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1995 ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOL LOGIC

Irvine, California March 30–April 2, 1995

The Annual Meeting of the Association was held at the University of California, Irvine, March 30–April 2, 1995. The program included eight invited hour talks and two special sessions, in model theory and recursion theory respectively, each consisting of six half-hour talks. Leo Harrington gave the sixth annual Gödel lecture, entitled *Gödel, Heidegger and direct perception (or, why I am a recursion theorist)*. In addition, there were twenty-eight contributed talks and four papers presented by title. The Program Committee consisted of C. A. Anderson, S. Buss, M. Foreman, C. Laskowski (chair), and M. Lerman. The local organizers were P. Eklof (chair), M. Foreman, P. Maddy, and A. Nesin.

The following invited talks were presented:

George Boolos (MIT), *The recent rehabilitation* (?) *of logicism*.

Barry Cooper (University of Leeds), *Turing definability and automorphisms*.

Andras Hajnal (DIMACS, Rutgers University), *Partition relations revisited*.

H. Jerome Keisler (University of Wisconsin), *Quantifier elimination in probability theory*.

Christos Papadimitriou (University of California San Diego), *Logic, complexity and artificial intelligence*.

William Reinhardt (University of Colorado), *On sets, set theory and higher type objects*.

Marion Scheepers (Boise State University), *Games that involve set theory or topology*.

Alasdair Urquhart (University of Toronto), *Russell and the foundations of logic, 1903–05*.

The following talks were presented in a Special Session on Model Theory:

Bradd Hart (McMaster University), *Stable varieties: A survey of results and open questions*.

Renling Jin (University of Illinois), *An easy way to use the isomorphism property*.

James Schmerl (University of Connecticut), *The reals in non-standard models*.

Chris Miller (University of Illinois-Chicago), *Polynomially bounded, o-minimal expansions of the field of real numbers*.

Alexander Kechris (Caltech), *Countable Borel equivalence relations and the classification problem for rank 2 torsion-free groups*.

Ambar Chowdhury (University of Connecticut), *The small index property and smoothly approximable structures*.

The following talks were presented in a Special Session on Recursion Theory:

Doug Cenzer (University of Florida), *Index sets for* Π^0_1 *classes*.

Tamara Hummel (Dartmouth University), *Effective versions of Ramsey's theorem*.

Carl Jockusch (University of Illinois) [with Rod Downey and Michael Stob], *Array nonrecursive degrees and genericity*.

William Calhoun (Kalamazoo College), *Structured programming for priority arguments*.

Andre Nies (University of Heidelberg), *Relativizations of structures arising from computability theory*.

Klaus Ambos-Spies (University of Heidelberg), *Resource bounded genericity concepts*.

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Abstracts of three invited talks, and all contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
CHRIS LASKOWSKI, CHAIR

Abstracts of invited talks in the Special Session in Model Theory

► RENLING JIN, *An easy way to use the isomorphism property*.
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Let V denote an ω_1-saturated nonstandard universe. V is said to be satisfy the isomorphism property if for any finite language $\mathcal{L}$, for any two $\mathcal{L}$-structures $\mathcal{A}$ and $\mathcal{B}$ which are internal in V, $\mathcal{A}$ is elementarily equivalent to $\mathcal{B}$ implies $\mathcal{A}$ is isomorphic to $\mathcal{B}$. The isomorphism property was suggested by C. W. Henson in early 70's for studying nonstandard hull of Banach spaces. Recently, Shelah and I discovered an equivalent form of the isomorphism property in terms of the realizability of some second-order types. This equivalent form makes the use of the isomorphism property much easier than before. In the talk I am going to give three examples to illustrate the idea.

I will show that the isomorphism property implies:

(1) the hyperreal line is not Scott complete,
(2) every unbounded Loeb measure space has a measurable subset with infinite outer measure and zero inner measure,
(3) every hyperfinite time line has a bad cut and a good type two cut.

► CHRIS MILLER, *Polynomially bounded, o-minimal expansions of the field of real numbers*.
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An expansion of $(\mathbb{R}, +, \cdot)$ is said to be *o-minimal* if every subset $\mathbb{R}$ (parametrically) definable in the structure is a finite union of points and open intervals, and is said to be *polynomially bounded* if for every definable $f : \mathbb{R} \to \mathbb{R}$ there is an $n \in \mathbb{N}$ such that $f(t) = O(t^n)$ as $t \to +\infty$. Sets and functions definable in polynomially bounded o-minimal expansions of $(\mathbb{R}, +, \cdot)$ have many of the nice analytic, metric and geometric properties of real semialgebraic sets and functions. Applications of this fact to particular polynomially bounded o-minimal expansions of $(\mathbb{R}, +, \cdot)$ yield new results in real analytic geometry.

► JAMES H. SCHMERL, *The reals in non-standard models*.
Department of Mathematics, University of Connecticut, Storrs, CT 06269, USA.

There are constructions of nonstandard models of Peano Arithmetic which yield models whose corresponding reals have certain desirable properties. We will discuss a method for extending these constructions to other theories which are very weak versions of set theory and second-order number theory.

Abstracts of contributed talks

► ALEXANDRU BALTAG, *Modal logic and topology: Multidimensional frames*.
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The project is to relate topology with modal logic, by giving a modal system, weak enough to be decidable, but with enough expressive power to account for elementary topological reasoning.

