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The X Latin American Symposium on Mathematical Logic was held at the Universidad de los Andes in Bogotá, Colombia, on July 24–29, 1995. Four invited short courses preceded the Symposium on July 17–22, at the Universidad Nacional de Colombia. The Symposium was organized by the Mathematics Departments of the Universidad de los Andes and the Universidad Nacional de Colombia. Financial support was received from both sponsoring universities, the Fondo Colombiano de Investigaciones Científicas (COLCIENCIAS), the Instituto Colombiano para el Fomento de la Educación Superior (ICFES), the Instituto Colombiano de Crédito Educativo y Estudios Técnicos en el Exterior (ICETEX), the Academia Colombiana de Ciencias Exactas, Fisicas y Naturales, and the Banco Ganadero. Technical support was received from the Sociedad Colombiana de Matemáticas.

Members of the Program Committee included Xavier Caicedo, Chair (Bogotá), Carlos A. Di Prisco (Caracas), Roberto Cignoli (Buenos Aires), Itala M. L. D'Ottaviano (Campinas), and Renato Lewin (Santiago). The local Organizing Committee consisted of Xavier Caicedo, Luis J. Corredor, Sergio Fajardo, Carlos Montenegro (Universidad de los Andes), José M. Muñoz, Rodrigo De Castro, and Clara H. Sanchez (Universidad Nacional de Colombia).

The Conference was dedicated to the memory of Rolando Chuaqui, who died in Santiago de Chile in March of 1994, in recognition of his tireless support of the development of logic in Latin America. He started the Latin American Logic Symposia in 1970 and participated actively in the organization of all of them. An opening address about his life and work was delivered at the X Symposium by Renato Lewin (Universidad Católica de Chile), and several of his former colleagues and students shared their remembrances of Professor Chuaqui.

Following are descriptions of the invited short courses and abstracts of invited talks, contributed talks, and papers submitted by title.

For the Program Committee
XAVIER CAICEDO, CHAIR

1079-8986/96/0202-0005/$03.30

Descriptions of invited short courses

► ANDREAS BLASS, *Cardinal characteristic of the continuum*.
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LECTURE 1. Combinatorial characteristics of the continuum, including bounding, dominating, splitting numbers, etc. The general framework of Galois-Tukey connections for relating characteristics.

LECTURE 2. Characteristics defined from measure and category. Combinatorial reformulations of these. Cichoń's diagram.

LECTURE 3. The distributivity and groupwise density numbers and applications.

LECTURE 4. Recent developments.

► WILLEM BLOK, *Abstract algebraic logic*.
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"Abstract algebraic logic" can be characterized as the process of reducing logical deduction, in its broadest sense, to equational logic, the particularly simple form of logical deduction that underlies most of modern algebra.

In these lectures we investigate the precise method by which this reduction can be either completely or partially effected. We show how this leads to close connections between logical systems and suitable classes of algebras, enriched with a predicate when the reduction is only partial.

We outline the theory of algebraization in a very general context, illustrating the ideas with numerous examples. Having isolated a precise notion of algebraizable logic, we show how it provides the proper framework for studying algebraic equivalents of metalogical properties as, for example, the property of "possessing a deduction theorem".

► JÖRG FLUM, *Finite model theory*.
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One of the major aims of Finite Model Theory consists in characterizing the queries in a given complexity class by means of a logic in which they can be described. In this way one obtains a new measure of complexity, the complexity of formal descriptions. Moreover, the characterizations allow to translate problems on complexity classes into purely model-theoretic problems, thus opening them to model-theoretic methods.

In general, this relationship between complexity classes and logic turns out to be useful for both complexity theory and logic.

We shall survey the results obtained in Finite Model Theory during the last 20 years.

► ANAND PILLAY, *The model theory of pseudofinite fields and applications to algebraic groups*.
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The basic point is that pseudofinite fields (infinite models of the theory of finite fields), as well as the more general class of perfect PAC fields with "small" absolute Galois group, have a good model theory, even though they are unstable.

There is a well-defined (finite valued) dimension (or rank) function, similar to but different from Morley rank. As a consequence, much of the finite Morley rank group theory can be developed for groups definable in such fields. This gives information about $G(F)$ where F is such a field and G an algebraic group defined over F (such as $G(F)$ is abstractly simple if G is simple and simply connected), as well as facts about maximal subgroups of finite simple groups, as well as new proofs of results in the arithmetic theory of algebraic groups (strong approximation). The results are due to Hrushovski and myself.

Abstracts of invited talks

▶ JUERGEN AVENHAUS AND KLAUS MADLENER, *Theorem proving in hierarchical conditional specifications*.
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Among the different specification languages used to describe software systems on the functional design level, the method of algebraic specifications has been intensively studied and used. Such a specification language should have enough expressive power, have a well-defined meaning (semantics) and be appropriate for proving properties of the specification. The use of pure equational specifications turned out to be inadequate in some of these aspects.

We present a language based on clausal specifications over a built-in algebra $\mathcal{A}$. A specification *spec* is a set of positive/negative conditional equations which define new (partial) operations on $\mathcal{A}$. The intended semantics is described by associating with *spec* and $\mathcal{A}$ a canonical algebra $\mathcal{A}_{spec}$, which, in order to capture partiality of functions, is defined in the setting of order sorted signatures and algebras. Notice that, since we allow negations in the conditions, the usual initial model semantics does not work and the notion of evaluating negative conditions has to be defined properly. We use the principle *negation by constructive foundation in* $\mathcal{A}$.

The corresponding algebra $\mathcal{A}_{spec}$ has properties similar to those of the initial model of Horn-clause specifications, i.e., it is a minimal model which is in fact the minimum of all models that do not identify more constructor ground terms than necessary. We present techniques for proving consistency of $\mathcal{A}_{spec}$ with respect to $\mathcal{A}$. Formulae which are valid in $\mathcal{A}_{spec}$ are called inductive theorems. An inference system for proving clauses to be inductive theorems is provided. The system allows one also to disprove assertions.

▶ JOAN BAGARIA, *Forcing axioms and generic absoluteness*.
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We present some results which relate forcing axioms with some forms of absoluteness under generic extensions. In particular, we show that Martin's Axiom is equivalent to absoluteness under ccc forcing extensions for Σ_1 formulas with parameters whose transitive closure is of size less than the continuum.

▶ ANDREAS BLASS, *Linear logic and games*.
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I plan to describe linear logic and several versions of game semantics for it. Topics will include the original version of game semantics (complete for additive linear logic); work of

Abramsky, Jagadeesan, Hyland, and Ong on completeness for multiplicative linear logic; Joyal's generalization of games based on category-theoretic considerations; and my efforts to obtain game semantics in a natural way from linear logic itself and its intuitive meaning.

► WILLEM BLOK, *Ideals in universal algebra*.
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In many of the familiar classes of algebras the congruences of an algebra are adequately represented by special subsets of the algebra; we call these classes *ideal determined*. This is true in classical algebras such as groups and rings, where normal subgroups and ideals respectively determine the congruences, as well as in the algebras of logic, such as Boolean algebras and Heyting algebras, where the ideals and filters respectively play the same role.

A notion of ideal for algebras with 0 was proposed by Ursini in the early seventies, and a considerable theory around the notion has developed since.

We propose a notion of ideal for arbitrary quasi-varieties of algebras, guided by insights obtained in the work on algebraization of deductive systems, and present a characterization of the ideal determined quasi-varieties.

We show that the class of varieties with 0 for which the ideals in the sense of Ursini had been shown to behave properly is exactly the class where that notion coincides with ours.

► JOSÉ CARLOS CIFUENTES, *Cauchy completion in abstract model theory*.
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In 1989, X. Caicedo directed a seminar on uniformly continuous operations in abstract model theory at the University of Campinas, exploiting the fact, already noticed by Fraïssé for first order logic, that the elementary topology defined by a logic in the space of structures is uniformizable. There arose the idea of studying the logical content of Cauchy completeness and completion of those spaces. A logic is compact if and only if its spaces of structures are Cauchy complete. By Łos' theorem, ultraproducts correspond to limits of Cauchy nets in the spaces of structures of first order logic, a fact verified by D. Mundici, A. M. Sette and the author, and also independently and simultaneously by X. Caicedo. In our doctoral dissertation we solve the problem of the completion (hence, the compactification) of an abstract logic, via a topological version of Łos' theorem, by introducing ideal structures that generalize ultraproducts. However, this is a partial solution from a logical viewpoint because the semantics for these generalized models deal only with the notion of "truth" of sentences, leaving unexplained the wider notion of "satisfaction" of formulae, a nontrivial problem which consists in finding the ontological domain of the variables with respect to the new models. In this talk, we explain some aspects of our work and sketch the semantics for the existential quantifier and the consequent notion of "satisfaction", from the functorial properties of the constructed compactification.

