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The George Hughes Memorial Conference of the Australasian Association for Logic was held at Knox College of the University of Otago, Dunedin, New Zealand, on August 26–29, 1994. About forty people participated in the conference, which was organized by Pavel Tichy, the retiring AAL President.

The conference was held in cooperation with the Association for Symbolic Logic. Abstracts of contributed talks given (in person or by title) follow.

For the ASL Committee on Logic in Australia
MARTIN BUNDER, CHAIR

Abstracts of contributed talks

▶ ROSS T. BRADY, *Gentzenization of relevant logics with distribution*.
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I present Gentzenizations of the common sentential relevant logics, B, DW, DJ, TW, TJ, RW, T and R. We use Inversion Rules and t-rules, in addition to the customary structural and connective rules, in order to eliminate the Cut-rule. In interpreting the Gentzenizations back into the logics, we need to add classical negation '$-$', fusions 'o' and 'o_i' and the sentential constant 't'. Either or both fusions will be deleted for some logics. We then show conservative extension results for each of the logics, with respect to these additions, so as to establish the Gentzenizations for the logics without these additions.

First, we show the conservative extension result for B, i.e., if A is provable in $\mathrm{Bt}^{--\mathrm{oo_i}}$ then A is provable in B, for formulae A without occurrences of 't', '$-$', 'o', and 'o_i'. The method uses a reduced Routley-Meyer semantics for B with the special postulate, $\mathrm{RT}bc \iff b = c$. The key step is the completeness argument for B, in which this postulate is ensured by adding two extra elements $\mathrm{B}'$and $\mathrm{B}^{*\prime}$ to the set of prime B-theories. Use is made of the work of Meyer, Slaney, Sylvan, Giambrone, Priest and Restall in establishing this for B and the other logics.

▶ MARTIN BUNDER AND NORMAN MEGILL, *Weaker D-complete logics*.
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To show that an implicational logic is D-complete it is enough to show that all substitution instances of its axioms are deducible in the corresponding logic from the condensed detachment rule D alone.

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It is well known that every logic with axioms from the set (I), (B), (B′), (C), (K) is D-incomplete. Meyer and Bunder (1988) showed that BB′IW is D-complete.

The main result of this paper is that there is an infinite sequence of independent D-complete systems that are all weaker than BB′IW. We also consider the question: Given a D-incomplete system, how many substitution instances of the axioms must be added to make the system D-complete? The answer to the question is a finite number (such as 1) in some cases, an infinite number in others and an unknown number in most.

► CRISTIAN CALUDE, *Gödel's incompleteness theorem: An information-theoretic perspective*.

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One of the main applications of *Algorithmic Information Theory* [1, 3] concerns the incompleteness phenomenon discovered by Gödel. The major results are due to G. Chaitin [4, 5]; they point out information-theoretic limits of Chaitin's results. Using topological tools as described in [2], we show that the incompleteness in Chaitin's sense is pervasive.

[1] C. Calude, ***Information and randomness: An algorithmic perspective***, Springer-Verlag, 1994 (in press).

[2] C. Calude, H. Jürgensen, and M. Zimand, *Is independence an exception?*, ***Appl. Math. Comput.*** (in press).

[3] G. J. Chaitin, ***Algebraic information theory***, Cambridge University Press, 1987. (third printing 1990)

[4] G. J. Chaitin, ***Information-theoretic incompleteness***, World Scientific, 1992.

[5] G. J. Chaitin, *The limits of mathematics: Course outline and software*, IBM Watson Center, December 12, 1993.

► PHILIP CATTON, *Problems with a deductivist image of scientific reasoning*.

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Deductivism as an approach to the characterisation of scientific inference has from the vantage point of several philosophers' concerns some significant merits. I briefly canvass these. Yet despite these merits, I believe the deductivist image of scientific reasoning is pie-in-the-sky. Actual sciences have not the perfection of logical organisation required for the deductivist image to ring true. The point moreover is not a merely psychologistic one, that actual scientists fail to reason logically. Rather it concerns how enormously difficult it is to achieve in a science the perfection of logical organisation required for the deductivist image to ring true. To argue this point, I look to where support for this point seems least likely. I look to the most exact and rigorous of sciences, mechanics. Just as it was historically extraordinary difficult so to formulate Euclidean geometry that proof procedures become discursive deductive logical, so the task of bringing to mechanics this degree of perfection of logical organisation is enormously challenging, so difficult, indeed, that to this day it has not been satisfactorily carried through. This has consequences for the perspicuity of mechanics, and by exploring these consequences I explore how far even of the most exact and rigorous of sciences the deductivist image fails to ring true.

