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1994–1995 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

San Francisco, California, January 6–7, 1995

The Winter Meeting of the Association was held in San Francisco, California, on January 6–7, 1995, in conjunction with the American Mathematical Society. The program committee consisted of Andreas Blass (chair), Julia Knight, Robert Soare, and W. Hugh Woodin. The program included seven invited talks, twenty-five contributed talks, and six papers presented by title.

The following invited talks were presented at the meeting:

John Steel (UCLA), *Recent results in core model theory*.

Alexandra Shlapentokh (East Carolina University), *Algebraic and Turing inseparability of rings*.

Valentina Harizanov (George Washington University), *Recursive model theory: Examining computability in the theory of theories*.

Tomek Bartoszyński (Boise State University), *Cardinal invariants and Borel sets*.

Alexander Kechris (Caltech), *A Glimm-Effros dichotomy for analytic equivalence relations and Ulm-type classifications*.

Ted Slaman (University of Chicago), *Recursion theory and second order arithmetic*.

Steffen Lempp (University of Wisconsin), *Coding and undecidability in computability theory*.

Abstracts of contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
Andreas Blass, Chair

Abstracts of contributed talks

▶ A. NAIT ABDALLAH, *Variational principles for reasoning about actions in the context of partial information*.

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The characteristic surface [1, 2] of a given syntactic path Φ representing an evolving physical system Π is defined by $\Sigma(\Phi) = \prod_{n\in I} (\Omega_{\min}(\models)(\Phi_n) \equiv_{\star}^{G(\cup_n \Phi_n)})$. The least action principle postulate that *"Of all the possible paths in $\Sigma(\Phi)$ that it might take to get from one state to another, the point representing a (physical) logic system takes a path (m_n) which is extremum, i.e., the smoothest possible on the characteristic surface representing the system."* This variational principle provides a solution to the frame problem in its temporal projection form.

Temporal explanation problems require a combination of the least action principle with

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abduction. An example is given by the Gun problem: *After performing an action, things normally remain the same. After a man is shot with a loaded gun, he dies. Fred was initially alive. After shooting Fred, Fred was dead. Was the gun originally loaded?* The Gun problem is not readily solved by the sole application of the least action principle. It requires a formalization of the following line of argument: *After shooting a person with a loaded gun, that person dies. After Fred was shot with a gun, Fred died. Therefore the gun must have been loaded.* To formalize this, one introduces the following inference rule, which expresses an *abduction principle*:

$$\frac{a'\ ,\ C[b]\ ,\ a \wedge a' \to b}{C[\star(a,a)]}$$

and uses it as part of a new variational principle. Let P be an ionic logic program specifying some physical system Π, and $H \subseteq P$ be the set of hard facts to be explained. The abductive variation δP of P with respect to H is the set of ions generated from H by means of the abduction principle. Program $P' = (P \backslash H) \cup \delta P$ is the "variational" logic program. Define syntactic path $D'_n = \{\varphi \in \mathit{lfp}(P') : \text{state-index}(\varphi) \leq n\}$, where $\mathit{lfp}(P')$ is the least fixpoint of P'. Let $\Sigma(D')$ be the characteristic surface of (D'_n). The abductive variational principle postulates that *"Among all possible paths that it might take to get from one state to another, the point representing system Π takes some extremum path (m_n) in $\Sigma(D')$ such that $H \subseteq \Omega(\models_{soft})(\lim_n m_n)$."* In other words the limit of the path taken satisfies in a soft sense every hard fact $\in H$. In summary: one introduces a variation, applies the least action principle, and then removes the variation; the final result obtained is the expected evolution path.

For the Gun problem, the characteristic surface of the "nearby" syntactic path D' has two minimal semantic paths, one of which converges towards a model where Fred is dead after the shooting (this is the "dangling semantic path" removal condition.) When the evolution of the system follows that path, the gun was originally loaded. In the other minimal semantic path, Fred is still alive after the shooting, and the gun was originally unloaded. Another example is the Murder Mystery problem: *After performing an action, things normally remain the same. After a man is shot with a loaded gun, he dies. Fred was initially alive. After shooting Fred and waiting, Fred was dead. When did Fred die? Was the gun originally loaded?* The characteristic surface of the "nearby" syntactic path (D'_n) here has two extremum semantic paths, one of which converges toward a model where Fred was dead after the waiting period (this is the "dangling semantic path" removal condition), where he died during the shooting, and where the gun was originally loaded. In the other minimal semantic path, Fred is still alive after the shooting, and the gun was originally unloaded.

[1] M. A. Nait Abdallah, *The dynamics of logic systems and its applications*, ***Logic Colloquium 94***, 1994.

[2] ———, ***The logic of partial information***, Springer Verlag, 1994.

▶ JEREMY AVIGAD, *Two effective conservation results involving subsystems of second-order arithmetic*.

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First we give an effective proof of the Harrington-Simpson-Brown result that $WKL{+}_0$ is conservative over RCA_0 for Π^1_1 sentences, and show that the increase in the length of proofs is polynomially bounded. In doing so we give general methods by which iterated-forcing arguments involving subsystems of second-order arithmetic can be formalized in the base theory. We also give an effective proof that ATR_0 is conservative over $\widehat{ID}_{<\omega}$ for arithmetic sentences, by showing that ATR_0's main axiom is equivalent ro a second-order version of $\widehat{ID}_{<\omega}$'s inductive definition scheme. In this case, however, there is a significant increase in the lengths of proofs, since ATR_0 has short proofs of the consistency of superexponentially-many inductive definitions.

► MATTHIAS BAAZ AND RICHARD ZACH, *Generalizing theorems in real closed fields*.
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Krajíček recently [2, Problem 23] asked the question: Is there a generalization result in the theory of real closed fields of the form: If $A(1 + \cdots + 1)(n\text{occurrences of } 1)$ is provable in length k for all $n \in \omega$, then $(\forall x)A(x)$ is provable? Questions and results about generalizations of proofs in mathematical theories depend substantially on the presentation of the theory. We investigate four distinct formalizations with respect to their generalization behavior. We show that there is a positive answer to Krajíček's question for (1) the usual axiom system *RCF* of Artin-Schreier with Gentzen's sequent calculus **LK** as underlying logical calculus, (2) *RCF* with the variant LK_B of **LK** allowing introduction of several quantifiers of the same kind in one step, and (3) LK_B and the first-order schemata corresponding to Dedekind cuts and the supremum principle. A negative answer is given for (4) any system containing the schema of extensionality.

The results (1) and (2) build on earlier work by Parikh and Krajíček-Pudlák, showing that the logical complexity of the formulas occurring in a proof in **LK** can be bounded in the length of the proof. Krajíček and Pudlák [3] use *unification* to prove this. For the axiom system investigated in (1) and (2), it follows that everything that can be proved in k steps in **LK** can already be proved in a finite subtheory of *RCF*. (Note that the logical complexity of the axiom asserting the existence of a root for all polynomials of degree $2n + 1$ grows as n gets larger.) Finite theories admit *finite term bases*, i.e.: For every $A(x)$ and k, there are $t_1, \ldots, t_m$ s.t. if $A(t)$ is provable in k steps, then t is a substitution instance of t_i for some i. From this the generalization result follows. A similar argument establishes (2), but a more careful analysis is needed.