[1] X. Caicedo, *Continuous operations on spaces of structures*, in ***Quantifiers: logics, models and computation***, (M. Krynicki, M. Mostowski and L. Szczerba, editors), Kluwer Academic Publishers, 1995, pp. 263–296.

[2] ———, *Compactness of topological spaces of models*, Relatório de Pesquisa RP 64/92 IMECC–UNICAMP, 1992.

[3] J. C. Cifuentes, *The problem of compactification in model theory*, Ph.D. Thesis, IMECC–UNICAMP, 1993.

[4] ———, *Compacidad y compactificación en teoría de modelos*, ***Proceedings of the IX***

Latin American Symposium on Mathematical Logic, Bahía Blanca, Argentina, Notas de lógica matemática, vol. 39 (1994), pp. 71–88.

[5] ———, *Cauchy completeness of Stone spaces in Boolean algebras*, ***Proceedings of the X Brazilian Conference on Mathematical Logic***, Coleção CLE, UNICAMP, vol. 14, 1995, pp. 93–105.

[6] J. C. CIFUENTES, D. MUNDICI, and A. M. SETTE, *Cauchy completeness in elementary logic*, to appear in ***The Journal of Symbolic Logic***.

▶ ROBERTO CIGNOLI, *Natural dualities for the algebras of Łukasiewicz finitely-valued logics*.

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Let $n \in \omega$, $n \geq 2$. We call n-valued MV-algebras the algebras in the subvariety of MV-algebras generated by the n-element chain $\mathbf{L}_n$. They are the algebraic counterpart of Łukasiewicz n-valued propositional calculus.

An n-valued Boolean space is a pair $\langle X, h\rangle$, such that X is a Boolean space and h is a meet-homomorphism from the lattice of divisors of $n-1$ into the lattice of closed subsets of X, sending $n-1$ into X. A *morphism* from $\langle X, h\rangle$ into $\langle Y, g\rangle$ is a continuous function $\varphi\colon X \to Y$ such that $\varphi^{-1}\big(g(j)\big) = h(j)$ for each divisor j of $n-1$.

THEOREM 1. *For each n-valued MV-algebra A, there is a unique (up to isomorphisms) n-valued Boolean space $\langle X, h\rangle$ such that A is isomorphic to the n-valued MV-algebra formed by the continuous functions $f : X \to \mathbf{L}_n$ such that $f\big(h(j)\big) \subseteq \mathbf{L}_{j+1}$, for each divisor j of $n-1$, with the MV-operations defined pointwise. The space X is homeomorphic to the Stone space of the maximal Boolean subalgebra of A.*

THEOREM 2. *The category of n-valued MV-algebras is naturally equivalent to the opposite of the category of n-valued Boolean spaces.*

▶ LUIS JAIME CORREDOR, *Conjugacy of the p-Sylow subgroups in some local subgroups of groups of finite Morley rank*.

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(Joint work with Alexander Borovik.)

Borovik and Poizat proved that the 2-Sylow subgroups of a group of finite Morley rank are locally finite and conjugated. The same statement should be also true for the p-Sylow subgroups for p arbitrary prime, but this is an open problem.

Let H be the intersection of two p-Sylow subgroups of G, a group of finite Morley rank. Under the basic assumption that in all definable sections of G the p-Sylow subgroups are locally finite and conjugated, we proved that the p-Sylow subgroups of $N_G(H)$ are conjugate. Note that $N_G(H)$, which is a p-local subgroup of G, is not necessarily definable. We show how this result can be used to give a generalization of the Alperin-Goldschmidt theorem to groups of finite Morley rank.

▶ J. CZELAKOWSKI AND DON PIGOZZI, *Interpolation and amalgamation in abstract algebraic logic*.

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The correlation between interpolation theorems of logic and certain properties of the class of models related to the amalgamation property is well known. In classical sentential and first-order logic it takes the form of a correspondence between Craig's interpolation theorem and Robinson's joint consistency lemma. In the algebraic versions of these logics the joint consistency property can be replaced by the amalgamation property for Boolean algebras and locally finite cylindric algebras. The connection between interpolation and amalgamation have also been explored in the context of intermediate and modal logics, equational logic, and general deductive systems in the sense of Tarski. More recently, logical interpolation results have been shown to have interesting applications for the specification of abstract data types, especially with regard to the important problem of modularization.

In the present paper we take advantage of the new domain of abstract algebraic logic to present a unified theory of interpolation, joint consistency, model extension, and amalgamation that comprehends all these results.

In these general context the Craig interpolation property ramifies into several different interpolation-like properties, one of which is closely related to the familiar congruence extension property of universal algebra.

We show how under quite weak conditions on the logical system, each interpolation property is equivalent to an extension or amalgamation-type property of the appropriate model class.

► JAIRO J. DA SILVA, *Translations between logics*.

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We characterize logics, in the most general sense, as sets with closure operators, and translations between logics as continuous functions between such closure systems. This paper presents some preliminary results in the theory of logical translations characterized in this way.

► RODRIGO DE CASTRO, *General combinatorial principles in second order theories of bounded arithmetic*.

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An attempt is made to study the mathematical strength of the weak second order theories of Bounded Arithmetic U_2^i and V_2^i, $i \geq 0$, introduced by S. Buss. It is first shown that U_2^1 can $\Sigma_1^{1,b}$-define the functions in the second class of Grzegorczyk, $\mathcal{E}^2$, or, equivalently, the class of $\Sigma_1^{1,b}$-definable functions in U_2^1 is closed under bounded recursion.

It is shown next that U_2^1 proves the $\Delta_1^{1,b}$-pigeonhole principle. Two general combinatorial principles, the $\Delta_1^{1,b}$-partition principle and the $\Delta_1^{1,b}$-equipartition principle, are obtained from it thereby demonstrating that the $\Delta_1^{1,b}$-PHP embodies a strong notion of cardinality. The introduction of these principles is motivated with some examples, notably by showing that Euler's theorem is probable in U_2^1. The provability of Euler's theorem in weak first order fragments of Peano Arithmetic is an open problem.

A theory of polynomials can be developed in U_2^1 and it is proved that $U_2^1 + B \vdash$"existence of primitive roots" where formula B asserts that a nontrivial polynomial of degree n can have at most n solutions modulo p if p is a prime. By an essential use of the $\Delta_1^{1,b}$-PHP and the $\Delta_1^{1,b}$-partition principle it is shown that $V_2^1 \vdash B$, hence $V_2^1 \vdash$"existence of primitive roots".

▶ MAX DICKMANN AND F. MIRAGLIA, *Boolean methods in the theory of quadratic forms*.
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(Presented by M. Dickmann.)

We introduce the notion of special group. These structures constitute a simple, first-order axiomatic framework to develop an abstract theory of quadratic forms, encompassing the usual algebraic theory over fields of characteristic not 2. Boolean algebras give rise to reduced special groups in a natural way. Furthermore, every reduced special group is canonically embedded in a Boolean algebra, its Boolean hull. This construction makes it possible to investigate the category of reduced special groups, and various complete first-order theories of reduced special groups. More importantly, using it we define two complete (and manageable) systems of invariants for isometry. One of them resembles Milnor's Stiefel-Whitney invariants, but has even better properties; the other has no analogue in quadratic form theory over fields. Several interesting computations can be carried out by use of these systems of invariants.

The category of reduced special groups and of abstract spaces of orders are related by a duality which generalizes the classical Stone duality.

▶ CARLOS AUGUSTO DI PRISCO, *Polarized partition relations and ultrafilters on ω*.
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A survey of results on partitions of the space of sequences of natural numbers will be presented.

Most of the partition properties considered follow from the property $\omega \to (\omega)^\omega$, but for some of them it is not known if the reverse implication holds. The problem of the consistency of these partition relations with the existence of ultrafilters on ω is related to this question.