► BRIAN F. CHELLAS, *S1, close relatives, classical precursors*.

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The paper reports research carried out jointly with Krister Segerberg and presents a uniform approach to the modal logics S1, S1°, S0.9, and S0.9° by way of classical, "prenormal" logics. Building on work of Cresswell, we develop semantics for prenormal logics and prove determination and decidability. Then we show that S1, S1°, S0.9, and S0.9° are non-classical extensions ("Lewis versions") of certain prenormal logics, determination and decidability for which are immediate. We also use generalizations of a key schema and equivalent inference rule to characterize a class of "Cresswell logics". In light of these we ask some questions about C. I. Lewis's concept of strict implication in S1.

[1] M. J. CRESSWELL, *The completeness of S1 and some related problems*, ***Notre Dame Journal of Formal Logic***, vol. 13 (1972), pp. 485–496.

[2] M. J. CRESSWELL, *S1 is not so simple*, in ***Modality, morality and belief; essays in honor of Ruth Barcan Marcus***, (Walter Sinnot-Armstrong, Diana Raffman and Nicholas Asher, editors), Cambridge University Press, Cambridge, 1994.

[3] ROBERT FEYS, ***Modal Logics***, edited with some complements by Joseph Dopp. Louvain: E. Nauwelaerts, and Paris: Gauthiers-Villars, 1965.

[4] E. J. LEMMON, *New foundations for Lewis modal systems*, ***Journal of Symbolic Logic***, vol. 22 (1957), pp. 176–186.

[5] C. I. LEWIS and C. H. LANGFORD, ***Symbolic Logic***, New York, 1932, second edition, Dover Publications, 1959.

► M. J. CRESSWELL, *Incompleteness and the Barcan formula*.
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A (normal) system of propositional modal logic is said to be complete if and only if it is characterised by a class of (Kripke) frames. One of the simplest incomplete logics is K+

$$\text{H} \qquad L(Lp \equiv p) \supset Lp.$$

When we move to modal predicate logic the question of completeness can again be raised. It is not hard to prove that if a predicate modal logic is complete then it is characterised by the class of all frames for the propositional logic on which it is based. Nor is it hard to prove that if a propositional modal logic is incomplete then so is the predicate logic based on it. But the interesting question is whether a complete propositional modal logic can have an incomplete extension. In 1967 Kripke announced the incompleteness of a predicate extension of S4 + $LMp \supset MLp$. It is now known that such incompleteness is common. Montagna in 1984 proved that the predicate extension of KW is one such, since the wff

$$\exists x M\phi x \,\&\, \forall x \exists y L(\phi x \supset M\phi y)$$

forces a transitive and irreflexive frame to have an infinite chain, yet its negation is not a theorem. It is perhaps not surprising that a non-compact logic has an incomplete predicate extension since a single wff of the lower predicate calculus (LPC) can often take the place of an infinite set of propositional wff. What is more surprising is that complete systems whose class of frames are definable by a single first-order sentence can have incomplete extensions. What is even more surprising is that it can make a difference whether these systems contain the *Barcan Formula*, which can be stated by the schema

$$\text{BF} \qquad \forall x L\alpha \supset L\forall x\alpha.$$

The purpose of the present talk is to present several such systems. In the first group it is the systems *with* the Barcan Formula which are incomplete, while those without it are complete. In the second group it is those *without* the Barcan Formula which are incomplete, while those with the Barcan Formula are complete. Yet the class of frames is definable in each case by a single first-order sentence. A point worth noting is that, unlike incomplete propositional logics, which often look as if they have been 'cooked up' simply to provide examples of incomplete logics, these incomplete predicate logics are based on propositional logics which have a history going back to the late fifties or early sixties.

