bounds on the proof-theoretic strength of a number of important theories, including RCA_0, WKL_0, ACA_0, $(\Pi^0_1 - CA)^{<\alpha}$, ACA, $\Sigma^1_1 - AC$, $\widehat{ID}_n$, ATR_0, and ATR.

► ALEXANDRU BALTAG, *Modal characterization for sets and Kripke models*.
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In this paper, we study several problems concerning a set-theoretical semantics for modal logic, defined and studied by Jon Barwise and Lawrence Moss in their book Vicious Circles: *On the Mathematics of Circularity* (to appear). Roughly speaking, the semantics is given by considering the sets in Aczel's "Antifounded" universe as "canonical" Kripke structures, where the "accessibility" relation is taken to be the converse membership relation. If we restrict ourselves to finitary modal logic L, then it is easy to see that the modal system K is strongly complete with respect to this semantics. The above-mentioned authors that, if we use infinitary modal logic L_∞, then for every set there exists a modal formula which characterizes it.

In our paper, we define and study an important modal characteristic, *the modal degree of a set*, which in some sense generalizes the rank function to non-wellfounded sets.

We also answer the following problems:

(1) When are two sets modally equivalent, with respect to *finitary* modal logic?
(The answer for infinitary logic is already given above: when they are identical.)

(2) In what conditions is a set characterizable by a finitary modal formula?
(3) What are the sets characterizable by a theory in finitary modal logic?
(4) The generalizations of the above problems for infinitary modal languages of "bounded" length L_κ (where κ is a regular cardinal).

We also present (weakly) complete deductive systems with respect to this semantics for the logics L_∞ and L_κ, for κ strongly inaccessible.

Each of the above mentioned results implies a corresponding result for (standard) Kripke models. The theorems by this "translation" appear to be new.

▶ PAUL BANKSTON, *Variable/operator languages*.
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We propose a formal linguistic framework that is general enough to deal with many of the finitary logical systems and functions-manipulating systems of current interest. By this we mean that propositional logic (classical, intuitionistic, modal), predicate logic (with extra quantifiers and Chang-style modal operators perhaps), and Church's lambda calculus can all be viewed as special cases of our formalism.

Our approach is not altogether unlike that of Montague, who laid out a general program for dealing precisely with the kinds of variable-binding operators that appear in (multi-variable) differential/integral calculus and elsewhere in mathematics. The main feature of this approach is that it makes a clear delineation between grammatical issues (e.g., formation rules, secure/insecure variable binding, denotational semantics) and derivation issues (e.g., logical calculi, calculi of function conversion).

▶ JOHN BYRNES AND WILFRIED SIEG, *A graphical presentation of Gandy's parallel machines*.
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As emphasized by Gandy, Turing's thesis that all effectively calculable functions are Turing computable is based on the analysis of mechanical computations by humans; cf. also [3]. For computations by machines, Gandy argues, parallelism must be considered. He provides, via a direct analysis of *machine computations*, an abstract machine that performs parallel computations [1]; such machines compute exactly the general recursive functions.

The states of Gandy's machines are hereditarily finite sets and the transitions are functionally given; both states and transition functions satisfy certain restrictive *principles*. Shepherdson writes that these principles are "rather complicated, involving many subsidiary definitions", but concludes "that that is in the nature of the situation ... " [2]. The complexity seems, however, to be due largely to the chosen form of mathematical description using hereditarily finite sets. By replacing hereditarily finite sets with labeled graphs, we achieve a much more perspicuous model.

DEFINITION. A *structural* set of finite labeled graphs $\mathcal{S}$ and a *structural* transition function $\mathcal{F} : \mathcal{S} \longrightarrow \mathcal{S}$ define a *Gandy machine* if, for some functions $\mathcal{G}_1$ and $\mathcal{G}_2$ on finite labeled graphs with finite domains and for every $x \in \mathcal{S}$:

(i) For every *causal neighborhood* y of x (a subgraph of x in the domain of $\mathcal{G}_1$), $\mathcal{G}_1(y)$ is a unique *determined region* in $\mathcal{F}(x)$ (again, a subgraph), in which vertices new to y are new to x as well.

(ii) For every set C (of restricted cardinality) of determined regions containing a given vertex there is a determined region in $\mathcal{F}(x)$ under $\mathcal{G}_2$ which is a supergraph of every region in C.

(iii) $\mathcal{F}(x)$ is the graph-theoretical union of all determined regions under $\mathcal{G}_1$.

THEOREM. *Gandy Machines compute exactly the Turing computable functions.*

In addition, we present a bisimulation between the graphical and set-theoretic machines, a simulation of cellular automata, and parallel computations of number-theoretic functions.

[1] GANDY, *Church's thesis and principles for mechanisms*, ***The Kleene symposium*** (J. Barwise, H. J. Keisler, and K. Kunen, editors), North-Holland, 1980, pp. 123–148.

[2] SHEPHERDSON, *Mechanisms for computing over arbitrary structures*, ***The universal Turing machine—A half-century of survey*** (R. Herken, editor), Oxford University Press, 1988, p. 586.

[3] SIEG, *Mechanical procedures and mathematical experience*, ***Mathematics & Mind*** (George, editor), Oxford University Press, 1994, pp. 71–114.

► PAUL CORAZZA, *The wholeness axiom and Laver sequences*.
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In this paper we introduce the Wholeness Axiom (WA), which asserts that there is a nontrivial elementary embedding from V to itself that is not definable in V; the undefinability restriction avoids Kunen's theorem, but the axiom remains strong enough to prove that the critical point of the embedding is super-n-huge for every n. We formalize WA in a language with one additional unary function symbol $\mathbf{j}$.

We use WA to give a direct construction of a Laver sequence; the construction leads to the observation that, assuming WA, there is a Laver sequence definable in V_κ if and only if $V = HOD$. A modification of the construction produces a *special* Laver sequence at κ—one that dominates, on a D-measure 1 set, all $f \in {}^\kappa V_\kappa$ definable in V_κ, where D is the projection of a normal ultrafilter on some $P_\kappa\lambda$ for which $x = i_U(f)\kappa$ and $|TC(x)| \leq \lambda$. We show that if there is a special Laver sequence at κ then it is consistent for κ to be the κth extendible cardinal.

We also use this construction to give a uniform definition of Laver sequences for a broad class of large cardinals. We show that WA implies the existence of Laver sequences for most of the familiar large cardinals not weaker than strong, and, in fact, show that a large cardinal admits some form of a Laver sequence if and only if it is strong. We refine the arguments to weaken the hypothesis WA as far as possible. One such argument improves a result of Barbanel by showing that if κ is super-2-huge, there is a superhuge Laver sequence at κ.

► SERGIO FAJARDO AND JAVIER PENA, *Games on probability spaces*.
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In this paper we introduce a family of games to be played on Probability Spaces. The original motivation for their was the purpose of finding a Back and Forth characterization of Adapted Equivalence, the notion corresponding to elementary equivalence within the Logic of Stochastic Processes, introduced by Keisler.

With these games we define new forms of comparing such spaces and part of our work explores issues related to such forms. On the other hand we solve a question of Hoover, and present some results concerning a question posed by Keisler about Robinson Consistency Properties in the Logic of Stochastic Processes.

▶ JOEL DAVID HAMKINS, *Small forcing makes every cardinal superdestructible*.
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Small forcing always ruins the indestructibility of an indestructible supercompact cardinal. In fact, after small forcing, any cardinal κ becomes *superdestructible*. That is, any further $<_\kappa$-closed forcing which adds a subset to κ will destroy the measurability and even the weak compactness of κ. Nevertheless, after small forcing indestructible cardinals remain resurrectible, but never strongly resurrectible.

▶ JEFF HIRST, *Reverse mathematics and ordinal exponentiation*.
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Reverse mathematics has been used to analyze the proof theoretic strength of formulizations of theorems of ordinal arithmetic [1, 2]. In general, statements concerning the multiplication of countable well-orderings are either provable in the weak system $\mathbf{RCA}_0$ or are provably equivalent to the much stronger system $\mathbf{ATR}_0$. For example, the following theorem shows that a version of the division algorithm for ordinal arithmetic is strong.

THEOREM. *The following can be proved equivalent in* $\mathbf{RCA}_0$:
(1) $\mathbf{ATR}_0$.
(2) *If α and β are countable well-orderings, then there are countable well-orderings ρ and μ such that $\rho + 1 \leq \alpha$ and $\beta = \alpha\mu + \rho$.*

Often, the strength of statements depends on the form of comparability used in their formalization. If there is an order preserving bijection between α and an initial segment of β, we say that α is *strongly less than or equal* to β, and write $\alpha \leq_s \beta$. Similarly, $\alpha \leq_w \beta$ denotes *weak comparability*, that is, the existence of an order preserving injection of α into β. $\mathbf{ATR}_0$ is needed to prove that multiplication preserves strong comparability, as shown by the following theorem.

THEOREM. *The following can be proved equivalent in* $\mathbf{ATR}_0$:
(1) $\mathbf{ATR}_0$.
(2) *If α, β, γ and δ are countable well-orderings such that $\alpha \leq_s \beta$ and $\gamma \leq_s \delta$, then $\alpha\gamma \leq_s \beta\delta$.*

The analog of (2) obtained by uniformly replacing $\leq_s$ by $\leq_w$ is provable in $\mathbf{RCA}_0$.

[1] H. FRIEDMAN and J. HIRST, *Weak comparability of well orderings and reverse mathematics*, ***Annals of Pure and Applied Logic***, vol. 47 (1990), pp. 11–29.

[2] J. HIRST, *Reverse mathematics and ordinal exponentiation*, ***Annals of Pure and Applied Logic***, vol. 66 (1994), pp. 1–18.