Some parameterized versions of these partition relations will also be considered. Relations with the perfect subset property will be mentioned.

▶ ITALA M. LOFFREDO D'OTTAVIANO, *What is a translation?*.
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In general, a *logic* is a set A of *structured* objects (the formulae of a language) together with a binary operation $R \subseteq P(A) \times A$, called the (syntactic or semantic) consequence relation. By a *translation* between logics we intuitively understand a map between the sets of objects of these logics that preserves "something". It seems that we have two possible "natural" choices for what this "something" must be. On the one hand, we could require that a translation should preserve the structure of the objects and see how these structure-preserving maps behave with respect to the consequence relation. On the other hand, we could require that a translation preserves the consequence relation regardless of its action on the structure of the objects, and consider as a special case translations that are also structure-preserving.

A simple glance at the history of the concept of logical translation is enough to convince us that translation have usually been thought of as consequence preserving maps. Originally, in the works of Kolmogoroff (1925), Glivenko (1929), Gödel (1933) and Gentzen (1933), the method of translating one logic into another (in general, classic into intuitionalistic logic) was developed in order to show relative consistence of the first with respect to the second, hence translations were defined as consequence preserving maps in such a way that inconsistencies in one logic would be translated into inconsistencies in the other logic.

In this paper we characterize logics, in the most general sense, as sets with closure operators. We propose that translations between logics are functions which preserve such consequence relations, that is, translations are continuous functions between the closure systems.

► ALI ENAYAT, *The significance of models of set theory and arithmetic for general model theory*.

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Besides its intrinsic interest, the study of models of arithmetic and set theory sheds light on many general results in model theory. For example, one can easily derive the countable compactness and "abstract completeness" of the logic $L(Q)$ (where Q is the quantifier "there exists uncountably many") from the well-known Keisler-Morley result on end extensions of models of set theory; or derive the corresponding result about $L(aa)$ (where aa is the "stationary quantifier") from Hutchinson's strengthening of the Keisler-Morley result.

More impressively, models of set theory and arithmetic have proved indispensable in the proofs of certain results in general model theory. Such cases include the crucial use of nonstandard models of arithmetic in Lindstrom's characterization of first order logic, and the key role played by models of set theory in the proofs of transfer theorems in model theory.

In this talk we present a survey of the aforementioned interconnections, including a recent result of the author concerning a new transfer theorem in the constructible universe, answering a question left open by the work of Schmerl and Shelah.

► JÖRG FLUM, *Model theory of fixed-point logics*.

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In finite model theory various fixed-point logics have been introduced in order to capture complexity classes. We present some results of their "infinite model theory".

► SY D. FRIEDMAN, *Fine structure and class forcing*.

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In this talk I will discuss aspects of my forthcoming book on the fine structure theory for L and the theory of class forcing. A modified Levy hierarchy can be used to give a smoother treatment of fine structure theory and we use it to establish square, morass with square, fine scales and gap-2 morasses in L. Class forcings do not in general preserve ZF and we isolate the condition of Tameness required for this to be the case. The four basic examples of tame class forcings are: Easton, Long Easton, Reverse Easton and Amenable forcing.

We then explore the question of generic class existence. L is rigid in class forcing extensions of L and we introduce $L[0^\#]$ as the least inner model in which L is not rigid. A class forcing is Relevant if it has a generic definable over $L[0^\#]$. Of the basic examples, Easton and Long Easton are relevant only if one avoids inaccessibles whereas Reverse Easton is relevant without this restriction.

Next comes the method of Jensen Coding which utilizes the fine structure theory introduced earlier. There are 2 proofs given, one assuming $-0^\#$, and the other more intricate but without the $-0^\#$ hypothesis. Jensen Coding is of the Long Easton variety and is thereby shown to be relevant.

We use class forcing to answer 3 questions of Solovay: If a real does not construct $0^{\#}$ then must it be generic over L? Is there a Π^1_2-*Singleton* strictly between 0 and $0^{\#}$ in L-degree? Is there a real R such that the R-admissibles are exactly the recursively inaccessible ordinals? The first question is answered immediately by Jensen Coding if generic is taken to mean set-generic; otherwise hyperclass forcing is used. The second question requires a mixture of Reverse Easton and Jensen Coding techniques. The third question uses a refinement of Jensen Coding called Strong Coding.

Finally, there are other applications of class forcing to descriptive set theory and to other notions of set-theoretic definability.

► MAX GARZÓN, *Symbolic logics in a new AI: new challenges, new perspectives*.
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In the beginning, logic arose as an attempt to understand, and later mathematically formalize, notions of apparent importance to thought processes such as deduction, inference, and 'objective' truth. Over a century later, we have amassed what amounts to a great body of knowledge about the nature and limitations of formal systems as tools in the understanding of these processes.

In the meantime, cognitive psychology, computer science, and AI have emerged and evolved as disciplines of their own. Artificial neural nets and connectionism, for example, are now well established models of a variety of cognitive phenomena in information processing systems. They have made patent a number of shortcomings of classical approaches and brought to the fore a number of more fundamental issues: in the adequacy of symbolic representations, the grounding problem, the learning and adaptive nature of cognitive systems, the vexing mind-body problem (genesis of intelligence from nonintelligent parts), among others. In this talk, we will attempt to articulate why these and other issues should be of central importance to a new logic, more on-line with recent developments directly and fundamentally concerned with the genesis and nature of thought processes, collective intelligence, and the emergence of objective logics from subjective cognitive systems.

► HECTOR GRAMAGLIA AND DIEGO VAGGIONE, *Birkhoff-like sheaf representations.*
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Krauss and Clark [3] define a subdirect product $A \subseteq \prod\{ A_i : i \in I \}$ to be *global* if there exists a topology τ on I such that

(1) τ contains the *equalizer topology on* I, i.e., the topology generated by the sets $E(x, y) = \{ i \in I : x(i) = y(i) \}$, $x, y \in A$; and

(2) If $x \in \prod\{ A_i : i \in I \}$ and $E(x, y) \in \tau$, for every $y \in A$, then $x \in A$.

Global subdirect products of mutually disjoint algebras are exactly the algebras of global sections of sheaves [3]. Given a variety of lattice ordered structures, we are interested in the existence of a class $\mathcal{F}$ with the following Birkhoff-like sheaf representational properties:

S1. *Every algebra A in the variety can be represented as* (*isomorphic to*) *a global subdirect product with factors in* $\mathcal{F}$.

S2. *Every nontrivial* $A \in \mathcal{F}$ *is globally indecomposable.*

In [1], [2] and [5] we showed that the following varieties (and its subvarieties) have a class $\mathcal{F}$ satisfying properties S1 and S2:

(1) distributive lattices.

(2) almost p-lattices.
(3) distributive double p-algebras of a finite range.
(4) semisimple varieties of lattice expansions such that the simple members form a universal class (bounded distributive lattices, De Morgan algebras, etc.).
(5) arithmetical varieties in which the finitely subdirectly irreducible algebras form a universal class (f-rings, vector groups, Wajsberg algebras, discriminator varieties, Heyting algebras, etc.).
(6) pseudocomplemented De Morgan algebras of a finite range.

As subproducts of our developments we obtain new characterizations of the (finitely) subdirectly irreducibles for the cases (2), (3) and (6), and for the cases (2), (3) and (4) we obtain results analogous to that of Nachbin saying that if every chain of prime filters of a bounded distributive lattice has at most length 1, then the lattice is Boolean [4].

► LAURI HELLA, *Implicit definability on finite structures*.
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The classical Beth definability theorem asserts that on the class of all finite and infinite structures, first-order implicit definitions can be replaced with explicit definitions. On the other hand, it has been known for a while that Beth's theorem fails when restricting to finite structures; a simple counterexample was given by Gurevich (1984). This failure can be seen as a negative result concerning the behaviour of first-order logic on finite structures. However, thinking positively it just indicates that $IMP(FO)$, the class of all queries on finite structures that are implicitly definable in first-order logic, is a proper extension of first-order logic and hence worth studying in its own right.