The results I shall be presenting are not new, although the importance of the Barcan Formula has not always been appreciated. However the proofs already in the literature use complicated mathematical techniques, and one of the purposes of this paper is to give alternative proofs using only methods standard in modal logic.

▶ JOHN N. CROSSLEY, *Natural deduction and linear logic*.
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We introduce a natural deduction system for linear logic equivalent to Girard's. The motivation is to be able to prove a strong normalization theorem. The proof will be outlined but there will also be a discussion of what is required of a natural deduction system and why the notion of substitution of proofs for proofs is so important—and difficult.

▶ RODERIC A. GIRLE, *The dynamics of belief revision*.
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Belief revision is the process of changing a set of beliefs by either adding beliefs, giving up beliefs, or doing both. In classical AGM belief revision theory the belief sets are logically consistent and closed under logical consequence. The addition of beliefs consistent with a belief set gives the expansion of a belief set, and the giving up of beliefs gives the contraction of a belief set. The addition of beliefs which are inconsistent with the belief set require the giving up of those beliefs inconsistent with the new beliefs, a contraction, and the expansion of the set. Such a process is called revision. If the set of beliefs is seen as a state description, the changes can be seen as state changes. Dynamic Logic provides a logic for reasoning about such processes of change. In the standard semantics for Dynamic Logic, the states are seen as possible worlds and the changes from state to state are seen as accessibility relations between worlds. In this paper we explore the extension of Dynamic Logic by adding to it a set of belief change processes to match the AGM processes of expansion, contraction and revision. We show that with the appropriate definitions of the three processes of belief revision we get a system equivalent to the AGM system.

▶ ROBERT GOLDBLATT, *Bisimulation and modal equivalence*.
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This talk will discuss the Hennessy-Milner modal logic of processes (finitary version) in relation to notions that have become standard in modal model theory (filtrations, bounded morphisms, modal saturation, preservation theorems).

A *labelled transition system* $(\mathcal{P}, \{\xrightarrow{\alpha} : \alpha \in \mathcal{A}\})$ comprises a collection of binary transition relations $\xrightarrow{\alpha}$ on a set $\mathcal{P}$ of *processes* (or *states*), indexed by a set $\mathcal{A}$ of *action labels* α. The associated modal language is the variable-free set of sentences A given by the abstract syntax

$$\mathrm{A} ::= \bot \mid \neg A \mid A_1 \wedge A_2 \mid [\alpha]A.$$

The satisfaction relation $P \models A$ between processes and states is defined in the usual way. Two states are *logically equivalent* if they satisfy the same modal sentences. A system has the *Hennessy-Milner Property* if this logical equivalence relation is a bisimulation (and hence is the maximal bisimulation on $\mathcal{P}$). Hennessy and Milner [1] showed that this always holds under the *image-finiteness* assumption that $\{ Q : P \xrightarrow{\alpha} Q \}$ is finite for all α and P.

Associated with a given transition system are new definable transitions $\overset{\alpha}{\Rightarrow}$ having $P \overset{\alpha}{\Rightarrow} Q$

if and only if $P \models [\alpha]A$ implies $Q \models A$. The system is $\mathcal{H}$-closed if $P \overset{\alpha}{\Rightarrow} Q$ implies $P \xrightarrow{\alpha} R$ for some process R logically equivalent to Q.

We prove

- Every image-finite system is $\mathcal{H}$-closed (converse false).
- Every $\mathcal{H}$-closed system has the Hennessy-Milner Property (converse false).
- Every $\mathcal{H}$-closed system has exactly one filtration (quotient).
- A system is $\mathcal{H}$-closed if and only if
  - (1) it has the Hennessy-Milner Property, and
  - (2) it has exactly one filtration.
- For any system $\mathcal{S}$, the following are equivalent:
  - (1) $\mathcal{S}$ satisfies the Hennessy-Milner Property;
  - (2) logical equivalence between $\mathcal{S}$ and its least filtration is a bisimulation;
  - (3) the quotient map from $\mathcal{S}$ to its least filtration is a bounded morphism.

