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1994 SPRING MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Kansas City, Missouri, May 6–7, 1994

The Spring Meeting of the Association was held in Kansas City, Missouri, on May 6–7, 1994, in conjunction with the Central Division of the American Philosophical Association. The program included a panel discussion, two invited lectures, nine contributed papers, and four papers presented by title. The program committee consisted of Larry Moss and Colin McLarty (chair).

The panel on *Category Theory and the Foundations of Mathematics and Logic* consisted of John Bell, Peter Freyd, and Vaughn Pratt; Colin McLarty was the moderator.

The following invited talks were presented:

Shaughan Lavine (Columbia University), *Understanding the infinite*.

L. Thorne McCarty (Rutgers University), *Indefinite information in commonsense reasoning*.

Abstracts of contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
COLIN MCLARTY, CHAIR

Abstracts of contributed talks

► STEVEN MICHAEL AWODEY, *Axiom of choice and excluded middle in categorical logic*.
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A notion of constructive, higher-order logic which includes the axiom of choice (AC) but not excluded middle (EM) cannot be modeled in the standard way in topoi, by a well-known result of Diaconescu. Certain predicative type theories are related to locally cartesian closed (LCC) categories the way higher-order logic is to topoi. A proof of AC is given in the LCC category setting, thus valid in any such type theory. Since topoi are LCC this provides a way of modeling constructive logic with AC in topoi where EM still fails in general. It is further shown that EM holds just in case AC holds under the standard interpretation of higher-order logic in topoi, and that this in fact obtains just when the two interpretations agree, i.e., when they give all the same logical truths.

► JON BARWISE, *Substructural logics and a model of information flow*.
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Informally, an information network is a constellation of components connected together in ways that systematically supports the extraction of information about components from

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information about other components. In a recent paper, "On the logic of information flow," Gabbay, Haronis, and I have explored Gentzen systems based on a formalization of this idea of an information network. Two of these systems bear striking similarities to the Lambek Calculus (LC). Indeed, they can be thought of as systems where the circle of the LC splits into an application operator and a composition operator. In this talk I will define and illustrate information networks and exhibit the complete calculus for these networks. This work grew out of noting remarkable formal similarities between my work on situation theory and Gabbay's work on labeled deductive systems, and also out of joint work of mine with Jerry Seligman on imperfect information flow.

▶ STEPHEN HARNISH, *An empirical set theory: Analyzing fractals via quantified resolutions*.
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Fractal images and many natural phenomena exhibit "complex structure" at "varying levels of detail." One way of making this observation precise follows. Assume "images" are arbitrary subsets of $\mathbf{R}^n$ or some other metric space, M. We'll consider simple conceptions of "image resolution" by introducing operators which simplify or "resolve" the images at certain levels of resolution. These operators have algebraic properties reminiscent of universal and existential quantification. Finally, we'll examine several schemes for representing the structure of images by directed graphs and interpreting the directed graphs as nonwellfounded sets.

An empirically chosen universe of sets results from the choices of the metric space, the scheme for graphical representation, and the criterion for set equality within a particularly nonwellfounded set theory. I'll focus on the case where M is the real plane, $\mathbf{R}^2$, and the decreasing-existential scheme of graphical representation is employed. The resulting graphs are trees with countable branching.

▶ ROBERT LUBARSKY, *Kripke models for independence results in IKP*.
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Classically, Σ_1-Dependent Choice, Resolvability, and Π_2-Reflection are all inequivalent over KP, but equivalent in L. However, using only intuitionistic logic, the inequivalence carries over even into L. The constructions all hinge on turning the classical forcing arguments for independence into constructions of Kripke models, and moreover into constructions of certain ordinals in Kripke models. The details of one of the proofs were given.

▶ CHRISTOPHER MENZEL, *Singular propositions and modal logic*.
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By *Russellian semantics* I mean the conjunction of the following three theses: (1) There is no more to the semantics of a name than the object referred to. (2) The meaning of a sentence F is an abstract entity, the *proposition* $[F]$ it expresses. (3) Propositions are structurally complex, that is, roughly, (i) they have an internal structure that corresponds directly to the syntactic structure of the sentences that express them, and (ii) the metaphysical constituents of that structure are the semantic values of the corresponding syntactic constituents of those sentences.

Consider a sentence containing a name—the sentence 'Quine is a philosopher', say. If

Russellian semantics is correct, it follows that the proposition this sentence expresses contains *Quine himself* as a constituent in that part of its metaphysical structure corresponding to the name 'Quine'. But this raises a puzzle. Intuitively,

$$\text{Quine might not have existed.} \qquad (*)$$

That is, the proposition [Quine does not exist] could have been true. But if Russellian semantics is correct, there would have *been* no such proposition as [Quine does not exist] if Quine hadn't existed, hence it wouldn't have been there to be true; rather, there would have been no information about Quine whatever. Since of course [Quine does not exist] also fails to be true when Quine does exist, it follows that, necessarily, whether he exists or not the proposition that he doesn't is not true. Thus, Quine's nonexistence is not really possible, and hence, contrary to our initial intuition, $(*)$ isn't true after all.

In this talk I argue for a logical solution to this puzzle that does minimal violence to classical quantified modal logic while preserving both the structured view of propositions and the intuition that $(*)$ is true. The framework for this solution is a first-order modal language enriched with complex predicates/terms that, intuitively, denote structured properties, relations, and propositions.