The systematic study of first-order implicit definability on finite structures was initiated by Kolaitis (1990). Kolaitis showed that $IMP(FO)$ is indeed quite interesting from the point of view of expressive power, since it subsumes least fixpoint logic. Moreover, he delineated the computational strength of $IMP(FO)$ by showing that, in the presence of linear order, $IMP(FO)$ captures the complexity class $UP \cap \text{co } UP$. Here UP refers to the class of all queries that are computable by nondeterministic polynomial-time Turing machines with at most one accepting computation path for each input.

In this talk, I will give a survey of some recent work on implicit definability in the context of finite model theory. In particular, I will discuss the intimate relationship between implicit definability, various fixpoint logics and infinitary logic with finitely many variables. Another topic that I will consider is the problem of implicit definability of linear order on classes of finite rigid structures.

► C. WARD HENSON, *Model theory for structures based on Banach spaces*.
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The main purpose of this talk is to explain how model theory can be made an effective tool for handling structures which come from functional analysis, topology, and geometry. By "model theory" we mean not just the use of saturated models (as done, for example, in Nonstandard Analysis) but also the use of more delicate aspects of model theory such as the omitting types theorem, quantifier elimination, stability, etc. The main innovation in the logic discussed in this talk is the introduction of an *approximate satisfaction relation* $\models_A$ and the restriction of the language to the class of *positive bounded formulas*. (These are the formulas in which only positive occurrences of the basic atomic formulas are allowed and in which each quantifier is restricted to range over one of the bounded balls of the underlying metric space.) The structures to which the logic applies are metric spaces equipped with additional named elements, operations, and real valued relations, which must be uniformly continuous on bounded sets. The properties of this logic have proved to be analogous to those of ordinary first order logic, and the pursuit of this analogy has led to a number of interesting developments already. Earlier discussions of this logic for Banach space structures can be found in [3, 4, 5, 6]; a full exposition will be given in [2] and a sequence of papers. In this talk, the emphasis is on the omitting types theorem and an application of it. The application discussed is the following strong homogeneity result for the Banach spaces L_p $(1 \leq p < \infty,\ p \neq 4, 6, \dots)$ and for the Gurarii space [1]:

THEOREM. *Let X be the Gurarii space or L_p for some p satisfying $1 \leq p < \infty,\ p \neq 4, 6, 8, \dots$. Fix $n \geq 1,\ \varepsilon > 0$ and $B > 0$. There exists $\lambda = \lambda(n, \varepsilon, B) > 1$ such that for any $a_1, \dots, a_n, b_1, \dots, b_n \in X$ all having norm $\leq B$ and with $(a_1, \dots, a_n)$ being λ-equivalent to $(b_1, \dots, b_n)$, there exists a linear isometry T of X onto itself such that $\|b_j - T(a_j)\| < \varepsilon$ holds for all $1 \leq j \leq n$.*

The conclusion of this result is true for any separable Banach space X which is separably categorical and admits quantifier elimination; indeed, the conclusion of the Theorem is true *precisely* for such X. Except for the Hilbert space case $(p = 2)$, the Theorem gives the strongest form of isometric homogeneity which these spaces can possess.

[1] V. I. GURARII, *Spaces of universal disposition, isotopic spaces and the Mazur problem on rotations of Banach spaces*, ***Sibirskii Mat. Zhurnal***, vol.7 (1966), pp. 1002–1013. (Russian; translated in Siberian Math. Journal.)

[2] C. W. HENSON and J. IOVINO, ***Banach space model theory I: Basics***, a lecture notes monograph in preparation.

[3] ———, *Nonstandard hulls of Banach spaces*, in Proceedings of the Abraham Robinson Memorial Conference, Yale University, May 1975, ***Israel Journal of Mathematics***, vol. 25 (1976), pp. 108–143.

[4] C. W. HENSON and S. HEINRICH, *Banach space model theory II: Isomorphic equivalence*, ***Mathematische Nachrichten***, vol. 125 (1986), pp. 301–317.

[5] C. W. HENSON and L. C. MOORE, JR., ***Nonstandard analysis and the theory of Banach spaces***, Lecture Notes in Mathematics, Number 983, Springer-Verlag, Berlin, 1983, pp. 27–112.

[6] ———, *The Banach spaces $\ell_p(n)$ for large p and n*, ***Manuscripta Math.***, vol. 44 (1983), pp. 1–33.

▶ JOSÉ IOVINO, *Stability in functional analysis*.
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We consider structures from functional analysis which are based on a Banach space. These include rich classes of structures such as Banach spaces, Banach lattices, C^*-algebras, and Hilbert spaces with operators. In recent years, the model theoretical analysis of these structures has led to the development of a rather powerful theory.

In this talk, we will focus on structures that are *stable* in the model theoretic sense.

We shall show that some familiar classes of structures are stable, and discuss some of the consequences of stability in analysis.

▶ RENATO A. LEWIN, IRENE F. MIKENBERG, AND GLORIA SCHWARZE, *Annotated logics are weakly congruential*.
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Annotated logics were introduced by V. S. Subrahmanian as logical foundations for computer programming. One of the difficulties of these systems from the logical point of view is that they are not structural, i.e., their consequence relations are not closed under substitutions. In [1] we give systems of annotated logics that are equivalent to those of Subrahmanian in the sense that everything probable in one type of system has a translation that is provable in the other. Moreover these new systems are structural. In this report we prove that these systems are *weakly congruential*, namely, they have an infinite system of congruence 1-formulas. Moreover, we prove that if the lattice of annotation constants is finite, the corresponding annotated logic is algebraizable (i.e., it has a finite system of congruence formulas).

[1] R. A. Lewin, I. F. Mikenberg, and M. G. Schwarze, *On the algebraizability of annotated logics*, manuscript submitted to ***Studia Logica***.

► RENATO A. LEWIN, IRENE F. MIKENBERG, AND GLORIA SCHWARZE, *Matrix semantics for an extension of $P\tau S$*.
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We extend the annotated logics PτS defined in [2] to systems SAL and define matrix semantics for which these new systems are complete.

[1] N. C. A. da Costa, V. S. Subrahmanian, and C. Vago, *The paraconsistent logics $P\tau$*, ***Zeitschrift für Math. Logic***, vol. 37 (1991), pp. 139–148.
[2] R. A. Lewin, I. F. Mikenberg, and M. G. Schwarze, *On the algebraizability of annotated logics*, submitted.

► JIMENA LLOPIS AND GISELA MÉNDEZ, *Minimising uncertainties in extended logic programs*.
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Categorical general logic programs provide a powerful tool for knowledge representation in situations which warrant the use of the closed world assumption. As argued in [2], since every ground query is answered *yes* or *no*, they do now allow one to represent incomplete knowledge about the world. Extended Logic programs (ELP's) [2] deal with this allowing a third possibility—the *unknown* answer. In the former paper, ELP's contain a second type of negation $\neg$ ("classical" negation) in addition to negation-as-failure not. ELP's use the answer sets semantic, and a literal l is true if and only if it is in every answer set of the program. If neither p nor $\neg p$ are true then we say that p is unknown. The possibility of having the *unknown* answer is very useful in knowledge representation, as it is very well discussed in [2].

We want to argue that in some cases the answer sets as defined in [2] are giving the unknown answer to literals l that are really being presumed to be true in order to obtain some of the conclusions.

Consider the following example (the penguin example): $b \leftarrow$; $f \leftarrow b$, $\mathsf{not}\ p$; $\neg f \leftarrow b, p$. The only answer set for this program is $\{b, f\}$. Note however that in order to conclude f we are assuming that the bird (b) is not a penguin $(\neg p)$. This in general cannot be obtained with the closed world assumption since in a symmetric example such as $b \leftarrow; f \leftarrow b, \mathsf{not}\,\neg p;$ $\neg f \leftarrow b, \neg p;$ we would be presuming p.

We propose here a notion of "generalized" answer set which will distinguish between "true" uncertainties and presumptions. This idea has strong connections with the 2-valued semantics of [1] and will allow us to sanction in the above examples b, f, $\neg p$ and b, f, p respectively, without losing efficiency.

[1] H. GEFFNER, J. LLOPIS, and G. MENDEZ, *Sound and efficient nonmonotonic reasoning*, Proceedings of IJCAI 95, Morgan Kauffman Publishers, pp. 1495–1501.

[2] M. GELFOND and V. LIFSCHITZ, *Logic programs with classical negation*, ***Logic Programming***, (D. Warren and Pete Szerdi, editors), Proceedings of the seventh international conference, 1990, pp. 579–597.