[1] MATTHEW HENNESSY and ROBIN MILNER, *Algebraic laws for nondeterminism and concurrency*, ***Journal of the Association for Computing Machinery***, vol. 32 (1985), pp. 137–161.

[2] ROBERT GOLDBLATT, *Saturation and the Hennessy-Milner property*, in ***Modal logic and process algebra***, (Alban Ponse, Maarten de Rijke and Yde Venema, editors), CSLI Lecture Notes No. 53, CSLI Publications, Stanford (1995), pp. 107–129.

► J. ROGER HINDLEY, *How many proofs can a formula have?*
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In the propositional logic of implication, formulae provable by the intuitionistic natural deduction rules correspond exactly to types in simple type theory that are inhabited by lambda-terms. And proofs and lambda-terms also correspond. Thus the well known decision-procedures for intuitionistic logic translate into lambda-calculus as algorithms to tell whether the set of inhabitants of a type is empty. This talk will describe an extension of these algorithms that will tell exactly how many irreducible members this set has. In particular, the algorithm will tell in finite time whether the number of such members is infinite. The algorithm is due to C. Ben-Yelles.

► DAVID LEWIS, *Humean supervenience debugged.*
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The thesis of 'Humean Supervenience' says that the whole truth about a world like ours supervenes on the spatiotemporal distribution of local qualities—for short, on history. Elsewhere I've argued that there was one 'big bad bug' for Humean Supervenience: chance, in other words single-case objective probability. But now, thanks to a suggestion of Michael Thau, I think I know how to get around the problem.

The problem: Suppose present chances supervene on the whole of history, future as well as present and past. (This would be so if present chances were given by actual frequencies throughout all time; but that's only one especially simple case.) Suppose two different futures each have non-zero chance of coming about; and suppose these two different futures would complete two different chance-making patterns, which in turn would make different present chances. Then there is some chance that the present chances would be other than they actually are. Then we get inconsistent results if we apply the 'Principal Principle', which tells us what our subjective probabilities for future events should be, conditional on hypotheses about present chances.

The old part of the solution: We know that the PP doesn't apply if we conditionalize on hypotheses that give us direct information about future history. Call this *inadmissible* information. If Humean Supervenience is true, then information about present chances is, in part, information about chance-making patterns in future history. So the PP *never* applies. That solves the problem; but it seems to clobber the PP altogether, leaving us no way at all to relate chance to degrees of belief.

The rest of the solution, due to Thau: A hypothesis about present Humean-Supervenient chances is never perfectly admissible, but it may come close enough. Inadmissibility admits of degree. For some uses of the PP, namely, the very ones we make in practice, the error introduced by not-quite-perfectly admissible hypotheses about chance can be utterly negligible.

It's an old story. Maybe nothing could *perfectly* deserve the name 'sensation' unless it were infallibly introspectible; or the name 'simultaneity' unless it were a frame-independent equivalence relation; or the name 'value' unless it were 'objectively prescriptive'; or the name 'chance' unless information about its present distribution were perfectly admissible. In each case, an imperfect candidate may deserve the name quite well enough.

▶ ED MARES, *A star-free semantics for relevant logic*.

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In this paper, I develop an intuitive semantics for R and some close neighbours which does not use the Routley star operator. This semantics is based on the Routley-Meyer semantics for negation-free R. It uses a set of worlds and a ternary relation. Moreover, it distinguishes between possible and impossible worlds and a false constant 'f' is said to hold at all and only impossible worlds. Negation is defined implicationally, viz. $\sim A =_{\text{def}} A \rightarrow f$. It is shown that by placing constraints on the ternary relation, a semantics can be constructed over which R is complete. In addition, I explore the semantics for some intuitionistic and intermediate relevant logics.

▶ ROBERT K. MEYER AND E. P. MARTIN, *S* (*for syllogism*).