▶ DARRYL JUNG, *On Russell's conception of logic: The notion of normal system in* Principia Mathematica.
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The modern notion of formal system descends directly from, among other sources, Russell's development of modern logic as articulated in *Principia Mathematica*. The conception of formal system that underlies *Principia*'s development, however, is in many respects at variance with the modern one. Indeed, some recent commentary has claimed that the earlier conception allows no room for metatheoretic considerations about logic. I examine closely Russell's conception of formal system in an effort to identify precisely the respects in which it differs from its modern analogue and then comment on the issue concerning the impossibility of metatheory.

▶ SALMA KUHLMANN, *Functional equations for lexicographic products*.
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Joint work with F. -V. Kuhlman and S. Shelah.

Let Γ and Δ be chains, and 0 a distinguished element of Δ. Consider the lexicographic power Δ^Γ, i.e., the set of all maps from Γ into Δ which have wellordered support in Γ. Ordered lexicographically, Δ^Γ is a chain, with distinguished element the zero map, which we also denote by 0. We study the following question:

When is Γ embeddable as a convex subset in Δ^Γ?

In [1], we have shown that such an embedding is impossible if Γ and Δ have no last element. In [2] we study those chains for which such embeddings exist. This question arose while studying the structure of exponential fields, and the most direct applications of possible solutions are to the construction of models of real exponentiation. Surprisingly, it turns out that the mere underlying ordering of an ordered field determines to a great extent its analytic structure. To illustrate this, consider the following functional equations

$$\Delta^\Gamma \simeq \Gamma, \quad (\Delta^\Gamma)^{<0} \simeq \Gamma, \quad (\Delta^\Gamma)^{\leq 0} \simeq \Gamma.$$

Here, $(\Delta^\Gamma)^{<0}$ (respectively, $(\Delta^\Gamma)^{\leq 0}$) denotes the set of all elements < 0 (respectively, ≤ 0). By [1], if Δ has no last element, then the first equation has no solution Γ. This in turn shows that no nonarchimedean power series field admits exponentiation. If Δ has 0 as its last element, then a solution Γ of the first equation exists, and if moreover ω^* (the order type of the negative integers) is a final segment of Δ, then this Γ solves the two other equations as well. A solution of the last equation exists for every Δ. These solutions turn out to be the ranks of nonarchimedean power series fields admitting a total (but not surjective) exponential function.

[1] F. -V. Kuhlmann, S. Kuhlmann, and S. Shelah, *Exponentiation in power series fields*, submitted.

[2] ———, *Functional equations for lexicographic products*, in preparation.

▶ BILLY JOE LUCAS, *Deontic evidential constraints on the logic of belief*.
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Take as a language L with $\bigcirc$ ($\wp$) the strong (weak) deontic operator, and $x\,\mathrm{B}$ ($x\,\mathrm{E}$) the strong belief (evidence) operator. A minimal deontic doxastic evidential frame (MDDE) is: $\langle \mathrm{W}, \mathrm{O}, \mathrm{B}, \mathrm{E}\rangle$, with $\mathrm{O}(\mathrm{B}, \mathrm{E})$ a binary serial relation defined on non-empty W. What MDDE models are is then self-evident. For α a proper deontic modality and β a proper evidential modality, wffs of the form $\alpha(x\,\mathrm{B}\,p \supset \beta p)$ make a moral claim that imposes on belief some necessary condition about evidence. When β is degree ≤ 1 the wff is an *externalist* theory, and for β of degree ≥ 2 an *internalist* theory. The canonical model for an MDDE frame is the canonical model for each of (1)–(12) when the corresponding condition is imposed on the MDDE frame.

Externalist Theories	Add to MDDE Frame:
(1) $\bigcirc(x\,\mathrm{B}\,p \supset x\,\mathrm{E}\,p)$	(1) $\forall x \forall y \forall z[(x\,\mathrm{O}\,y \cdot y\,\mathrm{E}\,z) \supset y\,\mathrm{B}\,z]$
(2) $\bigcirc(x\,\mathrm{B}\,p \supset {\sim}x\,\mathrm{E}\,{\sim}p)$	(2) $\forall x \forall y[x\,\mathrm{O}\,y \supset \exists z(y\,\mathrm{B}\,z \cdot y\,\mathrm{E}\,z)]$
(3) ${\sim}\bigcirc\,(x\,\mathrm{B}\,p \cdot {\sim}x\,\mathrm{E}\,p)$	(3) $\forall x \exists y[x\,\mathrm{O}\,y \cdot \forall z(y\,\mathrm{E}\,z \supset y\,\mathrm{B}\,z)]$
(4) ${\sim}\bigcirc\,(x\,\mathrm{B}\,p \cdot x\,\mathrm{E}\,{\sim}p)$	(4) $\forall x \exists y \exists z(x\,\mathrm{O}\,y \cdot y\,\mathrm{B}\,z \cdot y\,\mathrm{E}\,z)$
Internalist Theories	
(5) $\bigcirc(x\,\mathrm{B}\,p \supset x\,\mathrm{B}\,x\,\mathrm{E}\,p)$	(5) $\forall x \forall y \forall z \forall w[(x\,\mathrm{O}\,y \cdot y\,\mathrm{B}\,z \cdot z\,\mathrm{E}\,w) \supset y\,\mathrm{B}\,w]$
(6) ${\sim}\wp(x\,\mathrm{B}\,p \cdot x\,\mathrm{B}{\sim}x\,\mathrm{E}\,p)$	(6) $\forall x \forall y\{x\,\mathrm{O}\,y \supset \exists z[y\,\mathrm{B}\,z \cdot \forall w(z\,\mathrm{E}\,w \supset y\,\mathrm{B}\,w)]\}$
(7) $\bigcirc(x\,\mathrm{B}\,p \supset x\,\mathrm{B}{\sim}x\,\mathrm{E}{\sim}p)$	(7) $\forall x \forall y \forall z[(x\,\mathrm{O}\,y \cdot y\,\mathrm{B}\,z) \supset \exists w(z\,\mathrm{E}\,w \cdot y\,\mathrm{B}\,w)]$
(8) ${\sim}\wp(x\,\mathrm{B}\,p \cdot x\,\mathrm{B}\,x\,\mathrm{E}{\sim}p)$	(8) $\forall x \forall y[x\,\mathrm{O}\,y \supset \exists z \exists w(y\,\mathrm{B}\,z \cdot z\,\mathrm{E}\,w \cdot y\,\mathrm{B}\,w)]$
(9) ${\sim}\bigcirc\,(x\,\mathrm{B}\,p \cdot {\sim}x\,\mathrm{B}\,x\,\mathrm{E}\,p)$	(9) $\forall x \exists y\{x\,\mathrm{O}\,y \cdot \forall z \forall w[(y\,\mathrm{B}\,z \cdot z\,\mathrm{E}\,w) \supset y\,\mathrm{B}\,w]\}$
(10) ${\sim}\bigcirc\,(x\,\mathrm{B}\,p \cdot x\,\mathrm{B}{\sim}x\,\mathrm{E}\,p)$	(10) $\forall x \exists y \exists z[x\,\mathrm{O}\,y \cdot y\,\mathrm{B}\,z \cdot \forall w(z\,\mathrm{E}\,w \supset y\,\mathrm{B}\,w)]$
(11) ${\sim}\bigcirc\,(x\,\mathrm{B}\,p \cdot {\sim}x\,\mathrm{B}{\sim}x\,\mathrm{E}{\sim}p)$	(11) $\forall x \exists y\{x\,\mathrm{O}\,y \cdot \forall z[y\,\mathrm{B}\,z \supset \exists w(z\,\mathrm{E}\,w \cdot y\,\mathrm{B}\,w)]\}$
(12) ${\sim}\bigcirc\,(x\,\mathrm{B}\,p \cdot x\,\mathrm{B}\,x\,\mathrm{E}{\sim}p)$	(12) $\forall x \exists y \exists z \exists w(x\,\mathrm{O}\,y \cdot y\,\mathrm{B}\,z \cdot z\,\mathrm{E}\,w \cdot y\,\mathrm{B}\,w)$.

Relationships of proper inclusion, independence, and identity among these theories are as in Figures 1 and 2.

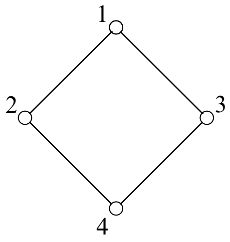

FIGURE 1

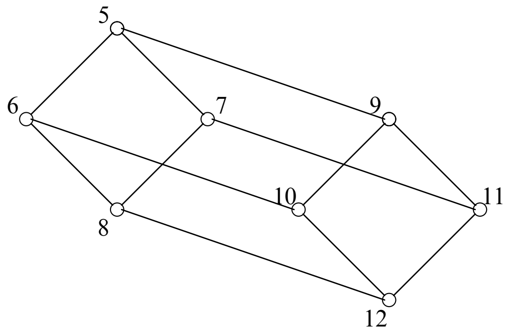

FIGURE 2

However, if we require that in an evidential model E be reflexive, these relationships change to those in Figure 3. It thus follows that folk philosophy errs in treating externalism and internalism as independent (or even contrary or contradictory) theories. E.g., once one stipulates that adequate evidence for the truth of a proposition must be sufficient to entail its truth, externalism itself generates internalist claims.