First, a modal logic with indexed modalities is proposed, having two natural interpretations: "n-dimensional Kripke frames" and n-dimensional topological spaces (a generalization of McKinsey's semantics for S4). I have a completeness and a decidability result for $n = 2$.

Second, generalizing a paper of R. Parikh and L. Moss, I am proposing a language with two types of indexed modalities (one for **knowledge** and one for **effort**), with topological and knowledge-theoretical applications.

► DOUGLAS BURKE, *Generic ultrafilters with well-founded ultrapowers*.

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A tower of height δ is a sequence $\langle F_x \mid X \in V_\delta \rangle$ where F_X is a normal filter on $\mathcal{P}(X)$ and $X \subseteq Y$ implies F_Y projects to F_X. A tower is based on stationary sets if each F_X is the club filter restricted to a stationary set. A tower is precipitous if the generic direct limit ultrapower is well-founded—forcing with the positive sets from the tower.

THEOREM 1 (Woodin). *Assume κ is regular and $\delta > \kappa$ is Woodin. Then there is a forcing of size δ such that it is forced that there is an elementary $j : V \to M$ with $c.p.(j) = \kappa$, $j(k) = \delta$, and $M^{<\delta} \subseteq M$.*

The forcing in this theorem comes from the tower of height δ where F_X is the club filter restricted to $\{a \subseteq X \mid |a| < \kappa\}$. We generalize this theorem as follows:

THEOREM 2. *Assume T is a tower of height δ, δ supercompact, and T is based on stationary sets. Then T is precipitous.*

We do not know if the large cardinal hypothesis can be weakened (down to a Woodin) but there are examples that show that the assumption "based on stationary sets" is necessary.

We use Theorem 1 to prove the following: Assume F is a countable complete filter and $\kappa > |F|$ is measurable or Woodin. Then there is a forcing P of size κ and a P name $\dot{G}$ such that it is forced that $\dot{G}$ is a V-ultrafilter extending F and

a. If κ is Woodin then (it is forced that) $\mathrm{Ult}(V, \dot{G})$ is well-founded.
b. If κ is measurable then (it is forced that) $j_{\dot{G}}(\kappa)$ is in the well-founded part of $\mathrm{Ult}(V, \dot{G})$.

Sometimes this lemma can be used instead of assuming F is precipitous. For example we use this lemma to answer a question of Abe ([1]):

THEOREM 3. *Assume there is a weakly normal (see* [1] *for the definition) filter on $\mathcal{P}_\kappa\lambda$ where $\kappa < cof(\lambda) < \lambda$ (and there is a measurable cardinal $> \lambda$). Then $\lambda^{<\kappa} = 2^{<\kappa} \cdot \lambda$.*

[1] YOSHIHIRO ABE, *Weakly normal ideals on $\mathcal{P}_\kappa\lambda$ and the singular cardinal hypothesis*, ***Fundamenta Mathematicae***, vol. 143 (1993), pp. 97–106.

[2] H. WOODIN, *Supercompact cardinals, sets of reals and weakly homogeneous trees*, ***Proc. Nat. Acad. Sci. U.S.A.***, vol. 85 (1988), pp. 6587–6591.

► JAMSHID DERAKHSHAN, *The Fitting subgroup of MC-groups and locally nilpotent substable groups*.

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This paper is concerned with the structure of groups with the chain condition on centralizers (denoted as MC-groups), and the structural group theoretical properties of substable groups. The following are proved. The Fitting subgroup of an MC-group is nilpotent, (this solves problems in Wagner (1992)). Any locally nilpotent substable group is hypercentral. Any uniformly locally nilpotent MC-group is nilpotent, (this generalizes a theorem of Poizat-Wagner (1993) on substable groups and a theorem of Bryant (1979) on MC-groups of finite exponent). Some consequences of these results are obtained which solve some other problems on the algebraic structure of stable groups. In this regard, among other results, an Engel theory is developed, and applications to skew linear groups are obtained.

[1] R. M. BRYANT, *Groups with the minimal condition on centralisers*, ***The Journal of Algebra***, vol. 60 (1979), pp. 371–383.

[2] B. FOIZAT and F. O. WAGNER, ***Journal of Symbolic Logic***, 1993.

[3] F. O. WAGNER, *The Fitting subgroup of a stable group*, ***The Journal of Algebra***, to appear, 1992.

▶ DERRICK DUBOSE, *Determinacy and the sharp function on all sets*.
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We characterize in terms of determinacy, the existence of indiscernibles for the least inner model of the sharp function on all sets.

Let $\#^k$ be the partial sharp function on objects of type k. The following result has been known for some time:

(i) [2, 3] The existence of indiscernibles for $L[\#^k]$ is equivalent to the determinacy of $(\Pi^0_k)^*_+$. (See the last paragraph for the definition of $(\Gamma)^*_+$.)

Let $\#^\infty$ be the partial sharp function on all sets. The following analogous result is now known:

(ii) The existence of indiscernibles for $L[\#^\infty]$ is equivalent to the determinacy of $(\Pi^1_1)^*_+$. Around 1986–1987, Martin showed the $\Leftarrow$ direction of (ii). Its proof is similar to that of the $\Leftarrow$ direction of (i), which in turn is similar to Martin's proof of the converse of (iii) below. "Recently" Martin and I independently showed the $\Rightarrow$ direction of (ii). The proof combines quasi-Borel Determinacy [4, 5] (formulated and proven by Martin) with the method of integrating strategies with respect to ordinal auxiliary moves, as done in Martin's proof of the following:

(iii) [Martin] $\beta - \Pi^1_1$ games are determined for $\beta < \omega^2$ if $0^\#$ exists (see [5] or [1]).

