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IN MEMORIAM: ROBIN OLIVER GANDY
1919–1995

Although Robin Gandy's career in mathematical logic might seem to have started with the appearance of "On the Axiom of Extensionality I" in THE JOURNAL OF SYMBOLIC LOGIC in 1956, his interest in the subject commenced through his wartime aquaintance with Alan Turing as far back as 1944. His twin interests in the foundations of science and mathematics were fostered by Turing. These interests moulded a career which always seemed larger than his publication list, although that grew in diversity with the years to include some seminal papers. Initially, he read widely and published little, but around 1960 he started to produce the major results which made him an international figure in the theory of abstract recursion and in effective, descriptive set theory. There were also major conceptual inovations such as the theory of inductive definitions, and a plethora of ingenious intricate proofs in different corners of the subject. He wrote papers on many areas of logic and was much in demand as a reviewer of books on the foundations of mathematics and science. He remained active and productive and was a familiar figure at international conferences until his sad and unexpected death (from an aortic rupture) on November 20th, 1995.

He was born on September 22nd, 1919, in Peppard, Oxfordshire, where his father, Thomas Gandy, was in general practice. His mother Ida Gandy earned a reputation for a sequence of books based on her early life in Wiltshire. Educated at Abbotsholme, a progressive public school, he went on to join that special élite at King's College, Cambridge, intellectual home of such famous names as E. M. Forster the novelist, J. Maynard Keynes the economist – and Alan Turing. He met Turing at a party in 1940 (Gandy's graduation year) but their early wartime careers were separate. Gandy was commissioned in the Royal Electrical and Mechanical Engineers and became an expert on military radio and radar, whilst Turing joined the cryptanalysts at Bletchley Park. They were brought together at Hanslope Park in 1944 to work on a speech decipherment system, christened 'Delilah' at Gandy's suggestion because it was intended to be a "deceiver of men".

Their friendship continued after the war, back at King's College where Turing had resumed his fellowship. Gandy took Part III of the Mathematical Tripos with distinction, then began studying for a PhD under Turing's supervision; his successful thesis on the logical foundation of physics ("On axiomatic systems in mathematics and theories in physics") was presented in

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1953 and can now be seen as a bridge between his wartime expertise and later career. When Turing died in 1954 he left his mathematical books and papers to Gandy, who, in 1963, took over from Max Newman the task of editing the papers for publication. That it became necessary in time to appoint four more editors illustrates the size and diversity of the task.

Between 1950 and 1961 Robin held lectureships in Applied Mathematics at first in Leicester then Leeds. During this period his commitment to logic evolved, and he developed a mathematics–philosophy course at Leeds with Martin Löb. In 1961, he moved to Manchester where the seemingly retiring but extremely astute Max Newman had, with James Lighthill, built up what was for a while perhaps the best mathematics department in Britain. Newman had worked on the Colossus project during the war, had brought Turing to Manchester in 1948, and now invited Gandy to develop logic and start up a mathematics–philosophy course. At last officially a logician, Gandy appointed new staff and invited many visitors from abroad. He was promoted to a readership in 1964 and a chair in 1967; he organised the European summer meeting of the Association for Symbolic Logic in Manchester in 1969, supported, as was usual then, by NATO funds. Turing had gently chided Robin in 1940 for his left-wing beliefs; now, ironically, he came to be attacked as right-wing for his support of NATO funding.

Gandy was a visiting associate professor at Stanford University from 1966 to 1967, and held a similar post at UCLA in 1968. During this period he was a member of the Council of the ASL. In 1969 he gave up his chair in Manchester for a readership in Mathematical Logic at Oxford, where he was to be based for the rest of his life. He was adopted by the young Wolfson College and soon had rooms in the college's fine new building. He occasionally complained about the 'tedious beat of heavy metal' from some other room but generally found college life very congenial. He was responsible for the mathematics–philosophy course, and with John Shepherdson from Bristol brought the British Logic Colloquium into being. Dana Scott was appointed to a new chair of mathematical logic at Oxford in 1972, Michael Dummett succeeded Sir Alfred Ayer to the Wykeham chair of logic in 1979 and Ronald Jensen moved to All Souls College in 1981: mathematical logic really came into its own in Oxford in those years and Robin's list of PhD students grew from three to around thirty.

He 'retired' in 1986 amongst fireworks and a full moon at the University of Wales' retreat at Gregynog in mid-Wales, féted at a conference in his honour by a gathering of international eminences and most of his PhD students; it particularly pleased him that the founder of recursive function theory, Stephen Kleene, was amongst the international figures. After retirement he not unnaturally found fresh energy and produced fine papers on foundational topics such as the nature of computability and ultrafinitism.

Of the three major results which bear Gandy's name, the earliest was first proved by Clifford Spector in 1962; Gandy published a second, simpler proof of it in the same year, a rare testimony to the theorem's significance by someone who generally published so little of his work.