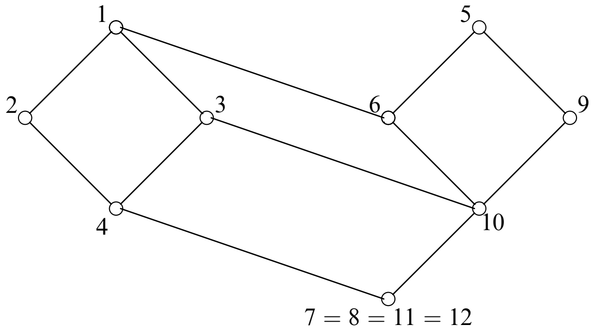

FIGURE 3

► KENNETH MANDERS, *Euclidean diagram reasoning*.
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In traditional Euclidean geometry, a diagram may license inference, just as entries in the text. This is now commonly held a defect of rigor. But the extraordinary career of Euclidean geometry justifies reconstructing the role of diagrams in its proof standards. (Italicized terms are defined more fully in the manuscript; their use in this sketch is merely suggestive, and unavoidably somewhat misleading.)

1. Diagram and text in Euclidean geometry.

Geometric cogency requires that each step be licensed, by prior entries in the text of features attributed to the diagram, or both. Some things may be "read off" directly from a diagram (*diagram-based*), some only inferred from prior entries in the text (*text-based*). Many attributions have components of both kinds; e.g., the infamous intersection of circles in Euclid I.1: that the curves are circles is available only from prior stipulation in the text; that they intersect arises from their relative situation, and is to be read off from the diagram rather than the text.

Directly *diagram-based* conditions are those with (a) *explicit perceptual cues* that (to make up for limited control in diagram production) are both (b) *stable* across a range of *diagram perturbation* and (c) not readily *eliminable by trouble-shooting* egregious failures of *diagram specifications*: precisely the inclusions and contiguities of distinct regions, segments and points, insofar as not clearly eliminable. Call this the *appearance* of a diagram.

2. Diagrams in Euclidean reductio proof.

1. The relationship of the diagram to any conditions in force upon entering the reductio context is unaffected. 2. The diagram *indicates* all diagram-based conditions from the reductio hypothesis. 3. The diagram may be *subject to* some text-based conditions from the reductio hypothesis—but not all.

3. Diagram Control breakdown.

Appearance control. Although appearance is reasonably stable under diagram perturbation, limited metric control in producing and judging a physical diagram (from one system of data) may leave us unable to agree on its appearance, even after trouble-shooting.

Case branching control. We produce but one diagram at a time; giving little grasp of the range of alternative outcomes. Euclidean reasoning lack resources to survey all systems of free data for a given diagram specification, let alone what appearances they give. Diagram use provides little to help establish whether given variant appearances exhaust the possibilities; or if not, to locate further alternatives.

4. Probing.

The threat of disarray posed by these failures is met by a shared responsibility for *probing* diagram-based demonstrations. This appears to lead to a stable demonstration eventually, but one cannot always determine or certify when. Diagram-based demonstration remains in principle open-ended.

► JUAN CARLOS MARTÍNEZ, *On thin-tall Boolean algebras*.
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If B is a Boolean algebra and α is an ordinal, the *Cantor-Bendixson ideal* I_α is defined by transfinite induction as follows: $I_0 = \{0\}$; if $\alpha = \beta + 1$, I_α is the ideal generated by $I_\beta \cup \{b \in B : b/I_\beta$ is an atom in $B/I_\beta\}$; and if α is limit, $I_\alpha = \bigcup\{I_\beta : \beta < \alpha\}$. A Boolean algebra B is *superatomic*, if every subalgebra of B is atomic. It is easy to check that B is superatomic iff $B = I_\alpha$ for some ordinal α.

We define the *height* of a superatomic Boolean algebra B by $\mathrm{ht}(B) =$ the least ordinal α such that B/I_α is finite (which means $B = I_{\alpha+1}$). For every $\alpha < \mathrm{ht}(B)$, let $\mathrm{wd}_\alpha(B)$ be the cardinality of the set of atoms in B/I_α. Then, we define the *width* of B by $\mathrm{wd}(B) = \sup\{\mathrm{wd}_\alpha(B) : \alpha < \mathrm{ht}(B)\}$. If κ is an infinite cardinal and $\alpha \neq 0$ is an ordinal, we say that a Boolean algebra B is a (κ, α)*-Boolean algebra*, if B is superatomic, $\mathrm{wd}(B) = \kappa$ and $\mathrm{ht}(B) = \alpha$. B is called κ*-thin-tall*, if B is a (κ, α)-Boolean algebra for some ordinal $\alpha \geq \kappa^+$. It was proved by Juhász and Weiss that for every ordinal α with $0 < \alpha < \omega_2$ there is an (ω, α)-Boolean algebra. However, it is not known whether there exists an ω_1-thin-tall Boolean algebra. The aim of my talk is then to present a proof of the following forcing analogue of the Juhász-Weiss theorem: "if κ is an infinite cardinal such that $\kappa^{<\kappa} = \kappa$ and α is an ordinal such that $0 < \alpha < \kappa^{++}$, then there is a cardinal-preserving notion of forcing that forces the existence of a (κ, α)-Boolean algebra." Consistency for any specific regular κ then follows as a corollary.

► FRANCISCO DIAS MATIAS, *A new generalization of Gödel's first incompleteness theorem*.
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In [1] I generalized constructively Gödel's first incompleteness theorem by exhibiting true undecidable formulas which were self-referential and diagonalized. In [2] I gave an effective way of constructing true undecidable formulas that were neither self-referential nor diagonalized. In the paper I show how we can generalize Gödel's first incompleteness theorem in a constructive way and that the true undecidable formulas are neither self-referential nor diagonalized. W_x is a recursively enumerable set whose Gödel number is x. Let $W_x \neq \emptyset$ and $V = \{x : A_x$ is true in the standard model of PA$\}$. PA is elementary arithmetic. By means of an algorithm (dovetailing and s-m-n theorem) we can find a recursive function ${}_xf$ such that range ${}_xf = W_x$. Let $D(y, z)$ represent ${}_xf$ in PA. On the other hand let $G(x, y)$ represent the recursive function g, that gives the Gödel number of a formula $A_u(u)$ which is the result of substituting in $A_u(y)$ the numeral u for all free occurrences of the variable y in $A_u(y)$. Now we construct the formula $\forall z \forall w \neg(D(z, w) \wedge G(p, w))$ whose Gödel number is $g(p)$, where p is the Gödel number of $\forall z \forall w \neg(D(z, w) G(y, w))$.

Of course $g(p) \notin W_x$ and if $W_x \subset V$, $g(p) \in V$. Now consider the constant functions $C(x) = g(p)$ for all x, and let $C(x)$ be represented in PA by $H(x, y)$. Then let us consider the formula $\forall w[\exists t \exists y(H(y,t) \wedge G(w,t) \Longrightarrow \forall y \forall z \neg(D(y,z) \wedge G(w,z)))]$. The whole procedure in constructing this formula from W_x was an algorithm, then by Church's thesis there exists a partial recursive function ψ such that, if $W_x \neq \emptyset$ and $W_x \subset V$, $\psi(x)$ is the Gödel number of the last formula and $\psi(x) \in V - W_x$. It is obvious that $\psi(x)$ is the Gödel number of a formula that is neither self-referential nor diagonalized. The generalization is done by means of the following well known result. $[A \neq \emptyset$ and $(\exists \psi$ partial recursive$) \forall x[W_x \subset A \,\&\, W_x \neq \emptyset \Longrightarrow \psi(x) \downarrow \,\&\, \psi(x) \in A - W_x]] \Longrightarrow A$ is productive. Here as it happened in [1] I did not find easy answers to the following questions, where $T_0 = \{X : \vdash_{\mathrm{PA}} A_x\}$ and $F = \{x : A_x \text{ is false}\}$.

(i) If $W_x \subset V - T_0$, $\psi(x) \in T_0$?
(ii) If $W_x \subset T_0$, $\psi(x) \in T_0$?
(iii) If $W_x \subset F$, $\psi(x) \in T_0$?

[1] FRANCISCO DIAS MATIAS, *On elementary arithmetic*, ***Journal of Symbolic Logic***, vol. 59, no. 1 (1994), pp. 716–717, Annual meeting of the ASL, abstracts of papers, Notre Dame, 1993.

[2] ———, *Self-reference and diagonalization*, this JOURNAL, vol. 1, no. 3 (1995), p. 388, Annual meeting of the ASL, abstracts of papers, Irvine, 1995.

▶ INGOLF MAX, *Types of quantifiers in two-dimensional logics*.
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(1) We syntactically extend the classical first-order logic by introducing an operator $[\ ]$ forming *pairs* of classical formulae: If A and B are any classical formulae of a first-order system, then $\left[\begin{smallmatrix}A\\B\end{smallmatrix}\right]$ is a formula. Furthermore, we must add special substitution rules called *reduction rules* to transform any expression of our language which contains *two-dimensional* bracket-expressions as subparts into an expression of the form $\left[\begin{smallmatrix}A\\B\end{smallmatrix}\right]$. Both classical functors and classical quantifiers can be characterized as acting uniformly in each dimension.