Result (3) follows from an analysis of the main theorem of [1], where a similar generalization result is proved for arithmetic with the least number principle for $\exists_1$-formulas.

Using a variant of Yukami's Trick [4], we give conditions under which an axiomatization of the theory of real closed fields containing the schema of extensionality does not admit generalization of theorems.

[1] M. Baaz and P. Pudl'ak, *Kreisel's conjecture for $L\exists_1$*, ***Arithmetic, proof theory and computational complexity*** (P. Clote and J. Krajíček, editors), Oxford University Press, Oxford, 1993, pp. 29–59.

[2] P. Clote and J. Krajíček, *Open problems*, ***Arithmetic, proof theory and computational complexity***, Oxford University Press, Oxford, 1993, pp. 1–19.

[3] J. Krajíček and P. Pudlák, *The number of proof lines and the size of proofs in first order logic*, ***Arch. Math. Logic***, vol. 27 (1988), pp. 69–84.

[4] T. Yukami, *Some results on speed-up*, ***Ann. Japan Assoc. Philos. Sci.***, vol. 6 (1984), pp. 195–205.

► ROY BENTON, *A simple incomplete modal extension of KT which is the logical union of two logics with the finite model property*.
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I present an incomplete modal extension of KT ($= K\{Lp \to p\}$) which has two points of interest: (1) it is simpler than incomplete KT-extensions which have appeared before (2) it can be realized as the union of two logics which have the finite model property (F.M.P.) and hence are themselves complete.

The resulting incomplete logic, with its three simple axioms, is simpler than van Benthem's classic examples ("Two incomplete logics", Theoria 44, 25–37). Van Benthem used two measures of simplicity of a logic, both of them involving the complexity of its characteristic axioms (i.e., axioms beyond those needed for the minimal normal modal logic K)—namely, modal depth and number of sentence letters. One of his logics extends KT, and its four axioms have maximum modal depth 2 and use only two sentence letters. My incomplete logic actually discards one of these four axioms—the only one with two sentence letters—so that the result uses only one sentence letter, and of course maintains modal depth at 2. Since van Benthem's other "simple" logic uses only one sentence letter, but requires a modal depth of 3 (and, incidentally, does not extend KT and cannot be easily modified to do so), my logic is strictly simpler on both counts. (More recently, other simple incomplete logics have appeared.)

I have previously showed that the class of normal logics with the F.M.P. do not form a lattice, by presenting another such failure of logical union. However, the resulting incomplete logic, while quite simple, did not extend KT.

My two "component" logics are $K\{MLp \to LMp\}$, previously shown to have the F.M.P.; and $K\{Lp \to p,\ Mp \,\&\, L(p \to Lp) \to p\}$, which I show to have F.M.P. using a technique similar to normal "forms."

► WILLIAM C. CALHOUN, *An untangled proof of the recursion theorem*.
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Kleene's Recursion Theorem has central importance in recursion theory. The usual proof of the theorem is very short, but rather cryptic. I will discuss a slightly longer "untangled" proof of the Recursion Theorem. I find the "untangled" proof more intuitive than the usual proof. It consists of three lemmas:

LEMMA 1. *If $\{\phi_i\}$ is an **acceptable numbering** of the partial recursive functions and the Recursion Theorem holds for $\{\phi_i\}$, then the Recursion Theorem holds for the standard numbering.*

LEMMA 2. *Define $\phi_i(x) = \phi_i(\langle i, x\rangle)$. Then $\{\phi_i\}$ is an acceptable numbering.*

LEMMA 3. *The Recursion Theorem holds for $\{\phi_i\}$.*

I will also discuss generalized versions of the Recursion Theorem.

► LUIZ CARLOS, P. D. PEREIRA, EDWARD HERMANN HAUSLER, MARíA CLAUDIA MERÉ, AND MARCELO F. FRIAS, *Bounded infinite parallel logic*.
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It is quite well known that the elimination of contraction and weakening gives rise to a splitting of the usual logical conjunction and disjunction into multiplicative and additive versions. It would be quite natural to think of a corresponding splitting with respect to quantifiers.

The purpose of this paper is twofold: (1) to present an **infinitary propositional system for classical linear logic** with infinite tensor and infinite par which satisfies the Normalization

Theorem; and (2) to show how to use this infinitary system in order to define multiplicative universal and existential quantifiers.

Infinitary propositional systems for classical and intuitionistic logic have been studied in several works [2, 3]. The natural infinitary generalization of the tensor operator does not introduce any significant additional difficulty. On the other hand the natural infinitary generalization of the par operator does not seem to allow for a natural infinitary generalization of the par operator does not seem to allow for a natural concept of $\square_\infty$-reduction. In a certain sense (as it was remarked by Blass [1]) our concept of sequent and our concept of derivation as a well-founded tree are not adequate for the definition of $\square_\infty$-reduction. In order to solve this well-foundness problem we introduce the concept of *bounded parallel inference*.

Instead of applying one rule at a time, we now have a unique rule "R-Sim" whose effect is the simultaneous application of a (possibly infinite) sequence of rules. We can show that: (1) If there is a finite bound for the length of the derivations of the minor premises of $\square_\infty$-elimination and (2) If there is a finite bound for the degree of the maximum formulas occurring in a (possibly infinite) list of formulas, then we can both define a $\square_\infty$-reduction and use it in order to prove normalization.

When we think of multiplicative quantifiers as infinite tensors and pars we see that the restrictions imposed on derivations in our system of parallel inference are perfectly adequate for the treatment of multiplicative quantification.

[1] A. Blass, ***Game semantics for affine logic***, Marseille, 1990.

[2] P. Martin-Löf, *Infinite terms and a system of natural deduction*, ***Composito Mathematica***, vol. 2 (1972), fasc. 1, pp. 92–103.

[3] L. C. P. D. Pereira, *On the estimation of normal derivations*, PhD thesis (Filosofiska institutionen), Stockholm, 1982.

► GREGORY CHERLIN AND NIANDONG SHI, *Graphs omitting a finite set of cycles*.
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We say that a class of graphs has a universal element if every graph in this class can be embedded in the universal one as a subgraph. We prove that the class of graphs omitting a finite set $\mathfrak{C}$ of cycles has a universal element if and only if the set $\mathfrak{C}$ consists precisely of all the odd cycles of length less than some specified bound. The sufficiency of this condition was proved by P. Komjath, A. Mekler and J. Pach.

► ALI ENAYAT, *A new transfer theorem in the constructible universe*.
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Recall that given two cardinals κ and λ, the transfer relation $\kappa \longrightarrow \lambda$ asserts that any theory (with a distinguished linear order) which has a κ-like model must have a λ-like model.

Theorem A. *If κ and λ are inaccessible cardinals and λ is* ***not*** *Mahlo, then $\kappa \longrightarrow \lambda$.*

Let μ be the function defined on inaccessible cardinals as follows:

$$\mu(\kappa) = \begin{cases} \omega, \text{ if } \kappa \text{ is } n\text{-Mahlo for each } n, \\ \text{The largest } n \in \omega \text{ such that } \kappa \text{ is } n\text{-Mahlo}, \textit{otherwise}. \end{cases}$$

THEOREM B ($V = L$). *Assume κ and λ are inaccessible cardinals. The following are equivalent:*

(i) $\kappa \longrightarrow \lambda$.
(ii) $\mu(\kappa) \geq \mu(\lambda)$.