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We discuss Logic Past, Logic Present, and Logic Yet to Come. Logic Past is Aristotelian logic; Logic Present is modern formal logic; Logic Yet to Come is, we claim, the system S, based on the prefixing and suffixing axioms. (These are B and B′, for combinatory logic fans.) Closely related is P-W (T-W, in some circles), which adds the I identity axiom $A \rightarrow A$. Martin in his Ph.D. thesis (ANU, 1978) showed that I is *totally* independent of the B and B′principles—that is, *no* instances of $A \rightarrow A$ are provable in S. We call such a formula *universally invalid*. By contrast, a formula is *potentially valid* if *some* instance of it is a theorem. If *every* formula of a system T is potentially valid, the system is *lax*. Most systems of implicational (or even positive) logic are in fact lax; classically or intuitionistically (or even in relevant logics like R and E), simply substituting $A \rightarrow A$ for all sentential variables p in an arbitrary positive formula produces a theorem. But, by Martin's result, $p \rightarrow p$ is universally invalid in S; as a corollary, we can find universally invalid formulas in P-W as well; for example, $p \rightarrow ((p \rightarrow q) \rightarrow q)$ is such a formula. In conclusion, S honours syllogistic principles, but nonetheless begs no questions; it is a *progressive* logic, validating only arguments that *go* somewhere.

▶ CHRIS NOWAK, *Logics of combinatorial possible worlds*.

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In this paper we will try to discuss logics of combinatorial possible worlds. These worlds are built up from states of affairs associating particulars (individuals) with universals (properties). We will first review results obtained in applying combinatorial worlds to an analysis of necessity and possibility; these include such concepts as concept lattice, concept entailment, accessibility and similarity between combinatorial possible worlds, and systems of similarity spheres. Similarity spheres appear not only in Lewis' counterfactual reasoning, but also in the area of belief revision. Combinatorial possible worlds seem to form an interesting framework for knowledge representation issues, including ontological ones. The first step is to develop logics of combinatorial possible worlds, but many issues relating to belief revision, nonmonotonic reasoning, and knowledge representation in general might be clarified as well.

► GRAHAM PRIEST, *On an observation of Hilbert and Bennays*.
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The paper is a discussion of a result in Hilbert and Bernays in their *Grundlagen der Mathematik*. Their interpretation of the result is similar to the standard interpretation of Tarski's Theorem. This and other interpretations of the result are discussed, and shown to be inadequate. Instead, it is argued, the result refutes certain versions of Meinongianism. In addition, it poses new problems for classical logic that are solved by dialetheism.

► KRISTER SEGERBERG, *Belief revision from the point of view of doxastic logic*.
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In 1985 Alchourrón, Gärdenfors and Mackinson presented their now classic theory (AGM) of theory change (belief revision). In 1988 Adam Grove, generalizing David Lewis' theory of counterfactuals, presented a model theory suitable for the AGM theory. Although AGM and Grove mentioned object languages, neither used them. But recently, Maarten de Rijke has shown how object languages can be brought into the picture. In the present paper we take de Rijke's idea further, addressing the question whether there is a particular doxastic or epistemic logic implicit in the AGM.

► PAVEL TICHY, *Cracking the natural language code*.
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Current grammatical theories either do not aspire to generate interpretations of well-formed English strings along with the strings themselves, or else attempt to do it indirectly by generating translations of those strings into some other language. In either case the coding system peculiar to the English language remains unexamined. To decipher the code of English, or any other natural language, one has to construct a generator of string-meaning pairs such that the second component of a pair is the actual denoting complex expressed by the first component. The paper reports on progress made towards developing a grammar of this sort.

Abstracts presented by title

► B. J. COPELAND, *Prior's role in the development of temporal logic and the possible worlds semantics*.
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Logics that form the basis of a number of currently much-studied software technologies were invented by the New Zealand philosopher A. N. Prior. For the $\Box$ (necessity) and $\Diamond$ (possibility) operators of standard modal logic he substituted a variety of other operators which he called quasi modal operators. An example is the K operator (it is known that) of epistemic logic. Prior's quasi modal operators migrated from philosophy to computer science via various mathematical logicians who were associated with departments of computer science. To date the tense operators have proved to be the most useful of Prior's inventions. His tense logic has found numerous applications in computer science, for example logical deduction involving time-dependent data, natural language processing, knowledge representation, planning, database management, specification and verification of concurrent programs, and VLSI. Prior's discovery of the so-called possible worlds semantics, now regarded as a representational system of great generality, was a direct result of his work on tense logic. (Priority is commonly assigned to Kripke but Prior, in collaboration with the Irish logician Carew Meredith, arrived at the essentials of the semantics several years ahead of Kripke.) It is noteworthy that two of the major forces in the genesis of these various software technologies were a love of ancient and medieval logic and a philosopher's concern to make conceptual room for freedom of the human will.