The classes $(\Pi^0_k)^*_+$ and $(\Pi^1_1)^*_+$ lie strictly between $\bigcup_{\beta<\omega^2}(\beta - \Pi^1_1)$ and $\triangle(\omega^2 - \Pi^1_1)$. The definition of $(\Gamma)^*_+$ is given in what follows. Suppose that the sequence $\vec{A} = \langle A_\alpha \mid \alpha < \omega^2 \rangle$ of Π^1_1 sets witnesses that A is $\omega^2 - \Pi^1_1$. Let us say that "x falls out of $\vec{A}$ for player I by stage n" if

$$\mu\hat{\alpha}(x \notin A_{\hat{\alpha}} \text{ or } \hat{\alpha} = \omega \cdot n) \text{ is odd.}$$

For $B \subseteq {}^\omega\omega \times \omega$, define $N^B(x)$ to be the least n such that $B(x, n)$; it is undefined if no such n exists. For $\vec{D}$ witnessing that some set is $\beta - \Pi^1_1$ for some $\beta < \omega^2$, let

$$x \in (B)^*(\vec{A}; \vec{D}) \text{ iff either (a) or (b) below holds.}$$

(a) $N^B(x)$ exists and x falls out of $\vec{A}$ for player I by stage $N^B(x)$.

(b) $N^B(x)$ is undefined and x falls out of $\vec{D}$ for player I.

Define $(\Gamma)^*_+$ to be the collection of all $(B)^*(\vec{A}; \vec{D})$ such that $B \in \Gamma$, $\vec{A}$ witnesses that some set is $\omega^2 - \Pi^1_1$, and $\vec{D}$ witnesses that some set is $\beta - \Pi^1_1$ for some $\beta < \omega^2$.

[1] D. A. DUBOSE, *The equivalence of determinacy and iterated sharps*, ***Journal of Symbolic Logic***, vol. 55 (1990), pp. 502–525.

[2] D. A. DuBose, *Determinacy and the sharp function on the reals*, ***Annals of Pure and Applied Logic***, vol. 55 (1992), pp. 237–263.

[3] D. A. DuBose, *Determinacy and the sharp function on the objects of type k*, ***Journal of Symbolic Logic***, to appear.

[4] D. A. Martin, *An extension of Borel determinacy*, ***Annals of Pure and Applied Logic***, vol. 49 (1990), pp. 279–293.

[5] D. A. Martin, his forthcoming book entitled "Borel and Projective Games."

► LEONA F. FASS, *Modeling perfect behavior in an imperfect world.*
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We have taken a formal, foundational approach to problems of reasoning about knowledge, with particular attention to determining a mathematical structure for a specified body of knowledge (and its identifiable components). Considering the knowledge to-be-acquired as a "perfect behavior" within an imperfect world, we need distinguish that which is the desired behavior from that which is not.

From our perspective (originally, in theoretical computer science) knowledge acquisition or learning is determination of a "device" that models a desired behavior perfectly. With suitable constraints on the process: guidance of the learning procedure; selection of illustrative information; and description of the goal "device" also can be made precise or, well-defined.

During the earliest development of formal theories for computer science there was much interest in designing such "abstract devices" and in reasoning about their "states". Work by E. F. Moore, Myhill, Nerode, Rabin and Scott, Buchi, J. W. Thatcher and J. Wright investigated the areas of: relationships between component-state structure and device behavior; necessity or elimination of device states; choice or certainty in state-to-state transitions; and achievement of goals through accessing of "final states". A typical Moore thought-experiment on an abstract device could involve determining what state it might or must be in, based on observations of how it has behaved. Much research was devoted to defining realizable functions and "behaviors", determining their processing devices and specifying their states. If a device could be shown to produce a precisely specified behavior, then it realized its "function", behaved "perfectly", and was verifiably correct.

As time progressed the focus of formal theory in computer science shifted from logical and algebraic-machine "foundations" to mathematics-of-the-feasible and time-and-space efficiency. However, there has been renewed interest in the foundational approach, in connection with artificial intelligence research into such issues as "human-like" reasoning, logical/philosophical epistemics, and computational learning theory. We ourselves have defined learning in terms of *modeling* a *behavior*, based on reasoning about informational observations to determine the model's *function*, *structure*, *choices*, and *states*.

Here we describe the evolution of our approach to reasoning and learning as we have sought to apply our theory to a variety of behavioral domains. We consider the case where a perfect behavioral model is *known* to exist and can be found effectively; where it is *suspected* that a perfect model exists; where it is suspected that *no* perfect behavioral model exists, so we accept a result that is approximately (probably) correct; and finally, the fuzzy domains where perfection, approximation and probability are replaced by possibility.

[1] L. F. Fass, *Perfect learning (more or less)*, presented at the ***1992 Annual Meeting of the Society for Exact Philosophy***, University of Southwestern Louisiana, Lafayette, LA, May, 1992.