► DON THOMAS FALLIS, *A defense of a probabilistic method of establishing mathematical truths*.
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One of the primary goals of mathematicians is to establish new mathematical truths. Toward this end, mathematicians are almost invariably theorem provers. However, there are several methods other than writing down a proof which seem to achieve this epistemic goal of establishing mathematical truths. For instance in [1], Michael Rabin described a probabilistic test for primality which establishes to an arbitrarily high degree of certainty that a number is prime. Nevertheless, the vast majority of mathematicians are unwilling to employ such probabilistic methods in their attempts to establish new mathematical truths. I will argue that mathematicians do not have good grounds for this rejection of probabilistic methods.

[1] M. O. RABIN, *Probabilistic algorithm for testing primality*, ***Journal of Number Theory***, vol. 12 (1980), pp. 128–138.

► MARCELO F. FRIAS, ARMANDO M. HAEBERER, AND PAULO A. S. VELOSO, *Metalogical properties of fork algebras*.
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In this abstract we present some results on the boundary between algebra and logic, which show the suitability of abstract fork algebras as a framework for doing logic in an algebraic setting.

DEFINITION (proper fork algebras). Consider a set $\mathcal{U}$ with an injective operation $\star : \mathcal{U} \times \mathcal{U} \to \mathcal{U}$ (constructing pairs $y \star z$). This induces an operation on subsets of $\mathcal{U} \times \mathcal{U}$:

$$r \lhd s = \{\langle x, y \star z\rangle \in \mathcal{U} \times \mathcal{U} : \langle x, y\rangle \in r \wedge \langle x, z\rangle \in s \wedge \langle y, z\rangle \in V\}.$$

A *proper fork algebra* (PFA) is a structure

$$\mathcal{F} = \langle F, \cup, \cap, |, \lhd, \sim, \prime, \emptyset, \triangle, V\rangle$$

such that F is closed under $\lhd$ and the reduct $\langle F, \cup, \cap, |, \sim, \prime, \emptyset, \triangle, V\rangle$ is a complete and atomic proper relation algebra.

DEFINITION (abstract fork algebras). An *abstract fork algebra* (AFA) is a structure

$$\mathcal{A} = \langle A, +, \bullet, ;, \triangledown, -, \smile, 0, 1, \infty\rangle$$

such that $\langle A, +, \bullet, ;, -, \smile, 0, 1, \infty\rangle$ is a complete and atomic relation algebra [1], and the following axioms hold:

(1) $r \triangledown s = (r;\pi) \bullet (s;\rho)$,
(2) $\pi \triangledown \rho \preceq 1$,
(3) $(r \triangledown s);(p \triangledown q)^\smile = (r;p^\smile) \bullet (s;q^\smile)$,
(4) $0 \neq t \preceq \infty \otimes \infty \to (\exists v)(\exists w)(0 \neq v \otimes w \preceq t)$

where *projections* and *cross* product are defined by

$$\pi = (1 \triangledown \infty)^\smile, \rho = (\infty \triangledown 1)^\smile \text{ and } r \otimes s = (\pi;r) \triangledown (\rho;s).$$

These projections satisfy the properties of quasi-projections [1], but are rather special. In fact, every atom of an atomic AFA is functional.

NOTATION. We will denote by $\mathcal{F}$ the set of sentences of classical first-order logic, and by $\mathcal{T}$ the set of fork-terms. In order to state the main results, we will use the following result which is proved in [3, 2].

THEOREM. *There exists a recursive function* $I : \mathcal{F} \to \mathcal{T}$ *such that* $\models \phi$ *iff* $\vdash_{=,\mathsf{AFA}-} I(\phi) \neq 0$. (*Where* $\vdash_{=,\mathsf{AFA}-}$ *means entailment using only equational reasoning, and the set of equational* AFA *axioms* (1), (2) *and* (3).)

THEOREM 1 (Strong Completeness). *Let* Γ *be a set of first-order sentences, and let us denote by* $I(\Gamma) \neq 0$ *the set* $\{I(\gamma) \neq 0 / \gamma \in \Gamma\}$. *Then*

$$I(\Gamma) \neq 0 \models_{\mathsf{PFA}} I(\psi) \neq 0 \textit{ iff } I(\Gamma) \neq 0 \vdash_{=,\mathsf{AFA}-} I(\psi) \neq 0.$$

The previous theorems allow the translation of the machinery of classical first-order logic into the framework of fork algebras, in such a way that proofs can be performed equationally. Thus fork algebras algebraize first-order logic, much as Boolean algebras do for propositional logic. Moreover, we have the analogous of the theorems of *compactness* and *Löwenheim-Skolem*.

DEFINITION. By a (*proper*) *fork model* of a set Σ of terms we mean a (proper) fork algebra $\mathcal{A}$ such that $\mathcal{A} \models t \neq 0$ for every $t \in \Sigma$.

Consider a (possibly) infinite set Γ of first-order sentences.

THEOREM 3 (Compactness). *If every finite subset of* $I(\Gamma)$ *has a proper fork model, then so does* Γ.

THEOREM 4 (Löwenheim-Skolem). *Suppose that the set* $I(\Gamma) \neq 0$ *has a* fork model *whose underlying set* $\mathcal{U}$ *is infinite. Then for each infinite cardinality,* α, $I(\Gamma)$ *has a proper fork model with underlying set of cardinality* α.

Notice that in order to prove the theorems it is not enough to use their versions for first-order logic since in that case the equationality in proofs is lost.

[1] A. TARSKI and S. A. GIVANT, *A formalization of set theory without variables*, ***American Mathematical Society Colloquium Publications***, vol. 41 (1987).

[2] P. A. S. VELSOS and A. M. HAEBERER, *A finitary relational algebra for classical first-order logic*, ***Bulletin of the Section of Logic***, ***Polish Academy of Sciences***, vol. 20 (1991), no. 2, pp. 52–62.

[3] P. A. S. VELOSO, A. M. HAEBERER, and M. F. FRIAS, *Fork algebras as algebras of logic*, ***Proceedings of the logic colloquium '94***, July 1994, p. 127, to appear also in: ***Journal of Symbolic Logic***.

► E. I. GORDON AND A. A. BUSHUEV, *On some extensions of IST and the predicates of relative standardness*.
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We consider one finite axiomatics for nonstandard set theory, which is based on the notion of class. This axiomatics is in the same relation to IST as NBG to ZFC. We add to NBG one unary predicate—St(X)—"class X is standard" and some axioms for this predicate. For example we require that classes which are constructed from standard classes by NBG axioms of classes existence, are standard themselves. The separation and substitution principles are formulated only for standard classes. The transfer, idealization and standardization principles are here axioms but not the schemes of axioms. So we have here for classes a possibility to be not the sets but subclasses of the sets. It is proved that a set does not contain such proper subclass iff its cardinality is standard natural number.