Let X be any two-dimensional formula, and Φ^1 and Φ^2 be the forms of unary and binary classical functors, then the reduction rules run as follows:

$$X \Longrightarrow X\left\{\Phi^1\left[\begin{smallmatrix}A\\B\end{smallmatrix}\right] / \left[\begin{smallmatrix}\Phi^1 A\\\Phi^1 B\end{smallmatrix}\right]\right\} \qquad X \Longrightarrow X\left\{\Phi^2\left[\begin{smallmatrix}A\\B\end{smallmatrix}\right]\left[\begin{smallmatrix}C\\D\end{smallmatrix}\right] / \left[\begin{smallmatrix}\Phi^2 AC\\\Phi^2 BD\end{smallmatrix}\right]\right\}$$
$$X \Longrightarrow X\left\{\Phi^2\left[\begin{smallmatrix}A\\B\end{smallmatrix}\right] C / \left[\begin{smallmatrix}\Phi^2 AC\\\Phi^2 BC\end{smallmatrix}\right]\right\} \qquad X \Longrightarrow X\left\{\Phi^2 C\left[\begin{smallmatrix}A\\B\end{smallmatrix}\right] / \left[\begin{smallmatrix}\Phi^2 CA\\\Phi^2 CB\end{smallmatrix}\right]\right\}$$
$$X \Longrightarrow X\left\{\forall\mathbf{a}\left[\begin{smallmatrix}A\\B\end{smallmatrix}\right] / \left[\begin{smallmatrix}\forall\mathbf{a} A\\\forall\mathbf{a} B\end{smallmatrix}\right]\right\} \qquad X \Longrightarrow X\left\{\exists\mathbf{a}\left[\begin{smallmatrix}A\\B\end{smallmatrix}\right] / \left[\begin{smallmatrix}\exists\mathbf{a} A\\\exists\mathbf{a} B\end{smallmatrix}\right]\right\}.$$

But we can also introduce n-placed *variable functors* of the form $\mathcal{V}^n$ which break the barrier of the two dimensions:

$$X \Longrightarrow X\left\{\mathcal{V}^n\left[\begin{smallmatrix}A_1\\B_1\end{smallmatrix}\right]\cdots\left[\begin{smallmatrix}A_n\\B_n\end{smallmatrix}\right] / \left[\begin{smallmatrix}\Phi^{2n} A_1\cdots A_n B_1\cdots B_n\\\Phi^{2n} A_1\cdots A_n B_1\cdots B_n\end{smallmatrix}\right]\right\}.$$

Using only classical validity "$\vdash$" we are able to define a notion of two-dimensional validity:

DEFINITION. $\left[\begin{smallmatrix}A\\B\end{smallmatrix}\right]$ is valid [symb.:$\models \left[\begin{smallmatrix}A\\B\end{smallmatrix}\right]$ iff $\vdash A$ and $\vdash B$.

Because of the THEOREM $\models \left[\begin{smallmatrix}A\\B\end{smallmatrix}\right]$ *iff* $\vdash (A \wedge B)$ we get—in respect to tautologies—a system reducible to ordinary classical logic.

(2) Let us interpret the A in $\left[\begin{smallmatrix}A\\B\end{smallmatrix}\right]$ as the *assertion*, and the B as the *presupposition* (correctness-expression) of some sentence without further qualification. Then a *presupposition/internal negation* can be explicated by using the variable functor "$\sim$" with the reduction

rule $X \Longrightarrow X\left\{\sim \left[\begin{smallmatrix} A \\ B \end{smallmatrix}\right] / \left[\begin{smallmatrix} \neg A \\ B \end{smallmatrix}\right]\right\}$, i.e., negating only the first (assertion) dimension.

In our two-dimensional system we are able to reconstruct the four forms of categorical propositions in an alternative way:

$$\text{universal affirmatives: } \left[\begin{smallmatrix} \forall x(Fx \supset Gx) \\ \exists x Fx \end{smallmatrix}\right] \quad \text{universal negatives: } \left[\begin{smallmatrix} \forall x(Fx \supset \sim Gx) \\ \exists x Fx \end{smallmatrix}\right]$$

$$\text{particular affirmatives: } \left[\begin{smallmatrix} \exists x(Fx \wedge Gx) \\ \exists x Fx \end{smallmatrix}\right] \quad \text{particular negatives: } \left[\begin{smallmatrix} \exists x(Fx \wedge \sim Gx) \\ \exists x Fx \end{smallmatrix}\right].$$

Then it can be shown that a corresponding reformulation of the *traditional square of opposites*—including the *empty domain*—yields a completely adequate reconstruction of the innerlogical relations.

(3) *Variable quantifiers* can be characterized by the following form of reduction rule:

$$X \Longrightarrow X\left\{\forall_i \mathbf{a} \left[\begin{smallmatrix} A \\ B \end{smallmatrix}\right] / \left[\begin{smallmatrix} \forall \mathbf{a}\Phi^2 AB \\ \forall \mathbf{a}\Psi^2 AB \end{smallmatrix}\right]\right\} \quad X \Longrightarrow X\left\{\exists_i \mathbf{a} \left[\begin{smallmatrix} A \\ B \end{smallmatrix}\right] / \left[\begin{smallmatrix} \exists \mathbf{a}\Phi^2 AB \\ \exists \mathbf{a}\Psi^2 AB \end{smallmatrix}\right]\right\}.$$

Variable quantifiers acting like those of Russell's type can be expressed by

$$X \Longrightarrow X\left\{\forall_R \mathbf{a} \left[\begin{smallmatrix} A \\ B \end{smallmatrix}\right] / \left[\begin{smallmatrix} \forall \mathbf{a}(B \supset A) \\ \forall \mathbf{a}(B \supset A) \end{smallmatrix}\right]\right\} \quad X \Longrightarrow \left\{\exists_R \mathbf{a} \left[\begin{smallmatrix} A \\ B \end{smallmatrix}\right] / \left[\begin{smallmatrix} \exists \mathbf{a}(B \wedge A) \\ \exists \mathbf{a}(B \wedge A) \end{smallmatrix}\right]\right\}$$

$$\models \forall_R x \left[\begin{smallmatrix} Gx \\ Fx \end{smallmatrix}\right] \equiv \forall x(Fx \supset Gx) \qquad \models \exists_R x \left[\begin{smallmatrix} Gx \\ Fx \end{smallmatrix}\right] \equiv \exists x(Fx \wedge Gx).$$

But there are a lot of other interesting quantifiers expressible in such a two-dimensional style.

(4) Finally, our two-dimensional construction allows the introduction of *mixed quantifiers* following another form of reductions:

$$X \Longrightarrow X\left\{\forall_\exists \mathbf{a} \left[\begin{smallmatrix} A \\ B \end{smallmatrix}\right] / \left[\begin{smallmatrix} \forall \mathbf{a}\Phi AB \\ \exists \mathbf{a}\Psi AB \end{smallmatrix}\right]\right\} \quad X \Longrightarrow X\left\{\exists_\forall \mathbf{a} \left[\begin{smallmatrix} A \\ B \end{smallmatrix}\right] / \left[\begin{smallmatrix} \exists \mathbf{a}\Phi AB \\ \forall \mathbf{a}\Psi AB \end{smallmatrix}\right]\right\}.$$

This offers us new insights into a reconstruction of *generalized quantifiers* in a classical style.

► ALEX M. MCALLISTER, *Completions of PA: Models and enumerations of representable sets*.

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In [1], Lachlan and Soare proved the following:

THEOREM 1 ([1], Theorem 2.1). *There is a model of true arithmetic whose degree is not a uniform upper bound for the arithmetic sets.*

In other words, there exists a model $\mathcal{A}$ of True Arithmetic ($\mathcal{TA}$) such that no enumeration of the family Rep($\mathcal{TA}$) of sets representable in $\mathcal{TA}$ is recursive in $\mathcal{A}$. We generalize this result to a large collection of other completions of Peano arithmetic ($\mathcal{PA}$) by describing conditions which give us completions $\mathcal{T}$ of $\mathcal{PA}$ and models $\mathcal{A}$ of $\mathcal{T}$ such that no enumeration of Rep($\mathcal{T}$) is recursive in $\mathcal{A}$.

An important step in generalizing this result is a new characterization of 1-generic sets as *coding sets* for the recursive subtrees of $2^{<\omega}$. In particular, $C \subseteq \omega$ is a *coding set* for the recursive subtrees of $2^{<\omega}$ iff for all recursive trees $T \subseteq 2^{<\omega}$, if χ_C is a path through T, then there exists $\sigma \in T$ such that $\sigma \subset \chi_C$ and every extension of σ is in T. Jockusch noted a connection between 1-generic sets and the coding sets for recursive subtrees of $2^{<\omega}$ and we show they are identical.

We specify the conditions for obtaining our completions and their models via the notion of a *coding pair*. We say that $(\mathcal{S}, \mathcal{J})$ is a *coding pair* iff $\mathcal{S}$ is a Scott set, $\mathcal{J}$ is a jump ideal $\mathcal{S} \subset \mathcal{J}$, and for all $X \in \mathcal{J}$, there exists $C \in \mathcal{S}$ where C is a coding set for the family of subtrees of $2^{<\omega}$ recursive in X and $C \not\leq_T X$. There exist uncountably many different coding pairs; for any countable jump ideal $\mathcal{J}$ and any nonrecursive $Q \in \mathcal{J}$, there exists a Scott set $\mathcal{S} \subseteq \mathcal{J} \setminus \{Q\}$ where $(\mathcal{S}, \mathcal{J})$ is a coding pair.

Finally, we have our main result:

THEOREM 2. *If* $(\mathcal{S}, \mathcal{J})$ *is a coding pair and* $\mathcal{T}$ *is a is a completion of* $\mathcal{PA}$ *such that* $\mathcal{S} = \mathrm{Rep}(\mathcal{T})$*, then there exists a model* $\mathcal{A}$ *of* $\mathcal{T}$ *such that* $\mathcal{J}$ *is the Scott set of* $\mathcal{A}$ *and no enumeration of* $\mathrm{Rep}(\mathcal{T}) = \mathcal{S}$ *is recursive in* $\mathcal{A}$*.*

The model $\mathcal{A}$ of $\mathcal{T}$ is a "generic" model of the completion of $\mathcal{PA}$.