▶ JORGE LOBO AND CARLOS UZCÁTEGUI, *Abductive change operators*.

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Change operators are operators that transform a theory K into a new theory in order to assimilate new incoming information. Take for example revision. Revision, given a theory K (for instance written in a propositional language $\mathcal{L}$) and a formula α, modifies K in order to incorporate α. In other words, the operator produces a theory $K \circ \alpha$ where $\alpha \in K \circ \alpha$ and $K \circ \alpha$ is "similar" to K (i.e., the changes to K are "minimal" and consistency is preserved). Alchourrón, Gärdenfors, and Makinson [1] have proposed a set of postulates (known as the AGM postulates) that an operator $\circ$ should satisfy in order to be considered a rational change operator. This is the best known formalism for the study of Belief revision. Other common change operators are contractions, expansions [1] and updates (introduced by Katsuno-Mendelzon).

In [3] we introduce new types of operators that use abduction as the basic reasoning mechanism to modify K. The basic idea is that in order to incorporate α we first look for an "explanation" γ of α and is γ and not α what is assimilated into K. The notion of explanation is defined with respect to a background theory Σ (called the domain theory) and in terms of a preferred collection of atoms (called abducibles). Thus γ is considered an abductive explanation of α if $\alpha \in K \circ (\Sigma \wedge \gamma)$ (where $\circ$ is an AGM operator). Given a selection function F that picks a formula from a set of explanations for α, we can define $\circ_a$ by $K \circ_a \alpha = K \circ \big(\Sigma \wedge F(\alpha)\big)$. When the selected explanation is the cautious explanation then these types of operators are characterized by a set of postulates similar to the AGM postulates. In fact, for the case of revisions, there is no need to refer to the AGM operator $\circ$, the abductive revision operators $\circ_a$ can be defined axiomatically. The axioms for the abductive operators have the AGM postulates as a limiting case (when every atom is abducible and the domain theory is empty).

We also present a representation theorem that characterizes abductive revisions in terms of a total pre-order over the collection of interpretations of the language (as in [2]).

[1] C. E. ALCHOURRÓN, P. GÄRDENFORS, and D. MAKINSON, *On the logic of theory change*, ***Journal of Symbolic Logic***, vol. 50 (1985), pp. 510–530.

[2] H. KATSUNO and A. O. MENDELZON, *Propositional knowledge base revision and minimal change*, ***Artificial Intelligence***, vol. 52 (1991), pp. 263–294.

[3] J. LOBO and C. UZCÁTEGUI, *Abductive change operators*, submitted for publication.

► E. G. K. LÓPEZ-ESCOBAR, *A continuation of Gentzen's N-systems*.
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Most Natural Deduction formalizations of logical systems are based on Gentzen's observation that the rules of inference for the logical atoms occur in pairs, the so-called "Introduction/Elimination" pairs.

This paper is based on a further remark of Gentzen, to wit:

> "it should be possible to display the E[limination]-inferences as unique functions of their corresponding I[ntroduction]-inferences."

In other words, we consider formalizations for logical systems in which only the I-inferences are primitive; the E-rules are derived (purely from the form of the I-rules) rules of inference. Not surprisingly the traditional logical systems can be so formalized.

In the process of deriving the normalization theorem for all such systems we introduce a system of combinators (and a variable-binding operator) particularly well-suited for the theory of proofs for arbitrary natural deduction systems.

► KERKKO LUOSTO, *Definability of a linear order in a random structure*.
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(Joint work with Lauri Hella and Phokion Kolaitis.)

Unlike in infinite structures, where the definability of a linear order means that the related theories are unstable and thus in a certain sense non-classifiable, in finite model theory linear orders are desirable, since they enable coding of Turing-machines in structures. This is manifested in the results that, fixpoint logic definable and PTIME computable model-classes (Immerman 1986, Vardi 1982), and respectively, partial fixpoint logic definable and PSPACE computable model-classes (Abiteboul and Vianu 1989) coincide on finite ordered structures. Note that if one can define a linear order in a finite structure $\mathcal{A}$, then $\mathcal{A}$ is necessarily rigid, i.e., the identity is its only automorphism.

Let $\mu = (\mu_n)_{n\in\mathbb{N}}$ be an asymptotic probability measure on $\mathrm{Str}(\tau)$. A binary query is called an *almost universal order with respect to* μ if it defines a linear order almost everywhere in terms of the asymptotic probability measure μ, and a *universal order* if it defines a linear order on rigid τ-structures.

In Hella, Kolaitis and Luosto 1994, we studied universal orders, e.g., we showed that there is a universal order which is definable in the weak Beth closure of partial fixpoint logic. If one replaces the class of rigid structures by any class of almost all structures this upper bound for definability can be considerably lowered: By a result of Babai, Erdös and Selkow, there is a PTIME almost universal order on graphs with respect to the uniform asymptotic measure.

A closer study of this and a related theorem of Karp reveals that there are almost universal orders which are definable in $\mathcal{L}_{\omega\omega}(R)$ and in $\mathrm{FP}(D_2)$ where R is the Rescher and D_2 the parity quantifier. I shall discuss applications of this result and possibilities of generalizing it to other asymptotic measures and to other quantifier logics.

► M. VICTORIA MARSHALL, *Types in class set theory and inaccessible cardinals*.
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If we add to KM's language the unary function symbol TW and the axiom ATW:

$$\forall R\forall S\big((\mathrm{WO}(R)\wedge \mathrm{WO}(S)\big) \to \big(R \approx S \leftrightarrow \mathrm{TW}(R) = \mathrm{TW}(S)\big),$$

where $\mathrm{WO}(R)$ is the formula that states that R is a well order type, we obtain the class set theory with well order types KM + ATW.

The following two questions were proposed by R. Chuaqui in 1980.

(1) Is it possible to define TW in KM?

And if the answer is negative,

(2) Is KM + ATW a conservative extension of KM?

With strong versions of the Axiom of Choice, we can define TW.

With the axiom of dependent choices for a collection of classes DC we know by a result of Reinhardt and Chuaqui (1983) that KM + DC + ATW is a conservative extension of KM + DC. We also have for this case by a result of Laver and Judah (independently, 1983) that if ZF+"there are two inaccessible cardinals" is consistent, then KM + DC + ATW is not an extension by definition of KM + DC.

Without any version of Axiom of Choice we have by a result of Judah and Marshall (1993) that if ZF+"there exists an inaccessible number of inaccessible cardinals" is consistent, then KM + ATW is not a conservative extension of KM.

In this paper I prove the converse of the last two results:

- *If* KM + ATW *is not an extension by definition of* KM (*and the model involved is well founded*) *then we can obtain a model of* ZF *with two inaccessible cardinals.*
- *If* KM + ATW *is not a conservative extension of* KM (*and the model involved is well founded*), *we can obtain a model of* ZF *with an inaccessible number of inaccessible cardinals.*

I also prove that if there is a well-founded model of KM, then we can obtain a model of ZF with an inaccessible cardinal.

▶ CARLOS H. MONTENEGRO E., *Weak versions of choice for collections of finite sets*.
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We consider weakenings of the Axiom of Choice by passing to infinite subsets. We denote by WC_n the statement that every collection of n-sets (sets of size n) has an infinite subcollection with a choice function on it. We denote by R_n the statement that every infinite set X has an infinite subset Y with the property that every set of n-subsets of Y has a choice function. It is easy to see that $R_2 \Longrightarrow WC_2$ and $R_3 \Longrightarrow WC_3$. We show that $R_4 \Longrightarrow WC_4$.

▶ MARCIN MOSTOWSKI, *Hierarchies determined by Henkin and finite-order quantification*.
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The work presented here employs the method of truth-definitions for studying hierarchies of concepts expressible in second and finite-order logics, and in the Logic of Branched quantifiers with Dualization (see [3] and [2]). Moreover it is employed also for the case of finite models and for positive results.

By concepts we mean closed on isomorphism classes of models.

By Σ^n_i we mean the class of all concepts (classes of models) definable by formulae of the form:

$$\exists \mathbf{x}_i \forall \mathbf{x}_{i-1} \exists \mathbf{x}_{i-2} \dots \varphi(\mathbf{x}_1, \dots, \mathbf{x}_i),$$

where $\mathbf{x}_1, \dots, \mathbf{x}_i$ are sequences of variables of the order n, and all other variables of

$$\varphi(\mathbf{x}_1, \dots, \mathbf{x}_i)$$

are of lower order than n.