► STAN J. SURMA, *On closure operators one-to-one associated with fixed object languages*.
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In its spirit, this paper is an exercise in algebraic logic and it aims at providing a unifying conceptual and terminological schema for generation of closure operators one-to-one associated with each reduct of an arbitrary but fixed object language. The main result of Part One is the following theorem.

Theorem 1 (Normal Form Theorem). *If* Cn *is a regular* $(\neg)$*-closure operator over the set* S *of all sentences of an object language with negation* $\neg$ *and an extra* m*-ary connective* #, *then the conditions below are equivalent*:

(i) Cn *is* (#)*-normal,*
[*i.e.,* $\mathrm{Cn}(X, \#A_1A_2\dots A_m) = \bigcap\{\,\mathrm{Cn}(X, A_1[f_j], A_s[f_j], \dots, A_m[f_j]) : j \leq 2^m$ and $f_j(\#A_1A_2\dots A_m) = 1\,\}$, *where* f_j *is a valuation function (a homomorphism) from* $(S, \#)$ *to* $(\{0,1\}, \underline{\#})$ *such that* $\underline{\#}$ *is an operation on* $\{0,1\}$ f_j*-correlated with* #, *and where* $A_i[f_j] = A_i$ *if and only if* $f_j(A_i) = 1$*; and* $A_i[f_j] = \neg A_i$ *if and only if* $f_j(A_i) = 0$ *for any* $A_1, A_2, \dots, A_m \in S$ *and any* $X \subseteq S$.]

(ii) Cn *is* (#)*-stratified.*
[*i.e., the fact that* $\#A_1A_2\dots A_m \in X$ *is equivalent to the fact that* $\#^*(A_1 \in X)(A_2 \in X)\dots(A_m \in X)$, *where* $\#^*$ *is the meta language correlate of* #, *for any* $A_1, A_2, \dots, A_m \in S$ *and any* Cn*-maximal* $X \subseteq S$.]

In Part Two a uniform way of weakening of the above concept of (#)-stratification is proposed so as to make a classical closure-theoretic set up a non-classical one. An intuitionistic, a Johansson-minimal and a modal closure-theoretic framework are given as an illustration.

The rest of the terminology used in this abstract is as follows. An operator Cn is a closure operator if and only if it is inclusive (i.e., $X \subseteq \mathrm{Cn}(X)$ for any $X \subseteq S$); monotonic (i.e., the fact that $X \subseteq Y$ implies that $\mathrm{Cn}(X) \subseteq \mathrm{Cn}(Y)$ for any X, $Y \subseteq S$); and idempotent (i.e., $\mathrm{Cn}(\mathrm{Cn}(X)) \subseteq \mathrm{Cn}(X)$ for any $X \subseteq S$). For any $A \in S$ and $X \subseteq S$, X is (Cn, A)-maximal if and only if $A \notin \mathrm{Cn}(X)$ and $A \in \mathrm{Cn}(X, B)$ for any $B \notin X$. X is Cn-maximal if and only if X is (Cn, A)-maximal for some A. An operator Cn is regular if and only if, for any $A \in S$ and $X \subseteq S$, the fact that $A \notin \mathrm{Cn}(X)$ implies that there is $Y \in 2_X$ such that $A \notin \mathrm{Cn}(Y)$ and $A \in \mathrm{Cn}(Y, B)$ for any $B \notin Y$. The expression $Y \in 2_X$ means that $X \subseteq Y$. Cn is a $(\neg)$-operator if and only if Cn is $(\neg)$-compact (i.e., $\mathrm{Cn}(A, \neg A) = S$ for any $A \in S$) and Cn has the $(\neg)$-cut property (i.e., $\mathrm{Cn}(X, A) \cap \mathrm{Cn}(X)$ for any $A \in S$ and any $X \subseteq S$).