[1] A. H. LACHLAN and R. I. SOARE, *Models of arithmetic and upper bounds of arithmetic sets*, ***Journal of Symbolic Logic***, vol. 59, no. 3 (1994), pp. 977–983.

▶ GRZEGORZ MICHALSKI, *A Friedberg-Muchnik type theorem from recursive theoretic model theory*.

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In [1], Knight gave conditions on a recursive $\mathbb{L}$-structure $\mathcal{A}$ with a pair of recursive subsets R and S, under which there is an isomorphism f from $\mathcal{A}$ to $\mathcal{B}$ such that $f(R)$ and $f(S)$ are Σ^0_β and independent over Δ^0_β, where $\beta \geq 1$ is a recursive ordinal. In response to a question of Knight which was motivated by [1], we give similar conditions on a recursive $\mathbb{L}$-structure $\mathcal{A}$ with two pairs of recursive subsets R_{i+1}, S_{i+1} $(i = 0, 1)$, under which there exists an isomorphism f from $\mathcal{A}$ onto a recursive $\mathbb{L}$-structure $\mathcal{B}$, such that $f(R_{i+1}), f(S_{i+1})$ are Σ^0_{i+1} sets with Turing degrees incomparable relative to $0^{(i)}$.

We treat this $0''$-priority argument as a test case for a level-two metatheorem—a one-step generalization of a metatheorem for finite injury priority arguments given in [2].

[1] J. F. KNIGHT, *Variants of the Friedberg-Muchnik theorem*, preprint.

[2] G. MICHALSKI, *A metatheorem for finite injury priority arguments*, preprint, abstract in this JOURNAL, vol. 1, no. 3, 1995.

▶ J. R. OTTO, *From Horn formulas to Makkai sketch resolution.*

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Signatures generalize to König (or K-) categories and Horn formulas to finite Makkai (or M-) sketches. A category is K- iff it is small, lacks infinite ascending chains of non-identity maps, and has finite fan-out. An M-sketch is a functor from a K-category to sets. Horn clauses properly generalize to maps between finite M-sketches. (Precisely the l.f.p. categories are so presented.) With such maps as axioms, deductions are the close up of axioms under isomorphisms, folding, pushing out, and composition, and resolutions are the cospans $d :\longrightarrow\longleftarrow: p$ with d a deduction. (Such p are partial answers.) Resolutions factor to the cospan compositions of a relatively few special resolutions.

▶ ANDRZEJ ROSLANOWSKI, *Ideals of perfect sets*.

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This is a joint work with Marek Balcerzak.

We look at the structure of the family of perfect sets from an ideal I on the real line $\Re$. We restrict our attention to ideals on $\Re$ which are dense on perfect sets (i.e., each perfect set contains a perfect subset from the ideal).

To compare different ideals from the point of view of their perfect members we use perfect isomorphism. A bijection f from $\Re$ onto $\Re$ is a perfect isomorphism if and only if it is measurable with respect to the field of Marczewski's sets. We describe canonical perfect isomorphisms of $\Re$ and apply this description to build some examples.

Further we look at intersections of families of ideals and their traces on perfect sets.

Some problems are posed.

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I examine some applications of $\mathcal{L}_{\omega_1 P}$ (Hoover and Keisler's logic with probability quantifiers) to foundational problems in statistics.

For example, an *experiment* is a parametrized family $\mathcal{E} = (\Omega, \mathcal{B}, P_t)_{t \in T}$ of probability measures. The class of experiments sharing a common parameter space T is typically preordered by saying that $\mathcal{E}_1 \sqsubseteq \mathcal{E}_2$ provided every decision based on $\mathcal{E}_1$ can be majorized by one based on $\mathcal{E}_2$. There are several equivalent characterizations of $\sqsubseteq$; I give another using $\mathcal{L}_{\omega_1 P}$: roughly, that (for a suitable sublogic $\mathcal{L}'$ of $\mathcal{L}_{\omega_1 P}$) $\mathcal{E}_1 \sqsubseteq \mathcal{E}_2$ if and only if for every $\mathcal{L}'$-structure $\mathfrak{A}_1$ based on $\mathcal{E}_1$ there is an $\mathcal{L}$-structure $\mathfrak{A}_2$ based on $\mathcal{E}_2$ such that $\mathfrak{A}_1 \equiv \mathfrak{A}_2$ (uniformly in $t \in T$).

▶ WIM RUITENBURG, *Intermediate logics over basic propositional calculus*.
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Basic Propositional Calculus (BPC) relates to modal logic K4 in essentially the same way that Intuitionistic Propositional Calculus (IPC) relates to S4. Intermediate logic theory over BPC naturally generalizes the one over IPC, and relates to a similar theory over K4.

Among other things, we will show:

> There are two maximal consistent intermediate logics, one of whom is Classical Propositional Calculus;

and

> the lattice of finitely axiomatizable atomless theories is closed under many morphisms that 'preserve' the binary logical operators and 'true.'

[1] M. Ardeshir and W. Ruitenburg, *Basic propositional calculus I*, ***Technical Report*** **418**, Marquette University, 1995.

[2] M. Ardeshir, *Aspects of basic logic*, ***Ph.D. thesis***, 1995.

[3] H. Barendregt, M. Bezem, and J. W. Klop (editors), *Dirk van Dalen Festschrift*, ***Quaestiones Infinitae***, vol. 5, Department of Philosophy, Ultrecht University, 1993.

[4] W. Ruitenburg, *Constructive logic and the paradoxes*, ***Modern Logic***, vol. 1 (1991), pp. 271–301.

[5] ———, *Basic logic and Fregean set theory*, in [3], pp. 121–142.

[6] A. Visser, *A propositional logic with explicit fixed points*, ***Studia Logica***, vol. 40 (1981), pp. 155–175.

▶ DAVID SANDBORG, *Pick's theorem: A case study in mathematical explanation*.
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Explanation has been extensively studied in the philosophy of science, but has received less attention in the philosophy of mathematics. Mathematicians however, often reprove results, sometimes in order to obtain an explanatory improvement. We report on a case study in mathematical explanation—proofs of Pick's theorem, some of which better explain the result than others, and analyze the explanatory differences. Pick's theorem states that the area of a lattice polygon is $i + b/2 - 1$, where i is the number of interior points and b the number of boundary points of the polygon. Two of the proofs, one explanatory, one less so, are inductive; each proving the theorem for a restricted class of polygons from which all lattice polygons can be composed, and proving that this composition preserves Pick's formula. Another more explanatory proof relates the area of the polygon with the visibility angle about each point within it.

Two independent differences between the proofs help account for their explanatory differences. First, in the inductive proofs, the choice of base case makes a crucial difference, which can be stated in a way which generalizes beyond inductive proofs—in the more explanatory proof, the inductive step involves less arbitrariness. Second, the more explanatory proofs provide better interpretations of the terms in Pick's formula. These differences affect our ability to answer natural why-quesions that arise about the theorem.

If we admit that explanation matters in mathematics, why has it been so neglected by philosophers of mathematics? They have primarily focused on justificatory aspects of proofs, but explanatory differences cannot be accounted for in these terms. Therefore, the study of mathematical explanation suggests a shift of emphasis in philosophical analysis of mathematical practice.

▶ T. SEIDENFELD, M. J. SCHERVISH, AND J. B. KADANE, *The Rudin-Keisler partial order of ultrafilters and non-conglomerability of finitely additive probability*.

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We consider how an unconditional, two-valued, finitely additive probability P on a countable set may localize its non-conglomerability (non-disintegrability) through some combinatorial properties of its associated non-principal ultrafilter U_P. We show that when ultrafilter U_P is not minimal in the Rudin-Keisler partial order of $\beta(\omega)\backslash\omega$, we may locate a partition in which P fails to satisfy deFinetti's principle of conglomerability by examining (at most) countably many partitions, $\{\pi^*, \pi_1, \pi_2, \dots\}$.

These partitions are chosen so that in each $\pi_i = \{h^i_j : j = 1, \dots\}\,(i = 1, \dots)$, that is except for partition π^*, one of two situations occur. Either there is a Rudin-Keisler equivalent U_P obtained by mapping the elements a partition member h^i_j of π_i to j, and then P is conglomerable in π_i. Otherwise, P cannot be made conglomerable in π_i and then P is singular with respect to all finitely additive probabilities that are conglomerable in π_i. If U_P has such Rudin-Keisler equivalents in each partition π_i then P is simultaneously conglomerable in all the π_i. However, then P fails the conglomerability principle in the remaining partition π^*.

This result is applied also to finitely additive probabilities that assume only finitely many values. By contrast, if ultrafilter U_P is Rudin-Keisler minimal, then P is simultaneously conglomerable in each finite collection of partitions, though not simultaneously conglomerable in all partitions.

▶ D. E. STEVENSON, *Developing a constructive theory of Euclidean geometry: The first twenty-eight theorems*.

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By all accounts, it starts with Euclid. In an effort to arrange the known results of geometry, Euclid wrote thirteen books. These books are taken as the beginnings of the "mathematical method" and the "axiomatic" method. Through the next twenty-five centuries, these books were—and still are—poured over by mathematicians and philosophers of mathematics and logic. Reading the commentaries of these investigators leads one to the conclusion that Euclid was a classical mathematician and that the constructive view just is not worthy of study. Yet, the first criticism of the very first theorem of Book I is one of an impredication on the intersection of two circles.

We discuss in our attempt to provide a constructive basis for Euclidean geometry. Our attempt is modest in that we concentrate on the first twenty-eight theorems thus staying out of the parallel postulate debate. We desire a computable meaning for each theorem of the type that might be used by computers. Our method is that of Errett Bishop:

- We require a procedure both for the arbitrary element of a set and for the test of equality of two elements;
- We have the usual prohibition of impredication and excluded middle; and
- We must strictly avoid omniscience.