THE SPECTOR-GANDY THEOREM. *A relation $P(n)$ on the set $\mathbb{N}$ of integers is Π^1_1 if and only if it satisfies an equivalence of the form*

$$P(n) \iff (\exists \alpha \in \Delta^1_1) R(n, \alpha),$$

where α ranges over the Baire space $\mathbb{R}$ (of infinite sequences of integers) *and $R(n, \alpha)$ is arithmetical.*

(Spector and Gandy established the "only if" part, the "if" direction having being proved earlier by Kleene.) Although not very difficult, the proof of the Spector-Gandy Theorem depends heavily on the following, classical *Representation Theorem* for Π^1_1 relations: *$P(n)$ is Π^1_1, if and only if there is a recursive function $f : \mathbb{N} :\to \mathbb{R}$ such that:*

(1) *For each n, $f(n)$* (recursively) *codes a linear ordering on $\mathbb{N}$;* and
(2) *$P(n) \iff f(n)$ codes a wellordering.*

It was later extended to several, abstract analogs of "classes of semirecursive relations", including the *inductive relations* (on an arbitrary, "acceptable" structure) and the Π^1_k relations on $\mathbb{N}$ (for odd k, under suitable determinacy hypotheses). Since the Representation Theorem just cited characterizes the Π^1_1 sets, it was not possible to establish these extensions by generalizing directly the proofs of Spector and Gandy, and the search for suitable "approximations" of the Representation Theorem which hold in more general situations (and imply the result, typically by new arguments) proved most seminal in the development of these "abstract recursion theories".

Gandy's second and most substantial contribution was to Kleene's *recursion in type* 2, and was exactly such an approximation to the Representation Theorem. An object $\boldsymbol{F} : \mathbb{R} \to \mathbb{N}$ is *normal*, if the existential quantifier ${}^2\boldsymbol{E}$ is recursive in $\boldsymbol{F}$.

THE GANDY STAGE COMPARISON THEOREM. *If a relation $P(\vec{x})$ with arguments in $\mathbb{N}$ and $\mathbb{R}$ is semirecursive in a normal, type-*2 *object $\boldsymbol{F}$, then there exists an assignment $\vec{x} \mapsto |\vec{x}|$ of ordinal numbers to the members of P, and an $\boldsymbol{F}$-recursive, partial function $\chi(x, y)$, such that:*

(1) *$\chi(x, y)$ is defined $\iff P(x)$ or $P(y)$;* and
(2) (with $|z| = \infty$, if $\neg P(z)$),

$$\chi(x, y) = \begin{cases} 1, & \textit{if } |x| \le |y|, \\ 0, & \textit{if } |y| < |x|. \end{cases}$$

This is one of those approximations to the Representation Theorem for Π^1_1 alluded to above, and its proof is long and complex. It contrasts greatly with the proof of its most significant Corollary, which now follows by a short and sweet Recursion Theorem argument:

THE GANDY SELECTION THEOREM. *If a relation $P(n, \vec{x})$ with arguments in $\mathbb{N}$ and $\mathbb{R}$ is semirecursive in a normal, type-2 object $\boldsymbol{F}$, then there exists an $\boldsymbol{F}$-recursive partial function $\nu(\vec{x})$ such that:*

(1) $(\exists n) P(n, \vec{x}) \iff \nu(\vec{x})$ *is defined;* and

(2) $(\exists n) P(n, \vec{x}) \Longrightarrow P(\nu(\vec{x}), \vec{x})$.

These two results were soon extended to all higher types, and they also inspired numerous generalizations and related theorems which form the core of higher-type and set-recursion. Gandy's basic idea of approaching a specific "abstract recursion theory" by looking first for a Stage Comparison and a Selection Theorem became the standard methodology of the field, and of its offshoots in effective descriptive set theory.

Gandy's third fundamental contribution was the introduction of Σ^1_1-*forcing*, or (equivalently) the *Gandy-Harrington topology*, which is simply the topology on $\mathbb{R}$ generated by the (countably many) Σ^1_1 sets. It is not a very nice topology from the geometrical point of view, but *it satisfies the Baire category theorem*, and it also has several additional regularity properties which make it a very powerful tool. Gandy invented it (probably in 1962) to prove that *every nonempty Σ^1_1 subset of $\mathbb{R}$ has a member with hyperdegree less than the largest Σ^1_1 hyperdegree*, a result he never published. The Gandy-Harrington topology was later used by Harrington, Louveau and Harrington-Kechris-Louveau to establish some deep theorems in descriptive set theory, including the first proofs by "effective" methods of fundamental and (sometimes) new properties of the classical Borel and co-analytic (Π^1_1) sets and equivalence relations on $\mathbb{R}$.

Robin Gandy had immense intellectual and personal qualities and utter dedication to his subject. He was a friend as well as mentor to his endless stream of PhD students, and numbered a wide range of international academics amongst his friends; he would sometimes jokingly refer to them as The Gandy Appreciation Society. A colourful and complex character who would arrive at work in motor-cycle leathers, and later dominate a crowd in the nearest pub with his foghorn voice, plumes of smoke and witty anecdotes, he was a great entertainer and his regular parties for visiting academics were eagerly anticipated events. He was much loved and his humour, enthusiasm, generosity, tolerance, hospitality, kindliness, irreverence, erudition and mouth-watering home-made ice-cream will be sorely missed.

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