Π_i^n is the class of those concepts which have their complements in Σ_i^n.

$\Delta_i^n = \Sigma_i^n \cap \Pi_i^n$.

For any class of concepts X, by $\mathrm{Bool}(X)$ (boolean combinations of X) we mean the smallest class containing X and closed on the operation $\cap$ and complements (to the class of all models of a proper vocabulary).

By LBD_k we mean the class of concepts definable by formulae with branched prefixes allowing at most k iterations of the dualization operator d. It means that LBD_0 is simply the class of concepts definable in the logic with Henkin quantifiers—it is known that this class contains $\mathrm{Bool}(\Sigma_1^1)$ (see [4], [1]) and is contained in Δ_2^1 (see [1]).

By various applications of the Tarskian method (also in the finite case) we obtain the following:

Theorem 1. *For each $i > 0$ in the following diagram*

$$\begin{tikzcd}
 & \Sigma_i^1 \arrow[dr, "3"] & & & \\
\Delta_i^1 \arrow[ur, "1"] \arrow[dr, "2"'] & & \mathrm{Bool}(\Sigma_i^1) \arrow[r, "5"] & \mathrm{LBD}_{i-1} \arrow[r, "6"] & \Delta_{i+1}^1 \\
 & \Pi_i^1 \arrow[ur, "4"'] & & &
\end{tikzcd}$$

every arrow represents a proper inclusion in infinite models.

In finite models the inclusions 1, 2, 3, 4, 5 *and the superposition of* 5 *and* 6 *hold. Additionally for $i = 1$ the inclusion* 6 *holds* (*see* [1]).

This theorem can be easily generalized for all higher (than second) orders. The only point is a proper generalization of branched quantifiers for higher orders.

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[1] H. B. Enderton, *Finite partially-ordered quantifiers*, ***Zeitschrift für Mathematische Logik und Grundlagen der Mathematik***, vol. 16 (1970), pp. 393–397.

[2] M. Krynicki and M. Mostowski, *Henkin quantifiers*, ***Quantifiers: Logics, Models and Computation, Volume One: Surveys*** (M. Krynicki, M. Mostowski, and L. W. Szczerba, editors), Kluwer Academic Publishers, 1995, pp. 193–262.

[3] M. Mostowski, *Quantifiers definable by second order means*, ***Quantifiers: Logics, Models and Computation, Volume Two: Contributions*** (M. Krynicki, M. Mostowski, and L. W. Szczerba, editors), Kluwer Academic Publishers, 1995, pp. 181–214.

[4] W. J. Walkoe, Jr., *Finite partially-ordered quantification*, ***Journal of Symbolic Logic***, vol. 35 (1970), pp. 535–555.

▶ DANIELE MUNDICI, *Twenty questions with many-valued answers*.

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We wish to give a modern presentation of the infinite-valued calculus of Łukasiewicz. We interpret propositions in the context of Ulam's game, the variant of the twenty questions game in which some of the answers may be erroneous.

We give a geometric proof of the completeness of the calculus, using techniques that were originally developed to desingularize toric varieties.

We show that the Lindenbaum algebras of the infinite-valued calculus, Chang's MV-algebras, stand to AF C^*-algebras as Boolean algebras stand to commutative AF C^*-algebras.

The many-valued counterparts of completeness and incompleteness are shown to have a nontrivial impact on C^*-algebraic quotient structures.

► JUHA OIKKONEN, *Game theoretic induction*.
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Inductive definability is in close connection to countable structures and the infinitary language $L_{\omega_1\omega}$. Both of these areas were under intensive and simultaneous study during the 70's.

In the 80's in Helsinki it was found out that there is a game theoretic language $M_{\omega_2\omega_1}$ which has in several respects an analogous relation to structures of cardinality ω_1 as $L_{\omega_1\omega}$ has to countable structures. The study of this kind of *infinitely deep* languages has later lead to interesting connections to Shelah's stability theory.

But also inductive definability is related to games; actually in many ways. In [1] Peter Aczel describes an interesting game in which an inductive definition is analyzed in a top-down fashion. This *Aczel game* has length ω. In [3] it is shown that there are more general longer versions of the Aczel game the length of which is measured by a tree.

Such games can be used to define a notion of *game theoretic inductive definability*. It is shown in [3] that game theoretic inductive definability has a similar connection to $M_{\omega_2\omega_1}$ as inductive definability has to $L_{\omega_1\omega}$. Moreover, many interesting structural theorems are proved in [3] about game theoretic inductive definability. These include versions of an abstract Kleene Theorem and restricted versions of the Stage Comparison Theorem and Reduction Theorem.

There is also a related notion of a game theoretic recursion definition studied in [2].

[1] Peter Aczel, *Inductive definability*, ***Handbook of Mathematical Logic*** (J. Barwise, editor), North-Holland, 1977.

[2] Juha Oikkonen, *A recursion principle of linear orderings*, ***Journal of Symbolic Logic***, vol. 57, no. 1 (1992), pp. 82–96.

[3] Juha Oikkonen and Jouko Väänänen, *Game-theoretic inductive definability*, ***Annals of Pure and Applied Logic***, vol. 65, no. 2 (1993), pp. 265–306.

► ANAND PILLAY, *Geometry of forking and groups of finite Morley rank*.
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The talk concerns the group theoretic consequences (in the finite Morley rank context) of various assumptions on the (geometric) behaviour of forking (the dependence relation in stable theories). A well-known such assumption is being 1-*based* which means that for any algebraically closed sets A and B, A is independent from B over their intersection. An old result (due to Hrushovski and the author) states that in a 1-based stable theory any connected definable group is commutative.

A weaker notion than 1-based is the notion of *CM-triviality*, introduced by Hrushovski. An interesting feature of this property is that it holds of all the new strongly minimal sets constructed by Hrushovski, as well as of Baudisch's new ω_1-categorical group. The condition is somewhat technical. It states that whenever $A \subseteq B$ are algebraically closed sets, c is a tuple, and $\mathrm{acl}(cA) \cap B = A$, then the canonical base of $\mathrm{tp}(a/A)$ is contained in the algebraic closure of the canonical base of $\mathrm{tp}(a/B)$. The property seems to be an essential one of the structures constructed by the new (amalgamation) techniques. The following result is proved:

THEOREM. *Any connected group of finite Morley rank definable in a CM-trivial stable theory is nilpotent. Equivalently, no simple group of finite Morley rank can be defined in a CM-trivial theory.*

▶ GONZALO E. REYES, M. LA PALME REYES, J. MACNAMARA, AND H. ZOLFAGHARI, *Models for non-Boolean negations in natural languages based on aspect analysis*.
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Since antiquity two different negations in natural languages have been noted: predicate negations ('not honest') and predicate term negation ('dishonest'). Aristotle tried to formalize them in his system of oppositions, distinguishing between affirmation and negation ('honest' and 'not honest') and contraries ('honest' and 'dishonest'). The Stoics replaced Aristotle's logic of terms by their logic of propositions. Although they considered three types of negation, none of them corresponded to Aristotle's predicate term negation. Frege and modern logic have followed the Stoics in either identifying predicate term negation with predicate negation or in casting predicate term negation out of logic into the realm of pragmatics. Although an extensive literature has arisen in these issues, we have not found mathematical models.

We propose category-theoretic models with two distinct negation operations, neither of them in general Boolean. We study combinations of the two ('not dishonest') and sentential counterparts of each. We touch briefly on quantifiers. The models are based on an analysis of aspects. For instance, to give an overall, global judgment of John's honesty we must agree on what aspects of John are relevant for that judgment: John qua person (global aspect), John qua social being (social aspect), John qua family man, John qua professional man, etc. We conceptualize this 'Aristotelian' analysis by means of a category of 'figures'. A model (for the negations) is obtained from the category of presheaves on this category. Although neither of the negations is Boolean, predicate negations turn out to be Boolean at the 'global' aspect (the aspect of the overall judgment) which may help to explain the persistent belief that logic is naturally Boolean.