[2] ———, *Modeling perfect behavior: A goal-driven learning analysis*, ***Notes of the AAAI—Stanford 1994 Spring Symposium on Goal-Driven Learning***, SS 94-02, AAAI Press, pp. 125–127.

► RAYMOND D. GUMB, *A free theory of types*.
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Partial functions can be formalized in a natural manner in free logics. The aim of the present work is to explicate the positive free theory of types underlying higher-order functional programs employing both strict and lazy evaluation. The logic is positive and intensional because, for example, the following sentences are simultaneously satisfiable: $\neg\exists X(X =$ error), zero(error) $= 0$, $1 \div 0 =$ error, and $\forall X(f(X) = g(X)) \wedge f \neq g$. The free theory is an extension of the standard theory, rather than an alternative to it.

► GEOFFREY HELLMAN, *Constructivization problems in space-time physics: Singularity theorems*.
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To what extent is constructive mathematics (á la Bishop, et al.) really able to recover mathematics of importance in the physical and other sciences? In previous work, we have already elaborated on some significant obstacles in connection both with quantum mechanics (Gleason's theorem, some important classes of unbounded linear operators acting in Hilbert space) [1] and with cases of ordinary reasoning in which the use of constructive logical connectives and quantifiers results in systematic distortions of intended meaning [2]. This paper calls attention to some related problems in connection with the singularity theorems of general relativity (Hawking, Penrose, et al.) and their applications to our universe.

The main points are the following:

1. The basic Robertson-Walker singularity theorem (Proposition 12.15, p. 348 of reference [3]) *can* be given a constructive proof. (This is the highly idealized case of completely isotropic "dust" models.) The key point is that a comparative order on the reals rather than a total linear order suffices.

2. Important parts of Lorentz manifold theory, including the main tools used to prove the more powerful and general Hawking (cosmological) singularity theorems, however, are essentially non-constructive. For example, (a) the "construction" of "quasi-limit" curves—limits of sequences of continuous causal curves—is not constructively legitimate (depending on the König Infinity Lemma); (b) the existence of length-maximizing geodesics relies essentially on sequential compactness. (Examples of non-constructive theorems are Theorems 9.4.4 and 9.4.5, pp. 236–237, of reference [4], which assert that the maximum value of an upper semi-continuous function on a compact domain is assumed.) Similar problems confront efforts to constructivize related singularity theorems of Penrose and of Hawking and Penrose.

3. Ordinary reasoning used in applying, says, the cosmological theorems, including even the constructive Robertson-Walker theorem, to our cosmos appear to involve non-constructive existence claims and rules in an ineliminable way. The data constrain the mathematical models only very approximately (e.g., the range of a positive Hubble constant, nearly vanishing radiation pressure "sufficiently after" the initial singularity, etc.), but we still conclude that, with reasonable probability, the universe had a "big bang" initial point at some time in the past (e.g., prior to recent, controversial recalculations, 1.5×10^{10} years ago give or take 5×10^{9} years!). Yet we have no method (and quite conceivably could never have any) for narrowing this to the point at which we could be said to instantiate this existential quantifier (as the generalized disjunction property of constructive "$\exists$' requires). Appeal to "interval-valued" or "fuzzy" magnitudes—however useful in more mundane contexts, such as measuring length, mass, frequency, etc.—can hardly help here.

[1] GEOFFREY HELLMAN, *Gleason's theorem is not constructively provable*, ***Journal of Philosophical Logic***, vol. 22 (1993), pp. 193–203, and *Constructive mathematics of quantum mechanics: Unbounded operators and the spectral theorem*, ***Journal of Philosophical Logic***, vol. 22 (1993), pp. 221–248.

[2] ———, *Never say 'never'! On the communication problem between intuitionism and classicism*, ***Philosophical Topics***, vol. 17, no. 2, pp. 47–67.

[3] BARRETT O'NEILL, ***Semi-Riemannian geometry***, Academic Press, 1983.

[4] ROBERST WALD, ***General relativity***, University of Chicago Press, 1984.

► GREG HJORTH, *On uncountably many minimal models*.
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Under $ZFC + \neg CH$, it is not decidable whether a countable complete theory can have exactly $\aleph_1$ many minimal models.

► RENLING JIN AND SAHARON SHELAH, *Can a small forcing create Kurepa trees?*.
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Hebrew University of Jerusalem, Jerusalem, Israel.

On one side, we prove that if one chooses a Levy's model (a model obtained by forcing with a countable support Levy collapsing order which collapses all cardinals between ω_1 and a strongly inaccessible cardinal) as his ground model, then it is extremely hard to create Kurepa trees by a cardinal-preserving forcing notion of size at most ω_1 (if it is ever possible). On the other side, we prove that by assuming the consistency of one inaccessible cardinal there exists a model of CH plus no Kurepa trees, in which there is an ω-distributive Aronszajn tree such that forcing with that tree creates a Kurepa tree in the generic extension.