The internal classes are defined in such a way that all sets are internal and the proper classes are internal iff they are sections of standard classes—relations. This definition of internal classes is connected with definitions of the predicates of relative standardness, which were introduced by the first author earlier (Siberian Journal of Mathematics—1989, N2). Using these predicates we prove that standardization principle and right-to-left implication in idealization principle are independent from other axioms of IST.

We consider also some consistent extensions of described theory, in which Alternative Set Theory can be interpreted.

► JOEL DAVID HAMKINS, *Lifting and extending measures through forcing*.
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Measures in V can relate to measures in a forcing extension $V[G]$ in two subtly different but connected ways. Namely, a measure μ in V might *extend* to a measure ν in $V[G]$ when $\mu \subset \nu$. Alternatively, μ might *lift* to ν when the corresponding ultrapower embedding j_μ lifts to j_ν. I will show how these similar notions subtly differ. In particular, a normal measure can extend to normal measures to which it does not lift, and a normal measure can extend to many measures, none normal, and hence not lift at all. I will also discuss how lifts and extensions respect or disrespect the Rudin-Kiesler and Mitchell orders.

► JEFFRY L. HIRST AND STEFFEN LEMPP, *Infinite versions of some NP-complete problems*.
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Some connections between NP-complete problems for finite objects and the complexity of their analogues for infinite objects have appeared recently. In particular, the problem of deciding which finite graphs have a Hamilton path is known to be NP-complete [3], while Harel [2] has shown that the problem of deciding which infinite recursive graphs have a Hamilton path is Σ^1_1-complete. Within the framework of reverse mathematics, Harel's construction can be used to prove that the existence of a "deciding function" for Hamiltonicity is equivalent to Π^1_1 comprehension [1].

We have considered some other infinite versions of NP-complete problems, particularly those related to graph theory, in terms of their complexity and proof-theoretic strength. The infinite analogues of NP-complete problems can behave in a variety of unexpected ways, as exemplified by the results below.

THEOREM 1A. *The following are provably equivalent over* $\mathbf{RCA}_0$*:*

(1) $\Pi^1_1 - \mathbf{CA}_0$.
(2) *For any sequence of ordered pairs of graphs,* $\left\langle G^L_i, G^R_i \right\rangle_{i\in\mathbb{N}}$*, there is a function* $f : \mathbb{N} \to 2$ *such that* $f(i) = 1$ *if and only if* G^L_i *is isomorphic to a subgraph of* G^R_i.
(3) *For any sequence of graphs,* $\langle G_i \rangle_{i\in\mathbb{N}}$*, there is a function* $f : \mathbb{N} \to 2$ *such that* $f(i) = 1$ *if and only if* G_i *contains an infinite clique.*
(4) *For any sequence of graphs,* $\langle G_i \rangle_{i\in\mathbb{N}}$*, there is a function* $f : \mathbb{N} \to 2$ *such that* $f(i) = 1$ *if and only if* G_i *has a co-infinite vertex cover.*

THEOREM 1B. *The index sets for the recursive graphs with the properties of* (2), (3), *or* (4) *above are all* Σ^1_1*-complete.*

By contrast, other NP-complete problems naturally lead to arithmetical problems in the infinite case.

THEOREM 2A. *The following are provably equivalent over* $\mathbf{RCA}_0$*:*

(1) $\mathbf{ACA}_0$.
(2) *For any sequence of graphs,* $\langle G_i \rangle_{i\in\mathbb{N}}$*, there is a function* $f : \mathbb{N} \to 2$ *such that* $f(i) = 1$ *if and only if* G_i *can be 3-colored.*
(3) *For any sequence of graphs,* $\langle G_i \rangle_{i\in\mathbb{N}}$*, there is a function* $f : \mathbb{N} \to 2$ *such that* $f(i) = 1$ *if and only if each connected component of* G_i *can be colored.*

THEOREM 2B. *The index set for the recursive graphs with the property in* (2) *is* Π^0_1*-complete, while the index set for the recursive graphs with the property in* (3) *is* Π^0_3*-complete.*

[1] W. GASARCH and J. HIRST, ***Reverse mathematics and recursive graph theory***, (in preparation).
[2] D. HAREL, *Hamiltonian paths in infinite graphs*, ***Israel J. Math.***, vol. 76 (1991), pp. 317–336.
[3] R. M. KARP, *Reducibility among combinatorial problems*, ***Complexity of computer computations*** (R. Miller and J. Thatcher, editors), Plenum Press, New York, 1972, pp. 85–103.

▶ GREG HJORTH, $\mathbf{\Pi}^1_1$ *sets universal for being non-Borel.*
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It is independent of ZFC whether there exists a $\mathbf{\Pi}^1_1$ set in the plane whose sections correspond exactly to the $\mathbf{\Pi}^1_1$ non-Borel sets in the line. This answers a question raised by Mauldin.

▶ HASEO KI, *Kechris-Woodin rank and Zalcwasser rank*.
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Zalcwasser introduced a rank that measures the uniform convergence of sequences of continuous functions on the unit interval. We apply the Zalcwasser rank to the Fourier series of a continuous function on the unit circle. In it is shown that on C (the set of series of a continuous functions, on the unit circle, with convergent Fourier series), the Zalcwasser rank is a $\mathbf{\Pi}^1_1$-norm which is unbounded below ω_1, i.e., functions in C are arbitrarily bad in terms of this rank. Kechris and Woodin defined a rank that measures the uniform continuity of the derivative of a differentiable function. We shall refer to this rank as the Kechris-Woodin rank. In fact, they have shown that on the set of all differentiable functions, the Kechris-Woodin rank is a $\mathbf{\Pi}^1_1$-norm which is unbounded below ω_1.

Ajtai and Kechris conjectured that the Kechris-Woodin rank is finer than the Zalcwasser rank, meaning that for any function f, the Zalcwasser rank is less than or equal to the Kechris-Woodin rank. There is a fair amount of evidence supporting this conjecture. For example, the Zalcwasser rank is 1, i.e., the smallest possible number, for all differentiable functions f, whose derivative f', is bounded. On the other hand, on the set of all differentiable functions with bounded derivatives, the Kechris-Woodin rank is unbounded below ω_1. Our main result is an affirmative answer to this conjecture of Atjai and Kechris.

Also Kechris and Louveau asked if Kechris-Woodin rank can be approximated by Zalcwasser rank. Recently, we have shown that it is true.

▶ CLAUDE LAFLAMME, *A few ordinal ultrafilters*.
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We discuss ordinal ultrafilters especially from the point of view of the Rudin-Keisler ordering. In particular, answering a question of J. Baumgartner, we show that consistently there is a downward closed ω^ω ultrafilter, i.e., an ultrafilter all of whose Rudin-Keisler predecessors are also ω^ω ultrafilters. We also show that there cannot be a downward closed $\omega^{\omega+\omega}$ ultrafilter but the question whether every $\omega^{\omega+\omega}$ ultrafilter has a ω^ω ultrafilter below it remains open.

▶ GEOFFREY LAFORTE, *Some phenomenon in the n-r.e. and n-REA degrees*.
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A set $A \subseteq \omega$ is said to be 1-r.e. if it is recursively enumerable, and $n+1$-r.e. if $A = C - B$ where C is r.e. and B is n-r.e. (Since 2-r.e. sets are differences of r.e. sets, we call them d.r.e.) If $W \subseteq \omega$, then A is said to be W-REA if A is recursively enumerable in W and $W \leq_T A$. Analogously to the n-r.e. sets, a set is 1-REA if it is recursively enumerable, and $n+1$-REA if it is W-REA for some n-REA set W. We examine some phenomena relating $n+1$-REA to $n+1$-r.e., n-REA, and n-r.e. sets.