▶ MARTA SAGASTUME AND ADRIANA GALLI, *Symmetric-intuitionistic connectives*.
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We define a notion of connective based on a new system of neighborhoods (increasing or decreasing) for each point on a given symmetric Kripke model. This idea has been motivated by the connectives of the Propositional Modal Symmetric Calculus of Moisil (that is, the basic intuitionistic connectives plus a De Morgan negation).

We define a possibility operator on this context and study an extension of Moisil Calculus including this operator and also its algebraic counterpart.

[1] H. GRAMAGLIA and D. VAGGIONE, *Birkhoff-like sheaf representation for varieties of lattice expansions*, ***Studia Logica***, accepted.

[2] H. GRAMAGLIA and D. VAGGIONE, *(Finitely) subdirectly irreducibles and Birkhoff-like sheaf representation for certain varieties of lattice ordered structures*, submitted.

[3] P. H. KRAUSS and D. M. CLARK, *Global subdirect products*, ***Memoirs of the American Mathematical Society***, vol. 210 (1979).

[4] L. NACHBIN, *Une propriété charactéristique des algèbres Booliénnes*, ***Portugaliae Mathematica***, vol. 6 (1947), pp. 115–118.

[5] D. J. Vaggione, *Sheaf Representation and Chinese Remainder Theorems*, ***Algebra Universalis***, vol. 29 (1992), pp. 232–272.

Abstracts of contributed talks

▶ OMAR DE LA CRUZ C., *Relations of implication and independency among some weak principles of choice*.
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Some choice principles are of the form $P = \forall X \Phi(X)$; others can be expressed in the following way: $P = (\forall F \in \mathcal{S})$(there exists a selector for F), where $\mathcal{S}$ is a class whose elements don't contain the empty set. Given a principle P stated in one of the two forms above, we can obtain a weaker principle doing the following modification: if the principle is in the first form, we write

$$P^{\Diamond} = (\forall X \text{ infinite})(\exists Y \subset X \text{ infinite})\Phi(Y);$$

if the principle is in the second form, we put

$$P^{-} = (\forall F \in \mathcal{S} \text{ infinite})(\exists F' \subset F \text{ infinite})(\text{there exists a selector for } F').$$

For example, $\mathrm{AC} = \forall X(X \text{ is well orderable})$; then

$$\mathrm{AC}^{\Diamond} = (\forall X \text{ infinite})(\exists Y \subset X \text{ infinite})(Y \text{ is well orderable}).$$

This restricted principle is equivalent to the well-known weak choice principle "every Dedekind-finite set is finite". On the other hand, if ACF is the axiom of choice for finite sets, then ACF^{-} is the statement "for every infinite family F of nonempty finite sets there exists an infinite subfamily $F' \subset F$ with a selector".

Starting with several principles P_i whose implication and independency interrelations have been already studied, such that every P_i can be expressed in at least one of the forms above, we obtain two new hierarchies formed by the principles $P_i^{\Diamond}$ and P_i^{-}. In this work we study several implication and independency relations among the principles of these new hierarchies, the ones of the original hierarchy and some other well-known choice principles.

[1] A. Blass, *Ramsey's Theorem in the Hierarchy of Choice Principles*, ***Journal of Symbolic Logic***, vol. 42, no. 3 (1977), pp. 387–390.
[2] T. Jech, ***The Axiom of Choice***, North-Holland, Amsterdam, 1973.
[3] D. Pincus, *Zermelo-Fraenkel Consistency Results by Fraenkel-Mostowski Methods*, ***Journal of Symbolic Logic***, vol. 37, no. 4 (1972), pp. 721–724.

▶ ANTONIO JOSÉ GONÇALVES DE FREITAS, *Karsner's generalized Galois theory and the results of Sebastião e Silva*.
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We compare Krasner's generalized Galois theory [4] with the results of Sebastião e Silva [6], [5] appearing in his so-called 'first Roma thesis'. We utilize for this comparison and presentation our work exposed in 1992 at the IX SLALM [2], especially the formalization and 'interpretation' of the concept of mathematical structure due to Sebastião e Silva, and the work of C. Gutierrez [3] on automorphism theorems on type theory found in the work of Sebastião e Silva. We have utilized also the comments by Franco de Oliveria [1] in the translation of the work of Sebastião e Silva [5]. Krasner's generalized Galois theory is in some sense a particular case of the theory presented by Sebastião e Silva.

[1] A. J. Franco de Oliveira, *On Automorphisms of Arbitrary Mathematical Systems*, ***History and Philosophy of Logic***, vol. 6 (1985), pp. 91–116.

[2] A. J. Goņalves de Freitas, *Hacia una teoría de modelos de orden superior en Sebastião e Silva*, ***Comunicación en el IX Simposio Latinoamericano de Lógica Matemática***, Bahía Blanca, Argentina, 1992.

[3] Claudio Gutiérrez, *Automorfismos de modelos en theoría de tipos*, ***Comunicación en el IX Simposio Latinoamericano de Lógica Matemática***, Bahí Blanca, Argentina, 1992.

[4] M. Krasner, *Une généralisation de la notion de corps*, ***Jour. Math. Pures et Appl.***, vol. 17 (1938), pp. 197–230.

[5] J. Sebastião e Silva, *Sugli automorfismi di un sistema mathematico qualunque*, ***Comm. Pontif. Acad. Sci.***, vol. 9 (1945), pp. 327–357.

[6] J. Sabastião e Silva, *Para uma teoría geral de homomorfismos*, ***En Obras Completas de José Sebastião e Silva***, vol. 1, Lisboa, 1985.

► JOSÉ M. MUÑOZ QUEVEDO, *Modal temporal schemata in increasing universes*.
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It is observed that for modal temporal predicate logic, interpreted in a natural way in Kripke models with increasing universes, many of the usual modal schemata which form part of the so-called normal systems are not valid. That is the case of the distributive law: $\Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)$. These principles are valid for propositional minimal logic but they have been extrapolated without justification to predicate logic. Restricted versions of these rules are shown to hold in increasing universes.

► ELENA OLIVOS H., *Interpretability lattices for classes of partial objects*.
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In partial algebras, the concept of identity between terms is not unique. Different alternatives and the resulting theories are studied. Specifically, the notion of interpretability between von Neumann varieties for classes of partial algebras is studied, obtaining various generalizations of the lattice of Gargía-Taylor.

► AQUILES PARAMO FONSECA, *Toward a denotative logic*.
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Based on a definition of *reference* built in terms of the classical model of set theory (ZF) I intend to show that it is possible to found and to formally develop a logical system for *denoting phrases*. Such a definition allows me to create an algebra of references which reflects to a certain degree the usage made by *natural languages* of denoting phrases. It also makes it possible to define some notions, functions, and set operations, which correspond to the principal syntactic resources used by natural languages for building these types of phrases. On the other hand, such a concept of reference is equivalent, in a fundamental sense, with Mostowski's notion of *generalized quantifier*—at least in Krajweski's version—, according to which I formulate the general hypothesis that every denoting phrase of a language is indeed an example of a logical generalized quantifier defined upon the undetermined set of all "beings".

Applied to the sphere of predication, denotative logic gives reason for restating first-order logic as a *copulative logic*, in which all atomic sentences are conceived as expressions of the form $\alpha \sim \beta$, where α and β are both denoting phrases, and "$\sim$" symbol represents the traditional copula; that is, all atomic sentences are viewed as entities formed each one by a subject and a predicate. I propose then a new validity principle issued from the notion of reference for sentential semantics. From this principle I inquire about the formal properties of the copula, present logic of categorical statements in a new way and establish the principal inference rules of denotative logic.

► CARLOS M. PARRA, *Bounded arithmetic in SC*.
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One of the first illustrations of the strong connection between Mathematical Logic and Complexity Theory was the characterization by S. Buss [2] of the poly-time computable functions as those defined in certain theories of Bounded Arithmetic. Since then, similar characterizations appeared of the functions in $NC = \bigcup NC^i$ [1] and [5] and consequently the separation of these classes has close connections with the study of how strong the corresponding theories TNC^i are, and also which ones are conservative extensions of the others.

In this paper we look at the theories T^oNC^o and TNC^o introduced in [5] and prove that the two theories are equivalent and consequently define the same functions.