► DARRYL JUNG, *The vicious-circle principle*.
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Prompted by Henri Poincaré, Bertrand Russell put forward his celebrated **vicious-circle principle** (vcp) as the solution to the modern paradoxes. (The logical theory of Russell's 1910 *Principia Mathematica* may be understood as the result of modifying the logical theory implicit in his 1903 *Principles of Mathematics* in order to abide by the vcp.) Gödel and Quine, among others, have raised two salient objections against Russell's vcp. First, Gödel has claimed that Russell's various renderings of the vcp really express distinct principles and, thus, distinct solutions to the paradoxes, a claim that gainsays one of Russell's positions on the nature of the solution to the paradoxes, *viz.*, that such a solution be uniform. Secondly, Gödel and Quine have protested that the vcp's proscription against impredicative specification is incompatible with a realistic conception of the domain of quantification, a conception that Russell certainly held. I examine Russell's vcp and defend it against these objections.

► THOMAS E. LEATHRUM, *More results concerning off-branch families*.
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In recent work, the author has defined two new "small cardinal characteristics," using variations on almost disjoint families. A set of nodes in the tree ${}^{<\omega}\omega$ (ordered by inclusion) is *off-branch* if it is disjoint from every branch in the tree; an *off-branch family* is an almost disjoint family of off-branch sets;

$$\mathfrak{o} = \min\{|\mathcal{O}| : \mathcal{O} \text{ is a maximal off-branch family}\}.$$

Previous results concerning $\mathfrak{o}$ include: (in ZFC) $\mathfrak{a} \leq \mathfrak{o}$, and (consistent with ZFC) $\mathfrak{o}$ is not equal to any of the standard small cardinal invariants $\mathfrak{b}$, $\mathfrak{a}$,$\mathfrak{s}$,$\mathfrak{d}$, or $\mathfrak{c} = 2^{\omega}$. (Most of these consistency results use standard forcing notions.) Several interesting consistency questions remained open, however—in particular, Con($\mathfrak{d} < \mathfrak{o}$) was answered positively (add random reals over a model of CH), but Con($\mathfrak{o} < \mathfrak{d}$) remained open. Results will be presented in this direction, based on Shelah's proof of Con($\mathfrak{b} < \mathfrak{s}$) [1].

In this model (in its purest form), Shelah showed that $\mathfrak{a} = \mathfrak{b}$. The current results investigate adapting that proof to show $\mathfrak{o} = \mathfrak{b}$ in the same model.

[1] SAHARON SHELAH, *On cardinal invariants of the continuum*, ***Contemporary Mathematics***, vol. 31, American Mathematical Society, New York, 1984, pp. 183–207.

► ADAM H. LEWENBERG, *Embedding partial groups in groups*.
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A **partial group** is a triple (X, p, i) where X is a set, $p : U \to X$ and $i : V \to X$, with $U \subseteq X \times X$ and $V \subseteq X$, are functions, such that, writing ab for $p(a, b$ and a^{-1} for $i(a)$, the following equations are true for all a, b, and c in X (when both sides of the equation are defined):

$$(ab)b^{-1} = a, a^{-1}(ab) = b \text{ and } a(bc) = (ab)c.$$

Using a nonstandard-analysis argument, I show that when a partial group is a group "almost everywhere," then the partial group can be embedded in a group of only slightly larger size.

► ISABEL LUENGO, *Symbols and icons in geometry proofs*.
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Diagrams used in geometry proofs are finite combinations of three kinds of objects: iconic objects, symbolic objects and labels. Iconic objects represent objects of geometry like points, lines, angles, polygons, etc. Symbolic objects are conventional devices used to represent properties and relations between geometry objects such as the property of being a right angle or the congruence relation among segments. Labels establish the link between the diagrams and the sentences at the semantic level. In this paper I am just going to consider strictly diagrammatic systems where no language is involved, so I am going to focus on the features of the systems arising from the distinction between iconic and symbolic objects.

In particular, any diagrammatic inference system has rules to transform the diagrams into new diagrams. The transformation rules of the system can be divided into two exhaustive classes: *construction rules* that add new iconic objects to the diagram, and *inference rules* that do not add new iconic objects to the diagram. That is, the latter only add symbolic and labels or they may erase information. Whereas the traditional notion of logical consequence is sufficient to account for the validity of inference rules we need a new notion of consequence that I call *geometric consequence* to account for the validity of construction rules.

► ALEX. M. MCALLISTER, *Coding sets for the recursive subtrees of* $2^{<\omega}$.
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A consequence of the lemma of Lachlan and Soare in [1] is: if $\mathcal{X}_{\mathbf{0}'}$ is a path through the recursive subtree of $2^{<\omega}$, then one can "code" a set using the paths through the subtree. In particular, there exists a recursive set B such that for all $h \in 2^B$, there exists a path f through the recursive subtree with $h \subseteq f$. The proof of this lemma requires the Recursion Theorem and is dependent on a specific definition of $\mathbf{0}'$.

However, there exists a different "coding strategy" that does not involve the Recursion Theorem and that enables one to construct other sets which provide a slightly stronger form of coding.

THEOREM 1. *There exists $C \subset \omega$ such that if T is a recursive subtree of $2^{<\omega}$ and C is a path through T, then there exists $\sigma \in T$ such that $\sigma \subset \mathcal{X}_C$ and for all $\tau \supseteq \sigma$, $\tau \in T$.*

Such a set C is called a coding set for the recursive subtrees of $2^{<\omega}$. Further, this coding strategy can be combined with a permitting argument to give us the following result.

THEOREM 2. *If $Y \leq_T \emptyset\prime$, then there exists $C \leq_T Y$ such that C is a coding set for the recursive subtrees of $2^{<\omega}$.*

One consequence of this theorem is that, surprisingly enough, there exist $\aleph_0$ many different coding sets for the recursive subtrees of $2^{<\omega}$.