By a result of Soare and Stob, if W is nonrecursive, then some W-REA set fails to be of r.e. degree. We show that this phenomenon fails in the d.r.e. degrees: there is a nonrecursive r.e. set W such that every W-REA set has d.r.e. degree. We conjecture that this result can be extended to $n > 2$ to construct, for example, an r.e. set W such that every W-REA set has 3-r.e. degree and some W-REA set is not of d.r.e. degree. An obstacle here is the lack of any direct method of constructing a 2-REA set with properly (that is, non-d.r.e.) 3-r.e. degree. Indeed, we show that any 3-r.e. set which is simultaneously 2-REA *as a set* must have d.r.e. degree. The existence of some 2-REA set with properly 3-r.e. degree, however, does not follow from some results of Cooper and Yi on isolated degrees.

If S is a class of subsets of ω, a set A is said to be *isolated in the S-degrees* by a set W if $W <_T A$ but there is no $C \in S$ with $W <_T C <_T A$. Cooper and Yi have shown that there is a d.r.e. set that is isolated in the r.e. degrees by an r.e. set below it. Turning to such isolation phenomena, we show that d.r.e. sets isolated in the r.e. degrees by an r.e. set below them occur in any nontrivial interval in the r.e. degrees.

[1] S. B. Cooper, S. Lempp, and P. Watson, *Weak density and cupping in the d.r.e. degrees*, ***Israel J. Math.***, vol 67(1989).

[2] S. B. Cooper and X. Yi, *Isolated d.r.e. degrees*, ***University of Leeds, Dept. of Pure Math., 1993 Preprint Series***, no. 17.

[3] R. I. Soare and M. Stob, *Relative recursive enumerability*, ***Proceeding of the Herbrand Symposium: Logic Colloquium '81*** (J. Stern, editor), North-Holland, 1982.

▶ ROBERT LUBARSKY, *V is probably L, intuitionistically*.
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We present a very simple construction, under very mild set-theoretic hypotheses, that, starting from a Kripke model of IZF built upon most any partial order, will put *any* set into L.

▶ GRZEGORZ MICHALSKI, *Relatively r.e. vs relatively Σ_1 in models of Peano arithmetic*.
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The usual setting for recursion theory is the standard model for second order arithmetic; i.e., the structure $(\mathbb{N}, \mathcal{P}(\omega))$, where $\mathbb{N}$ is the standard model for first order arithmetic, and $\mathcal{P}(\omega)$ is the family of all subsets of ω—the set of natural numbers. In recent years people (M.J. GROSZEK, P. HÁJEK, K. KONTOSTATHIS, A. KUČERA, M. E. MYTILINAIOS, P. PUDLÁK, T. A. SLAMAN, W. H. WOODIN, Y. YANG—see references) have considered recursion theory in other second order structures of the form $(\mathcal{M}, \mathcal{S})$, where $\mathcal{M}$ is a nonstandard model of Peano Arithmetic (*PA*) or some fragment of *PA*, and $\mathcal{S}$ is a family of subsets of $\mathcal{M}$. The purpose of these investigations in general has been to develop a better understanding of the properties actually needed in proving various results from recursion theory.

In the present paper, we consider the following elementary result:

PROPOSITION 1. For any sets X and Y, the following are equivalent
(1) X is r.e. relative to Y
(2) X is definable by a formula which is $\Sigma_1(Y)$; i.e., Σ_1 with a predicate for Y.

In structures of the form $(\mathcal{M}, \mathcal{S})$ where $\mathcal{M}$ is a model of (a sufficient fragment of) *PA*, (1) implies (2), and if $\mathcal{S}$ consists of amenable sets, then (2) implies (1). Here we show that without amenability, (2) need not imply (1). More precisely, we prove that for any countable model $\mathcal{N}$ of *PA*, there is a conservative extension $\mathcal{M}$, with a subset Y such that a certain $\Sigma_1(Y)$-formula defines a set which is not r.e. relative to Y.

The model $\mathcal{M}$ is produced using methods of Specker-MacDowell and Gaifman, and the set Y is produced by forcing, using $\mathcal{N}$-finite forcing conditions.

[1] M. J. GROSZEK and M. E. MYTILINAIOS, *Σ_2-induction and the construction of a high degree*, ***Recursion Theory Week, Proceedings Oberwolfach 1989*** (K. Ambros-Spies et al., editors), Lecture Notes in Mathematics, vol. 1432 (1990), Springer-Verlag, Berlin, Heidelburg, New York, Tokyo, pp. 205–223.

[2] M. J. GROSZEK and T. A. SLAMAN, *On Turing reducibility*, preprint.

[3] P. HÁJEK and A. KUČERA, *On recursion theory in $I\Sigma_1$*, ***Journal of Symbolic Logic***, vol. 534 (1989), pp. 576–589.

[4] K. KONTOSTATHIS, *The combinatorics of the Friedburg-Muchnick theorem*, preprint.

[5] M. .E. MYTILINAIOS, *Finite injury and Σ_1-induction*, ***Journal of Symbolic Logic***, vol. 54 (1989), pp. 38–49.

[6] M. .E. MYTILINAIOS and T. A. SLAMAN, *Σ_2-collection and the infinite injury priority method*, ***Journal of Symbolic Logic***, vol. 53 (1988), pp. 212–221.

[7] T. A. SLAMAN and W. H. WOODIN, *Σ_1-collection and the finite injury priority method*, ***Mathematical Logic and applications*** (K. Ambros-Spies et al., editors), Lecture Notes in Mathematics, vol. 1388 (1989), Springer-Verlag, Berlin, Heidelburg, New York, Tokyo, pp. 178–188.

[8] Y. YANG, *The existence of high incomplete r.e. set does not imply $I\Sigma_2$*, preprint.

▶ G. MINTS AND S. TUPAILO, *Epsilon-substitution method for ramified analysis*.
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We extend to Ramified Analysis (RA) the definition and termination proof of Hilbert's ε-substitution method [1, 2, 3]. Second order variables of RA are assigned ordinals as levels. *Critical formulas* are closed formulas

$$F[t] \to F[\varepsilon x F[x]] \quad (\phi_0) \qquad\qquad F[T] \to F[\varepsilon X F[X]] \quad (\phi_1)$$

with a suitable restriction for levels. ε-*substitution* assigns numerals to first order ε-terms and abstracts of lower level to second order ε-terms. The existence of values of closed terms under given substitution S is proved by induction on level.

The ε-substitution process is defined for a fixed system E of critical formulas as follows. Initial substitution assigns default values: $S_0(\varepsilon x A) = 0$, $S_0(\varepsilon Y B) = \lambda x(x = 0)$. If $S_0, \dots, S_i$ are already generated, and S_i is not yet a solving substitution (some critical formula in E is false), then S_{i+1} is found as follows. Take a false critical formula. If it is of the form (ϕ_0), set $S_{i+1}(\varepsilon x F) :=$ the least n such that $S_i(F[n]) =$ **true**, and set all terms of higher complexity to default value 0. In the case of (ϕ_1) set $S_{i+1}(\varepsilon X F[X]) = S_i(T)$.