A very interesting class between NC^1 and AC^1 is $L =$ LogSpace. Previously, G. Takeuti used the algebraic characterization of LogSpace in [3] to define a First-Order theory TLS of bounded arithmetic describing the functions in LogSpace. I extend this definition in a natural way to First-Order theories TLS^i and establish connections between the functions definable in these theories and the ones computable by a Turing Machine running simultaneously in space $\mathrm{Log}^i(n)$ and Polynomial Time, that is, the functions in the so-called "Steve's Class" SC.

[1] B. ALEN, *Arithmetizing uniform NC*, ***Annals of Pure and Applied Logic***, vol. 53, no. 1 (1991), pp. 1–50.

[2] S. BUSS, *Bounded Arithmetic*, ***Studies in Proof Theory***, vol. III, Bibliopolis, pp. 73–117.

[3] P. CLOTE, *A sequential programming language for Parallel Complexity classes*, Technical Report BCCS–88–07, Boston College, Computer Science Department, November 1988.

[4] G. TAKEUTI and P. CLOTE, *Bounded Arithmetic for NC, ALogTime, L, and NL*, ***Annals of Pure and Applied Logic***, vol. 56, pp. 76–117.

[5] G. TAKEUTI and P. CLOTE, *First-Order Bounded Arithmetic and small Boolean Circuit Complexity classes*, ***Feasible Mathematics II*** (P. Clote and J. Remmel, editors), Birkhäuser, 1995, pp. 154–218.

► ALEJANDRO PETROVICH, *Distributive lattices with an operator*.
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By a J-distributive lattice we understand an algebra $\mathbf{L} = (L, \vee, \wedge, j, 0, 1)$ of type $(2, 2, 1, 0, 0)$ such that $(L, \vee, \wedge, 0, 1)$ is a bounded distributive lattice and $j\colon L \to L$ is a join-homomorphism, i.e., j satisfies $j(0) = 0$ and $j(x \vee y) = j(x) \vee j(y)$.

It was shown in [1] (see also [2]) that there is a duality between the category of bounded distributive lattices endowed with a join-homomorphism and the category of Priestley spaces and Priestley relations. In this communication, I will show that there is a characterization of the congruence lattice of a J-distributive lattice in terms of the mentioned duality.

I will also give a characterization of the simple and subdirectly irreducible J-distributive lattices, and by means of this characterization, I will give a necessary condition, in algebraic terms, for a J-distributive lattice to be either simple or subdirectly irreducible.

[1] R. CIGNOLI, S. LAFALCE, and A. PETROVICH, *Remarks on Priestley duality for distributive lattices*, ***Order***, vol. 8 (1991), pp. 299–315.

[2] R. GOLDBLATT, *Varieties of Complex algebras*, ***Annals of Pure and Applied Logic***, vol. 44 (1989), pp. 173–242.

[3] A. PETROVICH, *Distributive lattices with an operator*, to appear in ***Studia Logica***.

▶ JOSÉ MARÍA TURULL, *Query completeness, distinguishability, and relational machines*.
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We show that (roughly) if every element in every structure of a class of finte structures is distinguishable among the others in its domain and if the defining properties are expressible in first-order logic with a constant number of variables (FO^k) then this is a sufficient condition for an order to be definable in that logic and for that class. This is what we define as 'strong (FO^k)-distinguishability' and '(FO^k)-orderability', respectively.

We use as a computation model the GM^{loose} of Abiteboul and Vianu (1991), and we show that this model is complete on structures of that kind, getting a stronger result than the one shown there which states that this is the case with order structures.

In Data Base Theory, this means that for a class of *db* instances which satisfies this property we can answer with a GM^{loose} the whole class CQ of the Computable Queries; and this is not the case if the class does not satisfy it.

We give examples of classes which satisfy the condition, yet not being ordered, and also of classes which, though being distinguishable, require a more expressive logic than (FO^k) for their defining formulas. Finally, we generalize the notion of (FO^k)-distinguishability and we define new machines which lie between GM^{loose} and GM (the complete machine). Then we get a general result about the completeness of every machine in the hierarchy when working on a corresponding type of class in a hierarchy of strongly distinguishable classes of finite structures.

We also show that the new machines are strictly included between GM^{loose} and GM considering them as classes of queries which are computable by the corresponding machines.

▶ ANDRÉS VILLAVECES, *End elementary extensions: Some independence results*.
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Let M be a transitive model of ZFC. Early results by Keisler and Silver provided certain conditions for elementary end extensions (*eee's*) of M to exist. In particular, they proved that κ is weakly compact if and only if given any S, $\big(\mathcal{R}(\kappa), \in, S\big)$ has an *eee*. This raises the natural question

QUESTION. *Do the existence of eee's for $\mathcal{R}(\kappa)$ and $o(M) = \kappa$ imply together that M has eee's*?

We prove that the answer to this question is independent of ZFC. We consider in particular a generalization of one direction of Keisler-Silver to *arbitrary* transitive models M that satisfy $o(M) = \kappa$, to get the following uniform result:

THEOREM. *The theory '*ZFC$+\exists\kappa(\kappa$ *measurable*$)+\forall\kappa(\kappa$ *inaccessible not weakly compact* $\rightarrow$ $\exists$*transitive* $M \models$ ZFC$\big(o(M) = \kappa \wedge M$ *has no eee's*$)\big)$*' is consistent relative to the consistency of* ZFC.

We also obtain some large-cardinal conditions which imply the existence of arbitrarily long chains in $(\mathcal{E}_{(M,\in,S)}, \prec_e)$ (the class of *eee's* of $(M, \in, S)$). Finally, we present some independence results related to the exact large cardinal strength of the property

$$U(\kappa)\colon \text{‘}(\mathcal{E}_{(\mathcal{R}(\kappa),\in,S)}, \prec_e) \text{ has arbitrarily long chains of } \textit{eee's}.\text{’}$$

Abstracts presented by title

▶ MAURICIO AYALA RINCÓN, *A deductive calculus for conditional equational systems with built-in predicates as premises*.
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Conditional equationally defined classes of many-sorted algebras, whose premises are conjunctions of (positive) equations and built-in predicates (constraints) in a basic first-order theory, are introduced. These classes are important in the field of algebraic specification because the combination of equational and built-in premises give rise to a type of clauses which is more expressive than purely conditional equations. A sound and complete deductive system is presented and algebraic aspects of these classes are investigated. In particular, the existence of free algebras is examined.

▶ OSWALDO CHATEAUBRIAND, *Sets, properties, and states of affairs*.
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I discuss three different conceptions of sets and some philosophical problems related to them. The background for the discussion is a hierarchy of properties and state of affairs.

First, sets may be conceived as abstracted from properties; i.e., a set is what is ‘common’ to all coextensive properties. A set in this sense has the same relation to coextensive properties as a cardinal has to equinumeric sets.

Second, sets may be conceived as certain states of affairs; namely, the states of affairs that ‘expresses’ the difference of the elements of the set. The set $\{\text{Frege}, \text{Russell}\}$, for instance, may be conceived as the state of affairs that Frege is different from Russell. A set in this sense, as a ‘collection’ as a plurality of entities into a unity, may be thought of as an ‘ontological code’ for the diversity of those entities.

Third, ‘pure’ sets may be conceived as abstract structures ‘made up’ of structure positions. The basic sets in this conception are the pure cardinality structures, with other sets arising from them by filling structure positions with sets of smaller rank.

There are interesting aspects to each of these conceptions as an analysis of some of what we mean by ‘set’, and there are also important connections between them.

▶ ANGELA MARIA PAIVA CRUZ, *Prior’s paradoxes and a relevant deontic propositional calculus*.
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The normative fragment of natural language is made of sentences that express acts and describe norms. In this fragment there is a criterion of logical truth and a relation of consequence between sentences which constitute a natural deontic logic. This paper adopts a translation function from the set of sentences of the normative fragment of natural language into the set of formulae in the formal language and claims that such a function translates logically true sentences of the natural language into provable formulae of the formal calculus.

In Von Wright's deontic calculus (1951) this does not happen and paradoxes are generated, known as Prior's Paradoxes. Cruz's paraconsistent deontic propositional calculus D_1 (1993) avoids some paradoxes, except that generated by the formula $OB \to O(A \to B)$. We build a relevant deontic propositional calculus that aims to avoid these paradoxes and keeps intact all other fundamental features of deontic operators, since the formula $B \to (A \to B)$ is improvable in some relevant calculi.