[1] A. H. LACHLAN AND R. I. SOARE, *Models of arithmetic and upper bounds of arithmetic sets*, ***Journal of Symbolic Logic***, vol. 59, no. 3 (1994), pp. 977–983.

► MONICA MCARTHUR, *Thresholds for 0-1 laws for $L^k_{\infty,\omega}$ on random graphs*.
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Using general theorems which characterize the existence of an $L^k_{\infty,\omega}$ 0-1 law (due to Kolaitis and Vardi) and an $L^k_{\infty,\omega}$ convergence law on a class of finite models with arbitrary measure, we establish some results about both 0-1 laws and nonconvergence for $L^k_{\infty,\omega}$ on $G_{n-\alpha}$, the random graph with edge probability $p(n) = n^{-\alpha}$, where α is rational and $0 < \alpha \leq 1$. Shelah and Spencer showed that for such $\alpha < 1$ these classes $G_{n-\alpha}$ do not have first-order convergence laws, and thus they do not have $L^\omega_{\infty,\omega}$ convergence laws. However, when we consider $L^k_{\infty,\omega}$ instead, we can obtain some bounds on k in terms of α which guarantee the existence of $L^k_{\infty,\omega}$ 0-1 laws; we also obtain bounds on how large k must be to guarantee nonconvergence for $L^k_{\infty,\omega}$ on $G_{n-\alpha}$. For $\alpha = 1/m$, m a positive integer, we obtain a complete characterization of the asymptotic of $L^k_{\infty,\omega}$ on $G_{n-\alpha}$: if $k \leq m$, there is an ∞, ω^k 0-1 law on $G_{n-\alpha}$; otherwise there is a $L^k_{\infty,\omega}$ nonconvergence. Thus we establish a single threshold for k such that there is $L^k_{\infty,\omega}$ convergence on $G_{n^{-1/m}}$. For $G_{n-\alpha}$, $\alpha = a/b$, a, b relatively prime positive integers with $a < b$, we establish bounds on the region in which such thresholds exist, but do no obtain a complete characterization.

These results fill in much of the remaining gaps in the knowledge of asymptotic behavior of $L^k_{\infty,\omega}$ on $G_{n-\alpha}$: Lynch has shown that $G_{n-\alpha}$ has an $L^\omega_{\infty,\omega}$ 0-1 law for $\alpha > 1$, and Lynch and Tyszkiewicz have announced a 0-1 law for $L^\omega_{\infty,\omega}$ on $G_{n-\alpha}$, α irrational, $0 < \alpha < 1$.

► GARVIN MELLES, *Set Turing machines*.
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Classification by set recursive functions and Shelah's classification theory are intimately related. For example a class of $\aleph_\varepsilon$ models of a superstable theory T has a set recursive classification relative to the reals and the cardinality function provable in ZFC if and only if T has $NDOP$. However, the notion of set recursion that is used was originally defined in a somewhat ad hoc way. Here we outline the proof that the class of set recursive functions and the more naturally defined class of set computable functions are equivalent.

► GRZEGORZ MICHALSKI, *A metetheorem for finite-injury priority arguments*.
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The paper presents a theorem which seems to capture essential combinatorial features common to all known finite-injury priority arguments in recursion theory and recursive model theory. In this sense the theorem can serve as a metatheorem for all such arguments, providing a general framework for them.

There are similar metatheorems of Ash and Knight (cf [1], [2], [3]) designed to do complicated α-nested priority arguments – they do not however apply to the classical Friedberg-Muchnik theorem. The statement of our metatheorem is as follows

THEOREM 1. *Suppose T is an r.e. tree, $I \subseteq T$ is an r.e. relation and $E : T \to \mathcal{P}^{<\omega}(\omega)$ is a partial recursive function. Suppose also that*

(1) *For every node $l \in T$ there is a node $l \in T'$ with $E(l) \subseteq E(l')$ which is either an immediate successor of l or satisfies $I(l, l')$*

(2) *There is no node $l \in T$ with a recursive sequence $(l_k)_{k\in\omega}$ of immediate successors of l such that for all $k \in \omega I(l_k, l_{k+1})$ and $E(l_k) \subseteq E(l_{k+1})$.*

Then, there is a $\triangle^0_2$ path p through T, such that $E(p) := \cup_{n\in\omega} E(p \restriction n)$ is r.e.

To show how this theorem works, we consider a sample of finite-injury priority arguments; including the Friedberg-Muchnik theorem, two more theorems from recursion theory, and the following theorem from recursive model theory (cf [4], [5]):

THEOREM 2 (Goncharon, Peretuat'kin). *Let $\mathbb{A}$ be a countable homogeneous structure and let $\mathcal{E}$ be an enumeration of the complete types realized in $\mathbb{A}$. Let f be a function such that if $\mathcal{E}_n = \mathcal{E}(\bar{U})$, x is the first variable not in $\bar{u}$ and $\exists x\varphi(\bar{u}, x) \in \mathcal{E}_n(\bar{u})$, then $\mathcal{E}_{f(n,\varphi)}(\bar{u}, x) \supseteq \mathcal{E}_n(\bar{u}) \cup \{\varphi(\bar{u}, x)\}$. If $\mathcal{E}$ and f are recursive, then there is a decidable $\mathbb{B} \cong \mathbb{A}$*

[1] C. J. ASH, *Labeling systems and r.e. structures*, ***Annals of Pure and Applied Logic***, vol. 47 (1990), pp. 99–119.