THEOREM 1. *The sequence $S_0, \dots, S_n, \dots$ always terminates.*

[1] D. HILBERT and P. BERNAYS, ***Grundlagen der Mathematik***, Bd. 2, Springer, 1970.

[2] G. MINTS,*Substitution method and Gentzen-type systems*, ***Proceedings of 9-th International Congress of Logic, Method. and Philos. of Sci.*** (Uppsala, Sweden), August, 1991 (to appear).

[3] G. MINTS and S. TUPAILO, *Epsilon substitution method for elementary analysis*, ***Report No. CSLI-93-175***, ***CSLI***, Stanford University, February 1993, submitted to Archive for mathematical logic.

▶ DANIEL E. TALAYCO, *Applications of cohomology to set theory: Todorčević trees*.
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Recent developments have lead to the description and investigation of certain set theoretic phenomena such as Hausdorff gaps and Aronszajn trees using (co)homological terminology and machinery. An example of this will be explored in this talk and a new class of Aronszajn trees, called Todorčević trees, that has arisen through this research will be described.

▶ ALAN D. VLACH, *Hyperarithmetical expansions of recursive structures: A new result and an application*.
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Let $\mathfrak{A}$ be a recursive $\mathcal{L}$-structure, P a relation symbol not in $\mathcal{L}$, ψ a recursive infinitary Π_2 sentence in the language $\mathcal{L} \cup \{P\}$, and let $0 < \alpha < \omega_1^{\mathrm{CK}}$. In [5] we found sufficient conditions under which we can construct a recursive $\mathcal{L}$-structure $\mathfrak{B} \simeq \mathfrak{A}$ such that $\langle \mathfrak{B}, P_{\mathfrak{B}} \rangle \not\models \psi$ for every $\triangle_\alpha^0$ relation $P_{\mathfrak{B}}$ on $\mathfrak{B}$. Here we sharpen the result, providing necessary and sufficient conditions which involve a notion of rank. The argument is a nested priority construction, making use of the metatheorems of C. Ash found in [1, 2, 3].

We also give an example of a structure $\mathfrak{A}$ of arbitrarily large recursive rank to which our new result applies. The structure $\mathfrak{A}$ will be a tree, isomorphic to a subtree of $\omega^{<\omega}$, with added unary predicates, for which ψ will say of the new unary predicate P that is a path.

[1] C. J. ASH, *Stability of recursive structures in arithmetical degrees*, ***Annals of Pure and Applied Logic***, vol. 32 (1986), pp. 113–135.

[2] C. J. ASH, *Recursive labelling systems and stability of recursive structures in hyperarithmetical degrees*, ***Transactions of the American Mathematical Society***, vol. 298 (1986), pp. 497–514. Errata, vol. 310 (1988), p. 851.

[3] C. J. ASH, *Labelling systems and r.e. structures*, ***Annals of Pure and Applied Logic***, vol. 47 (1990), pp. 99–119.

[4] HARTLEY ROGERS, JR., ***Theory of recursive functions and effective computability***, The MIT Press, Cambridge, Massachusetts, 1987. Reprint (with corrections) of the 1967 McGraw-Hill edition.

[5] ALAN D. VLACH, *Hyperarithmetical relations in expansions of recursive structures*, ***Annals of Pure and Applied Logic***, vol. 60 (1990), pp. 163–196.

► BENJAMIN WELLS, *Simple recursively-based pseudorecursive varieties of semigroups*.
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The equational theory of a variety may be decidable (recursive), as in the case of commutative semigroups, or undecidable, as in the case of relation algebras. In some sense, the *pseudorecursive* variety, announced elliptically in [1], is an intermediate notion: its equational theory is not recursive, but for every $n \in \omega$ the set of equations in the theory with n variables *is* recursive. (For relation algebras, even the variable-free equations in the theory form a nonrecursive set.)

Since 1983, six authors in four papers have constructed simple infinite recursive bases for pseudorecursive varieties of squarezero semigroups with additional operations. All have been inspired directly or indirectly by [2], the first published specification of a pseudorecursive variety. The author examines this growing cottage industry, reviews recent products, and reveals the latest models.

[1] A. I. MAL'CEV, ***The metamathematics of algebraic systems—Collected Papers: 1936–1967***, North-Holland, Amsterdam, 1971. Translator's notes by BENJAMIN WELLS, pp. 395, 473.

[2] BENJAMIN WELLS, *A simple pseudorecursive variety of infinite type*, ***Abstracts of Amer. Math. Soc.***, vol. 3 (1982), p. 592.

► DAN E. WILLARD, *Alternate versions of self-verifying axiom systems*.
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Define an axiom system A to be **self-verifying** iff

(i) one of the theorems implied by A is the statement of its own consistency, and
(ii) the axiom system is in fact consistent.

Kleene's Fixed Point Theorem [1] implies that every recursively enumerable axiom system A can be easily expanded into a broader system A^* which satisfies condition (i). The catch is that A^* can be inconsistent even while it asserts its own consistency, hence violating condition (ii). In the prior articles [3, 4], we showed how a self-verifying system, satisfying both conditions (i) and (ii), could prove all the Π_1 theorems of Peano Arithmetic without violating the constraints imposed by Gödel's Incompleteness Theorem. In this Abstract, we will describe two alternate systems which have additional nice properties.

Let d denote any cut-free rule of inference, such as Gentzen's cut-free sequent calculus, semantic tableaux or resolution. Let A denote any consistent axiom system. Define $f(x_1, x_2, \ldots, x_n)$ to be a **non-growth function** iff it satisfies $f(x_1, x_2, \ldots, x_n) \le \max(x_1, x_2, \ldots, x_n)$. Consider a self-verifying system, denoted as $\mathrm{IS}_d(A)$, that will consist of the five groups of axioms described below:

(1) An initial axiom group defining all the non-growth functions of A, as well as the growth functions of Addition and Scalar Multiplication. (Integer multiplication will not be a function under $\mathrm{IS}_d(A)$, but it can be implicitly represented as a relation through the non-growth function of Non-zero Division.)
(2) For any Π_1 sentence Φ, the sentence $\Phi \vee \neg\Phi$ will be an axiom.
(3) For any $\triangle_0$ formula Ψ and constant k, the sentence $\forall x \Psi(x) \supset \Psi(k)$ will be an axiom.
(4) Let $\mathrm{Prf}_A(\ulcorner\Phi\urcorner, x)$ be a formula asserting that x is the Gödel number of a proof of Φ from the axiom system A. Then for each Π_1 sentence Φ, $\mathrm{IS}_d(A)$ will have an axiom of the form $\exists \mathrm{Prf}_A(\ulcorner\Phi\urcorner, x) \supset \Phi$.
(5) $\mathrm{IS}_d(A)$'s Group-5 axiom will be a single sentence asserting that the entire $\mathrm{IS}_d(A)$

system (including this sentence looking at itself) is unable to support a proof of $0 = 1$.