Because Euclid did not identify non-logical axioms as such we rely on commentaries and later authors. For axioms we follow Hilbert's development, taking Euclid Theorem 4 as an axiom. However, we only accept Theorem 4 after seeing it is "obvious" from a constructive view point. We also formalize other aspects left untreated in Euclid's original work.

We describe interesting aspects that come to light as one insists on constructivity. We are left with the conclusion that the auxiliary diagrams are required for the constructive proofs.

▶ MIKLÓS ERDÉLYI-SZABÓ, *Decidability in the constructive theory of reals as an ordered* **Q**-*vectorspace*.

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We are investigating the following problems:

1. What are the decidable fragments of the constructive/intuitionistic theory of the real numbers?
2. What are the decidable fragments of the various topological models of the intuitionistic theory of the real numbers?

Let $L = \langle <, +, h_q, 1\rangle_{q\in\mathbf{Q}}$ be the language for the ordered **Q**-vectorspaces where h_q is a one-place function symbol corresponding to multiplication by q.

Our main tool is a constructive normal-form theorem which shows that every L-sentence is equivalent to one whose atomic formulas are of the form $x > 0$, x is a variable. The proof uses only facts true in Bishop's constructive analysis.

Using this result we show constructively that various fragments of the L-theory of the reals are decidable. The constructive truth of prenex universal sentences can be decided using intuitionistic propositional calculus. The methods applied in the proof of this fact go beyond Bishop-style constructivism; an intuitionistic theory of lawless sequences is used in the metatheory. We also give a decision method for sentences not containing universal quantifiers. The proof is syntactic and uses only facts accepted by every school of constructive mathematics. These two results can be combined to give a decision method for sentences where at most the innermost quantifiers are universal.

Regarding the second quesiton: I use Rabin's results on the decidability of some monadic second-order theories of several successor functions and the normal-form theorem mentioned above to show that the sets of L-sentences true in Scott's model and in Scowcroft's model are decidable. In the proof of the latter fact I also use a characterization of the continuum in this model for second-order arithmetic which has the same relationship, via Vesley's representation, to the model as Scott's model has to the model defined by Moschovakis. The proofs in this part are classical.

▶ ANDRES VILLAVECES, *Destroying end extensions and iterated forcing*.
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The existence of End Elementary Extensions of models M of ZFC is related to the ordinal height of M, according to classical results due to Keisler, Morley and Silver. In this talk, we further investigate the connection between the height of M and the existence of End Elementary Extensions of M. In particular, we prove that the theory

ZFC + GCH + there exist measurable cardinals + with all inaccessible non weakly compact cardinals are possible heights of models with no End Elementary Extensions

is consistent relative to the theory

ZFC + there exist measurable cardinals + the weakly compact cardinals are cofinal in the universe.

We also describe a simpler coding which destroys GCH but otherwise gives the same result.

▶ GEORGE WEAVER, *The second order theories of Dedekind algebras*.
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A *Dedekind algebra* is a mono-unary algebra (B, h) where h is injective. The *ancestors of* b in (B, h) consists of all d where $h^n(d) = b$, for some $n \geq 1$. The *configuration* of b is the smallest subset of B containing b and closed under h and h^{-1}. If A is a configuration, A is a *spoke* iff some member of A has no ancestors; A is a *copy of the integers* iff every element of A has infinitely many ancestors; A is a *loop* iff some member of A is among its own ancestors; and, for $n \geq 1$, A is an *n-loop* iff the cardinality of A is n.

The signature for (B, h) is the function $C(B, h)$ defined on the non-negative integers as follows: $C(B, h)(0)$ is the number of spokes; $C(B, h)(1)$ is the number of copies of the integers; and, for $n \geq 2$, $C(B, h)(n)$ is the number of $(n-1)$-loops.

THEOREM 1. (1) *Two Dedekind algebras are isomorphic iff their signatures are identical*; *and*
(2) *if for all $n \geq 0$, $C(B, h)(n)$ is either zero or a second order characterizable cardinal, the second order theory of (B, h) is categorical; and, if $C(B, h)$ is non-zero for only finitely many n, the second order theory of (B, h) is finitely axiomatizable.*

$\mathbf{D}_\beta$ is the class of Dedekind algebras of cardinality β. The *isomorphism problem* for $\mathbf{D}_\beta$ is the problem of whether the following is true: members of $\mathbf{D}_\beta$ which are equivalent in second order logic are isomorphic.

THEOREM 2. (1) *If all non-zero cardinals $\leq \beta$ are second order characterizable, the isomorphism problem for $\mathbf{D}_\beta$ has a positive solution*;
(2) *if $\beta \geq \aleph_{(2^{\aleph_0})^+}$, the isomorphism problem for $\mathbf{D}_\beta$ has a negative solution*; *and*
(3) *there are infinitely many β where β is not second order characterizable, but the isomorphism problem for $\mathbf{D}_\beta$ has a positive solution.*

▶ DAN E. WILLARD, *Self-justifying axiom systems and computational complexity*.
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This talk was motivated by the observation that it is possible to devise an axiom system that can verify its own Hilbert consistency and prove all the Π_1 theorems of Peano Arithmetic when these theorems *are written in a revised form* where the primitive operations of Subtraction and Division, replace Addition and Multiplication. We have previously [1, 2, 3, 4] explained how this observation was consistent with Gödel's Incompleteness Theorem.

Let $\lceil \Phi \rceil$ denote Φ's Gödel number, and $\text{Prf}_\alpha(x, y)$ be a Δ_0 formula indicating y is the Gödel number of a proof from axiom system α of the sentence with Gödel number x. The "**IDEALIZED**" Π_1 **Reflection Principle** for the axiom system α is the statement that for every Π_1 sentence $\forall v \Psi(v)$, that α can prove the assertion (1). Also, the "**SEGMENTED**" Π_1 **Reflection Principle** for α is the statement that for every Δ_0 formula $\Psi(v)$ and every fixed constant $\bar{c}$, that α can prove (2).

$$\forall y \{\text{Prf}_\alpha(\lceil \forall v \Psi(v) \rceil, y) \supset \forall v \Psi(v)\} \tag{1}$$

$$\forall y \{\text{Prf}_\alpha(\lceil \forall v \Psi(v) \rceil, y) \supset \Psi(\bar{c})\}. \tag{2}$$

We generalize Gödel's Incompleteness Theorem to show that *it is infeasible* for a consistent axiom system α to prove all of Peano Arithmetic's Π_1 theorems and support the Idealized Π_1 Reflection Principle. However, we show it is feasible for self-verifying systems proving all Peano Arithmetic's Π_1 theorems to support Segmented Π_1 Reflection.

Let $\text{OUTPUT}(e, t, x)$ be a Δ_0 formula asserting that a universal Turing machine will output a bit-stream x on its Turing Tapes at time t when its instruction tape initially stores an integer e (encoded as a string of $\text{Log}(e)$ bits). For an arbitrary axiom system A and for any non-decreasing function $R(y)$, using techniques analogous to [1, 2, 3, 4], we define an axiom system $\text{ISREF}^R(A)$, which can prove all A's Π_1 theorems, and for every Δ_0 formula $\Psi(v)$ and for every fixed constant $\bar{c}$, $\text{ISREF}^R(A)$ can axiomatically recognize the reflection principle below:

(3)
$$\forall y \forall x \leq y \, \forall t \leq R(y) \, \forall e \leq R(Y) \{\text{Prf}_{\text{ISREF}^{R}(A)}(\lceil \forall v \Psi(v) \rceil, y) \wedge \text{OUTPUT}(e, t, x)\} \supset \Psi(x).$$

Since (3)'s reflection principle lies midway between (1) and (2), it is unclear whether there exists a function $R(y) > \text{O}(\text{Polylog}(y))$ where $\text{ISREF}^R(A)$ is consistent whenever A is consistent. We prove $\text{P} \neq \text{NP}$ *if this conjecture is valid.*

We have also developed a second result of a similar type. Let $\text{ISIT}(A)$ denote an axiom system that

1. again uses Hilbert deduction and can verify all A's Π_1 theorems (once again we use the **NON-GROWTH** functions Subtraction and Division to replace Addition and Multiplication and assume that $A \supseteq$ Peano Arithmetic).

2. that for some pre-specified fixed real constant $\lambda < 1$, $\text{ISIT}(A)$ can axiomatically recognize that $\forall x$, an arbitrary Turing machine running for x^λ units of time is unable to output a proof of $0 = 1$ from $\text{ISIT}(A)$.

3. that defines the constant $\overline{N}$ with the axiom $\text{Predecessor}(\overline{N}) = \text{Predecessor}(\overline{N-1})$ and has no growth functions.

We have proven that $\text{ISIT}(A)$ satisfies a consistency-preservation property, analogous to our earlier theorems in [1, 2, 3, 4]. However, the same is not true, surprisingly, for an $\text{ISIT}^*(A)$ axiom system which contains an additional schema of *even-number axioms* of the form: $\overline{2N} - \overline{N} = \overline{N}$. For a pre-specified function $R(x)$, define $\text{ISIT}^R(A)$ to be a modified form of $\text{ISIT}^*(A)$ where item 2 only specifies that an arbitrary Turing machine running for $R(x)$ units of time is unable to output a proof of $0 = 1$ form $\text{ISIT}^R(A)$. It is unclear whether there exists a function $R(x) > \text{O}(\text{Polylog}(x))$ where $\text{ISIT}^R(A)$ is consistent whenever A is consistent. However, $\text{P} \neq \text{NP}$ *if this conjecture is valid.* We also prove that **EITHER** Peano Arithmetic is unable to prove that any fixed algorithm a can solve SAT problems in N^b time (for fixed b) **OR** ISIT(Peano) can prove for all fixed constants k that the bitwise concatenation of any k integers is not a proof of $0 = 1$ from ISIT(Peano).