[2] C. J. ASH and J. F. KNIGHT, *Mixed systems*, ***Journal of Symbolic Logic***, to appear.

[3] C. J. ASH and J. F. KNIGHT, *Mixed ramified systems*, ***Annals of Pure and Applied Logic***, to appear.

[4] S. S. GONCHAROV, *Strong constructivizability of homogeneous models*, ***Algebra i Logika***, vol. 17 (1978).

[5] M. G. PERETYAT'KIN, *Criterion for strong constructivizability of a homogeneous model*, ***Algebra i Logika***, vol. 17 (1978).

► CHARLES MORGAN, *Historicized forcing and pseudo-morasses*.
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In this talk I shall reconsider historicized forcing and explore its connections with pseudo-morasses. Some applications in general topology and combinatorics will be given.

► ITAY NEEMAN, *Woodin cardinals and determinacy*.
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There are essentially two different techniques used in proofs of determinacy from Woodin cardinals. The first, due to Martin-Steel, uses iteration trees and by itself suffices to show that the existence of n Woodin cardinals and a measurable cardinal above implies Π^1_{n+1}-determinacy. The other, due to Woodin, uses among other things the technique of stationary tower forcing (developed by Foreman, Magidor, Shelah and Woodin) to obtain such results as $L[R^*] \models AD$ assuming that δ is the sup of ω Woodin cardinals, and R^* represents the reals in the symmetric collapse up to δ. (More general results can also be obtained.)

I shall show how the Martin-Steel method can be adapted to yield some of Woodin's results.

▶ KRISTER SEGERBERG, *AGM as a dynamic, doxastic logic*.
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Dynamic doxastic logic is the result of allowing the agent of Hintikka type doxastic logic three types of doxastic action: expanding by a proposition, revising by a proposition and contracting by a proposition. Within this framework, theories of belief revision can be represented. In particular, one may represent the famous theory of Alchourrón, Gärdenfors and Makinson (AGM) in this way.

▶ SLAWOMIR SOLECKI, *A characterization of the meager ideal*.
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This is an account of a joint work with A. S. Kechris. We find all σ-ideals on a Polish space which are generated by closed sets and fulfill the countable chain condition. We derive from it the following characterization of the meager ideal: given a Polish group G, the meager ideal is the unique translation invariant σ-ideal on G which is generated by closed sets and fulfills the countable chain condition. This gives a partial solution to a problem of Kunen. Balcerzak and Rogowska found another application of the main result solving an old problem of Mauldin about the Baire order of families of functions whose sets of discontinuity points belong to a given σ-ideal.

▶ OTMAR SPINAS, *Partitioning products of $\mathcal{P}(\omega)/fin$*.
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We generalize the cardinal invariant a to products of $\mathcal{P}(\omega)/\mathrm{fin}$ and then sharpen Solomon's inequality $b \leq a$ by proving $b \leq a(\lambda)$ for every $\lambda \leq \omega$. Here $a(n)$, for $0 < n < \omega$, is the least size of an infinite partition of $(\mathcal{P}(\omega)/\mathrm{fin})^n$, $a(\omega)$ is the least size of an uncountable partition of $(\mathcal{P}(\omega)/\mathrm{fin})^\omega$ (there are always countably infinite partitions of $(\mathcal{P}(\omega)/\mathrm{fin})^\omega)$), and b is the least size of an unbounded family of functions from ω to ω ordered by eventual dominance. We also prove the consistency of $b < a(n)$ for every $n < \omega$.

▶ KATHERINE ST. JOHN, *Iteration in classes of finite structures*.
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On the standard model of the natural numbers, iteration and recursion are equally expressive (due to the Kleene Normal Form Theorem, see [1]). However, on classes of finite structures, iteration can be less expressive than recursion. Tiuryn [2], as the culmination of several results of others, for which references can be found in his paper, showed this for the class of complete binary trees. The class, $\mathcal{K}_T$, of complete binary trees is all of the finite Herbrand interpretations of the signature $\sigma = \{\mathrm{root}, \mathrm{left}, \mathrm{right}, =\}$, where *root* is a constant, *left* and *right* are unary functions, and $=$ is the standard equality on elements. His elegant proof showed that iterative functions could only "see" exponentially (i.e., all) elements in the structure (see [3]). We extend this result to any class of Herbrand interpretations of a finite functional signature.

THEOREM 1. *For any finite functional signature, for every $k \in N$, and for every iterative function S with k variables, there exists a polynomial, $p(n)$ such that*

$$|Val(S, \mathcal{A}_n, \vec{a})| < p(n)$$

holds for all $n \in N$ and all $\vec{a} \in (\mathcal{A}_n)^k$, where $\mathcal{A}_n$ is the finite Herbrand interpretation of height n, $|S|$ is the length of S, and $Val(S, \mathcal{A}_n, a)$ are the values "seen" by $S(a)$.

A corollary to these results gives polynomial bounds on the closure ordinals of iterative functions. Due to the work of Urzyczyn [3], this theorem is sharp.

[1] S. C. Kleene, ***Introduction to metamathematics***, D. Van Nostrand Company, Inc., Princeton, NJ, 1952.