Our first main theorem will state that if A is consistent then so must $\mathrm{IS}_d(A)$ be consistent. This theorem is stronger than our prior work [3, 4] because the latter did not include the Group-2 and Group-3 axioms. (It is a non-trivial generalization because Solovay [2] has shown essentially that the *cut-free* system $\mathrm{IS}_d(A)$ becomes inconsistent when its Group-2 axioms are extended to include for each sentence Φ, the axiom $\Phi \vee \neg\Phi$.)

It is also possible to develop versions of $\mathrm{IS}(A)$ that employ a substantially stronger notion of self-consistency than the Group-5 sentence above. Let Ψ denote an arbitrary sentence with n unbounded universal quantifiers, and $\Psi_k(p)$ denote a formula which differs from Ψ by requiring all Ψ's universal quantifiers to be bounded by k, and all its existential quantifiers to be bounded by the quantity pk. Let $\mathrm{IS}_d^R(A)$ be a modified version of $\mathrm{IS}_d(A)$ whose group-5 Class includes one axiom of the form $\forall x\{\mathrm{Prf}_{\mathrm{IS}_d^R(A)}(\ulcorner\Psi\urcorner, x) \supset \Psi_k(x)\}$, for each constant k and sentence Ψ. Then a stronger version of our theorem would state that if A is consistent then $\mathrm{IS}_d^R(A)$ must be consistent. Indeed, it is possible to develop a yet even slightly stronger system which asserts the preceding Group-5 reflection principle is valid in a fully uniform sense.

[1] S. C. Kleene, *On the notation of ordinal numbers*, ***Journal of Symbolic Logic***, vol. 3 (1938), pp. 150–156.

[2] R. Solovay, private communication, in a somewhat different notation, demonstrating how variants of $\mathrm{IS}_d(A)$(Peano) are inconsistent.

[3] D. Willard, *Self-verifying axiom systems*, ***Proceedings of the Third Kurt Gödel Symposium*** (1993), pp. 325–336, published in vol. 713, Springer-Verlag Lecture Notes in Computer Science.

[4] D. Willard, *Self-verifying axiom systems and the Incompleteness Theorem*, ***SUNY-Albany Computer Science TR 93-10*** (a 54 page manuscript, submitted for publication, whose results were orally presented at the 1994 Spring ASL conference in Gainesville, Florida).

▶ JINDŘICH ZAPLETAL AND SAHARON SHELAH, *Small forcings and Cohen reals*.
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Consistently all forcings of density $\aleph_1$ embed "$\aleph_1$ Cohen reals." We also derive this statement or its approximations from various forcing axioms.

Abstracts presented by title

▶ VLADIMIR KANOVEI, *On external Scott algebras and external definability in nonstandard models of Peano arithmetic*.
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D. Scott gave in [1] a necessary and sufficient condition for a countable set $A \subseteq \mathcal{P}(\omega)$ to be equal to SA^M, the *Scott algebra* of a **PA** model M, that is, the family of all sets $Z \subseteq \omega$ definable in M by a parameter-free **PA** formula.

It is evident that SA_M is exactly the collection of all arithmetical $Z \subseteq \omega$ provided M is an elementary extension of ω. One may, however, extend the *PA* language $\mathcal{L}$ by the unary predicate of standardness st interpreted, in all nonstandard models of **PA**, as being a member of ω. Let ESA^M, the *external Scott algebra* of M, denote the family of all subsets of ω definable in M by a parameter-free formula of the extended language $\mathcal{L}^{\text{st}}$. We give a necessary and sufficient condition for a countable family of subsets of ω to be an external Scott algebra for an elementary extension of ω.

THEOREM 1. *Let $\mathcal{Z} \subseteq \mathcal{P}(\omega)$ be countable. Conjunction of the following two conditions is necessary and sufficient for there to exist a countable $M \models$ **PA**, $M \succ \omega$, such that $ESA_M = \mathcal{Z}$:*
(1) *$\mathcal{Z}$ is arithmetically closed.*
(2) *$\mathcal{Z}$ contains $0^{(\omega)}$, the set of all (Gödel numbers of) true arithmetical sentences.*

The necessity is quite easy; the hard part is the sufficiency. To make every $Z \in \mathcal{Z}$ definable in M by a formula of the extended language, we use a coding system, which also gives a tool for the following definability theorem:

THEOREM 2. *Let $Z \subseteq \omega$, $n \in \omega$. There exists a countable model $M \models$ **PA**, $M \succ \omega$, such that Z is Σ^{st}_{n+2} in M but every $Y \subseteq \omega$, $Y \in \Sigma^{st}_{n+2}$ in M, is $\Sigma_{n+1}[0^{(\omega)}]$.*

By Σ^{st}_n we denote the class of all $\mathcal{L}^{\text{st}}$ formulas having Σ_n prefix, followed by a formula composed from recursive formulas by &,$\vee$, and quantifiers $\exists^{\text{st}}$ and $\forall^{\text{st}}$. $0^{(\omega)}$) in square brackets means that $0^{(\omega)}$) can participate as an extra parameter.

Notice that $0^{(\omega)}$) itself is $\triangle^{\text{st}}_1$ in M; moreover, any $Y \subseteq \omega$, $Y \in \Sigma_{n+1}[0^{(\omega)}]$ is Σ^{st}_{n+1} in any **PA** model $M \succ \omega$. Thus Theorem 2 tells us that a set Z of natural numbers can be made Σ^{st}_{n+2} but not Σ^{st}_{n+1} in an appropriate **PA** model $M \succ \omega$ unless it is essentially simple (belongs to $\Sigma_{n+1}[0^{(\omega)}]$.

The proof of both Theorem 1 and Theorem 2 is based on several forcing ideas, in particular, the notion of arithmetical forcing and generic ultrafilters.

[1] D. SCOTT, *Algebras of sets binumerable in complete extensions of arithmetic*, ***Recursive Function Theory, Proceedings of the Fifth Symposium in Pure Mathematics of the AMS***, 1962, pp. 117–121.

► GUN WON LEE, *CS ideal for properties and a definition of finiteness*.
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$\aleph_0 = \{0, 1, 2, \dots\}$ and $\beth_0 =$ the power set of $\aleph_0$ by definition. If $n \varepsilon \aleph_0$, n is said to be finite. Thus the *finiteness* has been defined. The poset of $\aleph_0$ is assigned for the properties of the elements of objects, whereas the elements of the set of natural numbers are assigned for the objects in question. However the properties are represented in terms of the equivalent sets of power set which are assigned to them. The equivalent sets of the power set in terms of their cardinalities are said to be the *ideal*.

There is an one-to-one mapping in between the ideal and the $\aleph_0$. Therefore the definition of the finiteness in the above can be an equivalent set of posets whose base is $\aleph_0$. This hypothesis assumes that the targets of measurements are properties, and the cardinalities of the objects are represented in terms of natural numbers. And the definitions of the finiteness can be fixed in this hypothesis at least. The operations admitted in this hypothesis are *closed* because of the infiniteness of the $\aleph_0$ set and also in the case of the $\beth_0$ set. This model can be posed as:

$$M = \{U, CS, LE, E, +\}.$$

► BILLY JOE LUCAS, *The logic of epistemic obligation, part 1*.
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Reasoning about epistemic obligation is hardcore philosophy, and goes back at least to Socrates, yet, oddly enough, reflection on the underlying principles behind such reasoning is quite rare. The basic motivation for this paper is to introduce a variety of alternative concepts of epistemic obligation and to study the logical structure of each such concept as well as to investigate how these concepts interact with each other. Two particularly important distinct families of such concepts are: *deontic* epistemic obligations, and *alethic* epistemic obligations.