[1] D. Willard, *Self-verifying axiom systems*, ***Proceedings of the Third Kurt Gödel Symposium***, Lecture Notes in Computer Science, vol. 713, Springer-Verlag, 1993, pp. 325–326.

[2] ———, *Self-verifying axiom systems and the incompleteness theorem*, talk presented at the 1994 *annual meeting of the ASL*, short summary of talk in this Journal (1995), pp. 136–137, and full-length version of paper to appear in the ***Journal of Symbolic Logic***.

[3] ———, *Alternative variants of self-verifying systems*, talk at 1995 *ASL annual winter meeting*, see this Journal (1995), pp. 371–372.

[4] ———, *Introspective semantics of self-verifying systems*, talk at 1995 *ASL summer conference*.

Abstracts presented by title

► JOSEPH BARBACK, *On regressive isols and a comparability of summands property, I*.
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By error this abstract was included as a contributed talk at the meeting; the author intended it to be published as a contribution by title.

All of the isols that we consider are assumed to be regressive. An isol c is said to be *hereditarily odd-even* (HOE), when all $u \leq c$ have parity (being either even or odd). The existence of an infinite isol that is HOE was proved by L. Harrington; a proof appears in [4]. Let c be HOE. Then it is known that c has the following property: Whenever $c = u + v$, then either $u \leq^* v$ or $v \leq^* u$. This fact is proved in both [1] and [4].

We say that an isol c has *direct comparability of summands* (DCS), if c has the following property: Whenever $c = u + v$, then either $u \leq v$ or $v \leq u$. We have proved the following facts:

Theorem 1. *There exists an infinite isol c that has DCS.*

Theorem 2. *Let c have DCS. Then c is HOE. Moreover, every $u \leq c$ also has DCS.*

Theorem 3. *Let c have DCS. Then $c + 1$ and $2c$ also have DCS.*

L. Harrington used a reasoning with recursive binary trees to find an infinite isol that was HOE. Later, E. Ellentuck applied a similar approach in [3] to find an infinite hyper-torre isol. We now know that hyper-torre = HOE by the work in [1], and it is also proved in [4]. Our proof of Theorem 1 also uses a reasoning with recursive binary trees.

[1] J. Barback, *Hyper-torre isols and an arithmetic property*, ***Proceedings of a conference on aspects of effective algebra (Monash University, Australia, August, 1979)***, Upside Down A Book Company, Yarra Glen, Victoria, Australia, 1981, pp. 53–68.

[2] ———, *On hereditarily odd-even isols and a comparability of summands property*, ***Pacific Journal of Mathematics***, vol. 118 (1985), pp. 27–35.

[3] E. Ellentuck, *Hyper-torre isols*, ***Journal of Symbolic Logic***, vol. 46 (1981), pp. 1–5.

[4] T. G. McLaughlin, ***Regressive sets and the theory of isols***, Marcel Dekker, New York, 1982.

► JOSEPH BARBACK, *On regressive isols and a comparability of summands property, II*.
Mathematics Department, State University of New York College, Buffalo, NY 14222, USA.

All of the isols that we consider are assumed to be regressive. An isol c is said to have *direct comparability of summands* (DCS), if c has the following property: Whenever $c = u + v$, then either $u \leq v$ or $v \leq u$. The isol c has *special DCS* (SDCS), if every isol of the form $c + r$ that is regressive, has DCS. If an isol has SDCS then it is easy to show that it must be infinite. We have proved the following fact:

THEOREM 1. *There exists an isol c that has SDCS.*

If r is any recursive function, let r_Λ be its extension to the isols (the extension of J. Myhill and A. Nerode). An isol c is called *recursively strongly torre* (RST), if $r_\Lambda(c)$ is an isol for every recursive function r. Isols that are RST were first shown to exist by A. Nerode in [2]. An isol c is called *hereditarily RST* (HRST), if every $u \leq c$ is RST. HRST isols were introduced by E. Ellentuck in [1], though at that time it was not known whether there were any isols that were infinite and HRST.

We have proved the following result:

THEOREM 2. *Let c be an (infinite) isol that has SDCS. Then c is HRST. Also, if f is any increasing recursive function, then $f_\Lambda(c)$ is HRST.*

[1] E. ELLENTUCK, *Diagonal methods in the theory of isols*, ***Zeitschrift für Mathematische Logik und Grundlagen der Mathematik***, vol. 26 (1980), pp. 193–204.

[2] A. NERODE, *Diophantine correct non-standard models in the isols*, ***Annals of Mathematics***, vol. 84 (1966), pp. 421–432.

► JOHN CORCORAN, *Semantic omega properties and mathematical induction*.
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The semantic omega properties, semantic omega-completeness, SOC, and semantic omega-inconsistency, SOI, occur in the context of the 1931 Gödel incompleteness article. The 1931 article involves an interpreted higher-order language whose only non-logical constants are the zero symbol (0) and the successor symbol (here s) and whose universe is the class of natural numbers. Accordingly the principle of mathematical induction is expressed by a single sentence, PMI (Gödel's axiom I3). PMI expresses a property-universal proposition: every property belonging to zero and to the successor of every number to which it belongs also belongs to every number. The numerals are: 0, $s0$, $ss0$, $\ldots$. A numeral instance of a number-universal sentence is the result of deleting the initial quantifier and substituting one and the same numeral for all freed occurrences of the variable. A set of sentences (such as the set of arithmetic truths) is SOC if and only if it implies every number-universal sentence all of whose numeral instances it implies. This paper centers on the fact that *a set of sentences is SOC if and only if it implies PMI*. This fact is established by reference to well-known semantic features of PMI and to logical features of a sentence called alternative mathematical induction, AMI. AMI expresses the proposition that every number has every property belonging to zero and to the successor of every number to which it belongs. AMI is a number-universal sentence equivalent to PMI. Moreover, AMI is paratautological in the sense that it is a number-universal sentence every numeral instance of which is tautological. Thus, AMI, and equivalently PMI, is implied by every set which is SOC. A set of sentences is SOI if and only if it implies every numeral instance of a number-universal sentence whose negation it implies, i.e., it implies the negation of a universal sentence whose negation it implies. Reasoning similar to the above is used to establish the fact that *a set of sentences is SOI if and only if it implies the negation of PMI*. The negation of PMI is equivalent to a paracontradictory sentence, a number-existential sentence every numeral instance of which is contradictory, or unsatisfiable. First-order analogues of the above are also mentioned.

► GUN-WON LEE, *On Craig lemma*.
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As an intuitive glance in [1], if a sentence is followed by another sentence, there is a third sentence which can follow from the first and the second can follow from this third. If the premise that "A implies C" is given, we may take it as a premise which can be used as a proposition from which we can find the third to meet the conditions specified. The condition, namely, that the names in the third B are in both of the first and the second can be eliminated, if the propositions in question are on the same domain, say, the domain of natural numbers where all the names are fixed for the any predicates in use here. With this condition that the domain is fixed, this question can be treated as a question in a pattern as follows, I think:

(1) $B = (A \vee C)$. Namely that "A or C" can be interpolated to meet the conditions specified in the above as far as the domain is fixed in this particular discourse.

Proof (sketch). To have that A implies B in this case, we can have B by using 'addition rule' to the proposition connecting by a disjunction, which is accepted as a rule usually and also confirmed by the truth table too. However the second condition that B implies C cannot be confirmed by the proposition itself, but it can be proved from the original proposition that A implies C. As in the model theory [2], if we formulate the Craig Lemma as that A makes that C true, it certainly confirms that we can use the proposition in the proof that B implies C. By using the deduction theorem, we can have that B implies C using the proposition that A implies C. It runs as follows (sketch):

Using the commutation rule on the interpolated proposition, say, 'A or C', we have 'C or A'. By using implication rule on the above, we can have 'if not C then A'. By using a hypothetical syllogism on both of 'A then C' and 'not C then A', we can have that 'not C then C' which is 'C' by the tautology rule. Here if we are allowed to use the deduction theorem, we can have that 'A or C' which is 'B implies C'.

(2) Now about the last condition, say, " ... which contains only such names as are contained in both A and C" [1, p. 234], if the domain is fixed, and in that reason, if we are allowed to use the predicates without bothering the variables in use here, it may be formulated even without mentioning it at all as in Robinson [4, p. 116]. However if the domain is not fixed, this condition makes the proof in the above (1) meaningless. For in that case, it should be treated as a predicate logic in which the predicates and the individual names are considered explicitly. Here the 'elimination of variables' is not accepted and to deal the predicates only without considering the individual names are not accepted. Therefore the above mentioned argument has a concrete limit, say, that A and C should be the propositions which have the same names, say, the set of natural numbers. And the Craig Lemma in the domain where the arithmetizations of syntax are also admitted as in the incomplete proof in [3], the above treatment of the Craig Lemma cannot be admitted. For in there [3], we cannot guarantee that the same names are occurring in all the propositions.

(3) Finally, about the use of the deduction theorem in the proof in (1), if it is true, we cannot prove without using the deduction theorem, and hence the lemma reveals the special characteristics of the satisfaction conditions in the proof theory. As we can see, that B implies C cannot be true by itself, which can be verified easily by the truth table methods. Only in the fixed domain and the premise that 'A implies C', is used as the given truth, we can prove that 'B implies C'. The proposition that 'A implies C' can give a special limitation on the names used in the propositions, and it can change the truth value of the proposition 'B implies C', enough to prove in terms of the deduction theorem as in (1).