[2] Jerzy Tiuryn, *Simplified proof of $DDL < DL$*, ***Information and Computation***, vol. 81 (1989), pp. 1–12.

[3] Pawel Urzyczyn, *A necessary and sufficient condition in order that a Herbrand interpretation be expressive relative to recursive programs*, ***Information and Control***, vol. 56 (1993), pp. 212–219.

▶ JINDRICH ZAPLETAL, THOMAS JECH, AND BOHUSLAV BALCAR, *Approximations to Cohen algebras*.
Department of Mathematics, Pennsylvania State University, University Park, PA 16802, USA.

We define several properties of complete Boolean algebras which are weakenings of the property "being a Cohen algebra" and investigate their relationships. We prove for example that the completion of a measure algebra from the ground model can be a Cohen algebra in an $\aleph_1$-preserving generic extension.

Abstracts presented by title

▶ MATIAS FRANCISCO DIAS, *Self-reference and diagonalization*.
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I give a new effective way of constructing true undecidable formulas of elementary arithmetic (PA), and remark that they are neither self-referential nor diagonalized. Let W_x be any r.e. set. fx is the recursive function whose range is W_x and $D_x(y, z)$ is the formula of PA that represents f_x. $g(u)$ is a recursive function that gives the Gödel number of a formula $A_u(u)$ which is the result of substituting in $A_u(Y)$ the numeral u for all free occurrences of the variable y. Let $G(x, y)$ represent in PA the function g. Ψ is a productive partial recursive function for the set $V = \{x : A_x \text{ is true in the standard model of PA}\}$, such that for a r.e. set $W_x \# \emptyset$ and $W_x CV$, $\Psi(x0$ gives the Gödel number of $\forall z \forall w \neg(D_x(z, w) \vee G(p, w))$ where p is the Gödel number of $\forall z \forall w \neg(D_x(z, w) \vee G(y, w))$. So $\Psi(x) = g(p)$. Of course the formulas obtained in this way are diagonalized and self-referential. However my next step is to construct true undecidable formulas that are neither diagonalized nor self-referential. Given W_x as above let us construct a r.e. set W_y in the following way: first define the sets $W_x \bigcup \{\Psi(x)\} = W_{x_1}$, $W_x \bigcup \{\Psi(x_1)\} = W_{x_2}$ and so on. Finally define $W_y = \{\Psi(x), y(x_1), \Psi(x_2), \dots\}$. $W_y CV$. Now let h be a recursive function whose range is W_y and let h be represented in PA by $H(z, w)$. Then consider the formula $\forall w[\exists t \exists y(H(y, t) \wedge G(w, t) \Rightarrow \forall y \forall z \neg(^{D_x}(y, z) \wedge G(w, z))]$. We should note that such formulas are neither self-referential nor diagonalized, although they are true and undecidable. Once more diagonalization and self-reference do not seem to be a good criterion for undecidability in PA.

[1] Matias Francisco Dias, *On elementary arithmetic*, ***Journal of Symbolic Logic***, vol. 59, no. 1 (1994), pp. 716–717, annual meeting of the ASL abstracts of papers: Notre Dame, 1993.

▶ V. KANOVEI AND M. REEKEN, *Internal approach to external sets and universes*.
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It has been considered as a problem for nonstandard set theories how external sets can be conveniently incorporated in the internal universe. To solve this problem, we use *Bounded set theory* **BST**, a nonstandard set theory which differs from **IST** of Nelson so that the "standard" variable in Idealization is restricted by a standard set as the domain, and the axiom which postulates that every set belongs to a standard set is added. **BST** is a conservative and equiconsistent extension of **ZFC**.

Let **HST** be the external theory NS_1 of Hrbaček, plus Choice in the case when the choice function has a set of standard size as the domain. (A *set of standard size* is an image of a set of the form $\{x \in X : \text{st}x\}$, where X is standard.) In particular **HST** contains Separation and Replacement for all st-$\in$-formulas, and Saturation for families (of internal sets) of standard size.

THEOREM 1. ***HST** has an interpretation in **BST** such that the class of all internal sets in the sense of the interpretation coincides with the ground **BST** universe.*

In other words, a **BST** spectator can legitimately consider things so that his universe is embedded, as the class of all internal sets, in a universe of **HST**, a theory convenient enough to deal with external sets.

▶ GUNWON LEE, *Silk ball orders*.
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We pursue the orders which can be verifiable in our every day life such as the four dimensional coordinates in use. The points can be ordered as enumerations of the coordinations in the form of random collections. The extensions of the diagonal methods in the form of a silk ball are the finite subsets of the spatio-temporal continuum of Aristotle-type. The connections inbetween silk ball orders and the continuum are uncleared.

▶ ZHOU XUNWEI, *Brief introduction to geometrical logic*.
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Geometrical logic is constructed by the author. It is a geometrical approach to logic. Geometrical logic deems that human knowledge is divided into four levels: terms, fact, empirical connection and logical connection, which can be graphically represented by four coordinate spaces which constitute a hierarchical coordinate system.

Human knowledge is denoted by prepositions, geometrical figures are made by the locus of moving points. In geometrical logic, one-to-one correspondence is established between propositions and geometrical figures through the hierarchical coordinate system, so that human knowledge is described by locus of moving points.