The results in Part 1 presuppose as base systems a minimal deontic epistemic logic (MDE) and a minimal logic of alethic epistemic obligation (MAE). Let L be a language with $\sim$, $\vee$, $\bigcirc$ (it ought to be true that), and sK (s knows it is true that), as the only primitive operators. An MDE frame is a triple $\langle X, R, S\rangle$, with R serial and S reflexive. An MDE model is the usual sort of thing, with $\bigvee(\bigcirc p, x) = T$ iff $\forall y(xRy \supset \bigvee(p, y) = T)$ and $\bigvee(sKp, x) = T$ iff $\forall y(xSy \supset \bigvee(p, y) = T)$. Let L' be a language with these primitive operators: $\sim$, $\vee$, sOK (s has an alethic epistemic obligation to know it is true that). An MAE frame is a couple $\langle X, T\rangle$, with T reflexive, and an MAE model has $\bigvee(sOKp, x) = T$ iff $\forall y(xTy \supset \bigvee(p, y) = T)$. For languages that include both L and L', we use MADE frames (and their extensions): $\langle X, R, S, T\rangle$, with R, S, T, and truth clauses for $\bigcirc$, sK, and sOK as above. In this paper I'm concerned with what we can learn about those concepts of epistemic obligation which can be articulated on MDE (MAE) frames and extensions of them formed in just two ways: adding first-order definable conditions (separately) on any of R, S, and T; adding first-order definable principles that link at least two of R, S, and T. Subsequent parts of this study will focus on the epistemic obligation for additional extensions of MDE (MAE) frames, as well as for systems in which various principles sound on MDE (MAE) frames fail.

At least since Aqvist [1], deontic logics that extend *KD* (including *KD* itself) have been suspect as candidates for being the appropriate deontic systems to be used in combination with epistemic logics, and since not long after Hintikka [3], epistemic logics that extend *KT* (including *KT* itself) have been suspect as candidates for anything. MDE models make all of these suspect assumptions, and it turns out that study of such systems can substantially enrich our understanding of the logic of epistemic obligation. For example, Aqvist's paradox of epistemic obligation can be avoided without mutilating the basic deontic logic in the ways said to be required in Goble [2], and a philosophically *useful* interpretation of '*sK*' can be found relative to which a normal modal logic for knowledge has some plausibility.

[1] Lennart Aqvist, *Good Samaritans, contrary-to-duty imperatives, and epistemic obligations*, ***Nous***, vol. 1 (1967), pp. 361–379.

[2] Lou Goble, *A logic of good, should, and would*, ***Journal of Philosophical Logic***, vol. 19 (1990), pp. 169–199.

[3] Jaakko Hintikka, ***Knowledge and belief***, Cornell University Press, 1962.

► MARíA CLAUDIA MERÉ AND PAULO AUGUSTO VELOSO, *Definability in extensions by sorts*.
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The process of implementing a formal specification on another one is very important in formal software development and it can be described in terms of simple basic logical concepts. Formal specifications are presentations of theories in many-sorted logic, and an implementation of a formal specification A on another formal specification C amounts to an interpretation of A into a conservative extension B of C.

An extension consists of the addition of new symbols and axioms. The new symbols to be introduced correspond to sorts, functions, or predicates. When performing an extension one expects an increase in both deductive and expressive powers. On the other hand, one often does not wish to disturb the smaller theory. When the extension only adds properties of the new symbols, without disturbing the old ones, one has a **conservative extension**, which does not increase deductive power for the properties of the old symbols. Similarly, the increase in expressive power may be inessential, in which case our extension has the **property of eliminability**.

Extensions by the addition of function and predicate symbols are well studied in the literature. In particular, **extensions by definitions** of function and predicate symbols have some nice properties. Consider such an extension of theory T over language L to T' over L'. We then have conservativeness and eliminability. Consequently, we have the following properties:

- **Interpretability**. T' can be interpreted back into L.
- On the model-theoretic side, we have **Invariance under elementary equivalence**. Two models of T' are elementarily equivalent whenever their reducts to L are so.
- **Definability**. A relation is definable in a model of T' iff it is definable in its reduct to L.

The many-sorted case has not been deeply studied because of the existence of the well known reduction to unsorted logic. Our purpose here is to analyse conservative extensions that introduce new sorts. This is of importance because it occurs often in implementing formal specifications, when the new sorts are "constructed" from the concrete ones. We specify and analyse some well-known sort introduction constructs akin to those found in programming languages ([1]), namely cartesian product, discriminated union, subsorting and quotient.

In a previous work [2] we have axiomatised the specifications and established their adequacy in the sense of exhibiting the intended behaviour. In this work we centre our attention in establishing that these extensions are like extensions by definition of a new sort, even though we have proved that we have only unique expandibility up to isomorphism.

In the case of introduction of a sort, we have new variables over the new sort, and new universes realising the new sort. To cope with these, we use two ideas

- **connection**: a local translation of variables ranging over the new sort to the old sorts.
- **internalisation**: a view of relations involving the new sorts.

With these new ideas we will be able to generalise the above properties of extensions by definitions of predicate or function symbols to our sort introducing constructs. Actually, we will prove that there are no new definable relations, up to internalisation, which corroborates the intuitive feeling that these amount to extensions by definition of new sorts.

[1] C.A.R. HOARE, *Notes on data structuring*, ***Structured programming*** (O. J. Dahl, E. W. Dijkstra, and C.A.R. Hoare, editors), Academic Press, 1974, pp. 83–174.

[2] M. C. MERÉ and P. A. S. VELOSO, *On extensions by sorts*, ***Monografias em Ciência da Computação, No. 38/92 D.I., PUC-Rio***, 1992.

▶ ALBERT A. MULLIN, *Applications of recursive number theory to literature,* dedicated to the memory of Prof. S. C. Kleene $(23 \cdot 83;2 \cdot 997)$.
506 Seaborn Drive, Huntsville Alabama, USA.

Recall that the author has shown that *popular* Japanese poetry (e.g., Haiku and Tanka) is Eastern poetry in prime numbers with its own additive problems and recursive results for prime-number theory. We show that more *private* Japanese poetry (e.g., Sedōka and Dodoitsu) is Eastern poetry in bi-composites; e.g., with 38 and 26 syllables, respectively. Here recursive number-theoretic results on the number of essentially different representations of bi-composites provide estimates on the number of essentially different kinds of Sedōka and Dodoitsu. The methodology used is analogous to the author's representation theory for Haiku and Tanka which depends on use of primality results rather than bi-compositeness results. Poetry in *odd* bi-composites provides yet another motif for the lyrics of Japanese folk songs.

PROBLEM 1. Give an example of a (validromic) verse in one natural language which backwards is prose in a different natural language.

PROBLEM 2. Give an example of a WFF in one formal language which backwards is a WFF in a different formal language.