However if this Craig Lemma can be found correctly, it may be in the context of 'definability' as it has been treated in [1], [2], and [4]. If it is in the context of definability, this definitions are following the definitions in the context should be admitted as a propositions from which we can prove in the future. For all definitions are lambda-definitions, I think. If the definitions are restricted in a fixed domain, say, the set of natural numbers, and all the others are the same, we may freely use this Craig Lemma as in (1), I think. Can it be extended in the arithmetizations of syntax? It is open, I think.

[1] BOOLOS et al., ***Computability and logic***, Cambridge, 1980, pp. 234–249.

[2] CHANG et al., ***Model Theory***, North-Holland, 1973, p. 84.

[3] Feferman et al. (editors), ***Kurt Gödel***, vol. 1, Oxford, 1986, pp. 162–170.

[4] ROBINSON, ***Introduction to model theory and to the metamathematics of algebra***, North-Holland, 1986, p. 116.

▶ BILLY JOE LUCAS, *Some internalist alternatives in the logic of consequentialist belief*.
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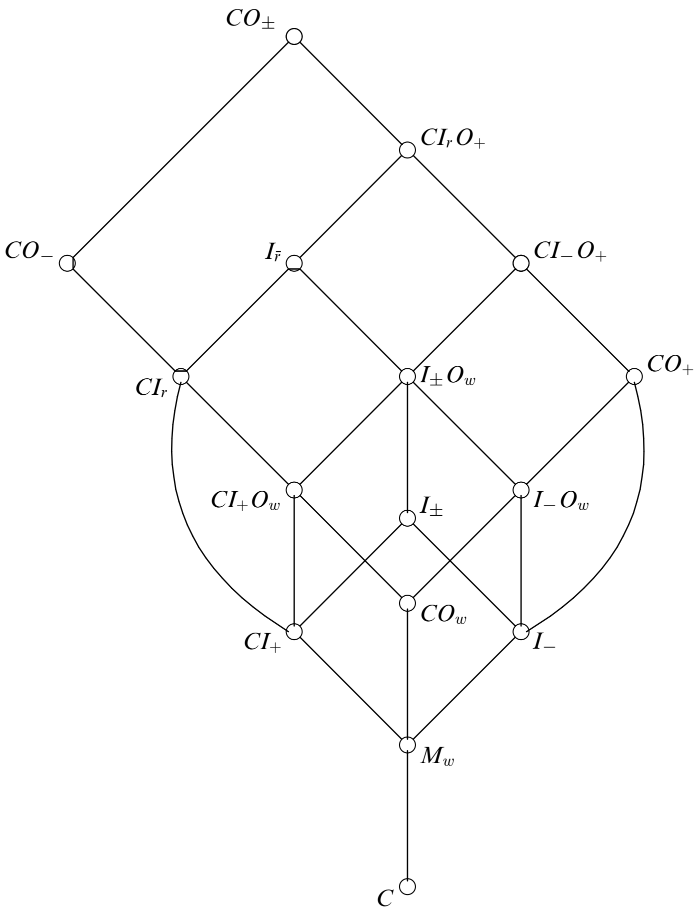

Let x B be the modal operator such as the □ in [1]. We get by an easy proof in the minimal normal modal logic K: $x\,\mathrm{B}(p \cdot \sim p) \supset x\,\mathrm{B}\,q$. It thus follows that one could genuinely believe that some specific contradiction was true (contra Aristotle) only if one also believed *everything*. An interesting logic of belief which is closed under classical truth-functional inference is thus one containing Consistency $(C) = x\,\mathrm{B}\,p \supset \sim x\,\mathrm{B} \sim p = R$ serial. Modeling of less rational concepts of belief using normal modal logics can then be done using some variation of the methods suggested for such work in AI in [2]. The main question investigated: what are the plausible options for additional rationality constraints on a normal modal logic of rational belief? The principles here described (each with a corresponding condition on frames) turn out to have an interesting structural relationship to each other: proper inclusion and independence among extensions of normal logics with Consistency via the addition of any of the seven *internalist* rationality conditions are as depicted below. Internalist rationality co nditions: Positive Omniscience $(\mathrm{O}_+) = x\,\mathrm{B}\,p \supset x\,\mathrm{B}\,x\,\mathrm{B}\,p = \mathrm{R}$ transitive, Negative Omniscience $(\mathrm{O}_-) = \sim x\,\mathrm{B}\,p \supset x\,\mathrm{B}\sim x\,\mathrm{B}\,p = \mathrm{R}$ Euclidean, Positive Infallibility $(\mathrm{I}_+) = x\,\mathrm{B}\,x\,\mathrm{B}\,p \supset x\,\mathrm{B}\,p = \forall x \forall y[x\,\mathrm{R}\,y \supset \exists z(x\,\mathrm{R}\,z \cdot z\,\mathrm{R}\,y)]$, Negative Infallibility $(\mathrm{I}_-) = x\,\mathrm{B}\sim x\,\mathrm{B}\,p \supset \sim x\,\mathrm{B}\,p = \forall x \exists y[x\,\mathrm{R}\,y \cdot \forall z(y\,\mathrm{R}\,z \supset x\,\mathrm{R}\,z)]$, Reflexive Infallibility $(\mathrm{I}_r) = x\,\mathrm{B}(x\,\mathrm{B}\,p \supset p) = \forall x(\exists y y\,\mathrm{R}\,x \supset x\,\mathrm{R}\,x)$, Weak Omniscience $(\mathrm{O}_w) = x\,\mathrm{B}\,p \supset x\,\mathrm{B}\sim x\,\mathrm{B}\sim p = \forall x \forall y[x\,\mathrm{R}\,y \supset \exists z(x\,\mathrm{R}\,z \cdot y\,\mathrm{R}\,z)]$, Weak G. E. Moore $(\mathrm{M}_w) = \sim x\,\mathrm{B}(\sim p \cdot x\,\mathrm{B}\,p) = \forall x \exists y \exists z(x\,\mathrm{R}\,y \cdot y\,\mathrm{R}\,z \cdot x\,\mathrm{R}\,z)$.

[1] B. CHELLAS, ***Modal logic***, Cambridge University Press, 1980.

[2] K. O'HARA, H. REICHGELT, and N. SHADBOLT, *Avoiding omnidoxasticity in logics of belief*, ***Notre Dame Journal of Formal Logic***, vol. 36 (1995).

► CYRUS F. NOURANI, *Functorial models and infinite language categories*.
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The abstract presents Infinitary Language Categories, INFLCs, and shows their preliminary properties. INFLCs are not the same as a categorical interpretation for logic in the categorical logic sense. Thus INFLCs have their own theoretical properties and application areas defined by the present author since [5]. We define a model theory for functors which are really Logische Syntax der Sprache on an Infinitary language $\mathrm{L}\,\omega 1, \omega$. Starting with a well-behaved countable fragment of an infinitary language $\mathrm{L}\,\omega 1, \omega$ defined by H. J. Keisler [1]. The term fragment is not inconsistent with the terminology applied in categorical logic. Let us refer to the above fragment by $\mathbf{L}\boldsymbol{\omega}\mathbf{1}, \mathbf{B}$. Arrow diagrams, as in commutative diagrams are referred to by A-Diagrams. To model-theoretic diagrams we refer to by M-Diagrams. The G-diagrams are algebraic M-diagrams, defined by this author around the 1980's, to be diagrams definable by a specified function set, e.g., $\Sigma 1$ Skolem functions. Define a small-complete category $\mathbf{L}\boldsymbol{\omega}\mathbf{1}, \mathbf{B}$ from $\mathrm{L}\,\omega 1, \mathrm{B}$. The category is the preorder category defined by the formula ordering defining the language fragment.

(A) Talking Revers To A Set. We take the Op category formed by $\mathbf{L}\boldsymbol{\omega}\mathbf{1}, \mathbf{B}$. Present a functor to take the Logische def Sprache from this category to the category **Set**, creating a limit corresponding to a model for the language defined from the syntax.

LEMMA. $\mathbf{L}\boldsymbol{\omega}\mathbf{1}, \mathbf{B}$ *is a small-complete category.*

A functor $F : \mathbf{L}\boldsymbol{\omega}\mathbf{1}, \mathbf{B}^{\mathrm{Op}} \longrightarrow \mathbf{Set}$ is defined by a list of sets Mn and functions fn.

(B) Language categories, not categorical interpretations. A second distinguishing part of our mathematics is that instead of having a direct interpretation for the Infinitary logic as categorical logic has practiced, we define an embedding from a model definable in $\mathbf{D}\langle\mathbf{A}, \mathbf{G}\rangle$—the category of models definable with generalized diagram $\mathbf{D}\langle\mathbf{A}, \mathbf{G}\rangle$, presented by this author in [4], similar to the diagrams defining generic models for set theory, to the limit created from the functor from the Infinitary language category $\mathbf{L}\boldsymbol{\omega}\mathbf{1}, \mathbf{B}$ to **Set**. Creating limits amounts to defining generic sets on $\mathrm{L}\,\omega 1, \omega$ for $\mathbf{L}\boldsymbol{\omega}\mathbf{1}, \mathbf{B}$, since the present theory creates limits for Infinitary language sets to define models. These techniques had been applied by this author to define Positive Forcing [3]. Let us refer to the above functor by the generic model functor since it defines generic sets from language strings to form limits and thus models.

THEOREM. *The generic model functor defines a generic model.*

The techniques were defined by this author this year to define new computing theories and to present a solution set theorem for category theory [4, 5].

[1] H. J. KEISLER, ***Model theory for infinitary logic***, North-Holland.

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