► JAMES OTTO, *The linear time hierarchy* (Δ_0) *via tiers and monoidal categories*.
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We find the functions computable in linear time among the primitive recursive functions. We do this via tiers, to restrict the definability of nested loops, and via weakening product to tensor, to make duplication of variables definable only by explicit loops.

We then find that the linear time hierarchy relations, which Woods [9] has shown are the bounded arithmetic relations (Δ_0), are the tier 0 quantifications of the linear time relations, and that the recursively enumerable relations are the tier 1 existential quantifications of the linear time relations.

Our characterization of the linear time functions differs from similar work of Bloch's [3] by not allowing diagonal inside loops.

Our characterization of the linear time hierarchy relations differs from Wrathall's [11] in replacing bounded quantifications with (not explicitly bounded) tier 0 quantifications. Bellantoni [1] has a related characterization of the polynomial time hierarchy functions.

As is usual in recursion theory, we characterize the linear time functions as the smallest set of maps closed under certain rules. However the maps are in set^2 rather than in *set*. This is our understanding [7] of tiers [2, 5].

As our rules include composition and identities, we collect them into a doctine $\mathcal{D}$ (= restricted class) of categories $\mathcal{C}$. However, rather than have $\mathcal{C}$ be FP (= having finite products) as is often implicit in recursion theory, we take $\mathcal{C}$ to be monoidal [6]. In particular $\mathcal{C}$ is closed under tensor $\otimes$ rather than under product $\times$. We further take $\mathcal{C}$ to have 2 tiers of dyadic numbers and to have flat recursion [5, 4] and left very safe recursion [3, 8].

The image in set^2 of the initial category $\mathcal{I}$ in our doctrine $\mathcal{D}$ is then both what is definable in set^2 by our doctrine and the smallest set of maps in set^2 closed under our rules. Our rules restrict the rules of Román [10] and Paré-Román [8] for numbers in FP categories and monoidal categories.

[1] S. BELLANTONI, ***Ph.D. thesis***, Department of Computer Science, University of Toronto, 1992.
[2] S. BALLANTONI and S. COOK (editors), ACM STOC Proceedings, 1992.
[3] S. BLOCH, e-mail, 1992.
[4] R. COCKETT, conversation, 1993.
[5] J. MARION and D. LEIVANT, e-mail, 1992.

[6] S. MacLane, ***Categories for the working mathematician***, Springer-Verlag, 1971.
[7] J. Otto, *Kalmar, linear space, and P*, 1993; Notes on Pspace, 1993.
[8] R. Paré and L. Román, ***Studia Logica*** (1989).
[9] J. Paris, A. Wilkie, and G. Wilmers (editors), ***Logic colloquium 84***, 1986.
[10] L. Román, ***Journal of Pure and Applied Algebra*** (1989).
[11] C. Wrathall, ***SIAM Journal on Computing*** (1978).

► MASARU SHIRAHATA, *The extension of linear set theory with equality*.
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In this paper, we study set theory based on linear logic with the equality relation. In particular, we present the system AZF with equality, which is an extension of the author's system LZF and a conservative extension of Zermelo-Fraenkel set theory without the regularity or ZF^-.

The motivation to formulate set theory in linear logic is that we can have the unrestricted comprehension in such a system without being inconsistent. However, Grishin showed that the equality relation induces the contraction over equations and, combined with the extensionality, the contraction over any formulas. He concluded from this that linear set theory with the extensionality axiom is inconsistent.

Given this result, we formulate AZF with the equality relation but without the extensionality. AZF has the unrestricted comprehension, and the contraction over equations as a rule of inference from the beginning. We then show by the cut-elimination argument that AZF is a conservative extension of ZF^- and therefore consistent.

With equality at hand, it becomes possible to define in general basic mathematical constructions such as ordered pairs, relations and functions. We work out such constructions in some detail and see the peculiarities unique to linear set theory, as well as the limitation imposed by the lack of extensionality. Finally, to clear the way for the future work, we show that Grishin's result does not hold if we formulate the extensionality with the exponentials, which were not available to him.

► LAWRENCE NEFF STOUT, *Generalizing categorial logic—What can be gained?*
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Category theory can be applied to logic at several different levels: proof theory at the propositional level can be viewed in terms of cartesian closed categories; first order logic can be obtained using locally cartesian closed categories to get universal and existential quantification; higher order logic can be obtained using subobject classifiers. Each level can be generalized.

We can obtain propositional logics with alternative forms of conjunction by considering monoidal closed structures. The first order logic can arise from factorization systems (as in Monro's article *Logic, sheaves, and factorization systems*, ***The Journal of Symbolic Logic***, vol. 58, 1993) coupled with a local monoidal closed structure. Monoidal systems can introduce noncommutativity or nilpotence. Factorization systems can introduce nondistributivity.

Existential quantification comes from factorization, universal quantification comes from the local monoidal structure. If the factorization system respects the monoidal structure a satisfactory typed proof theory can be given for first order logic. This logic is best thought of as dealing with inferences about predicates rather than statements about individuals. It can take into account limitations on resources and multiple use of the same data.

Subobject classification is a very strong condition even with the subobjects are generalized using factorization systems, essentially forcing the logic to resemble topos logic. Weakening the conditions on subobject classification by allowing nonunique classifying maps and a generic subobject which is not the terminal object will allow for higher order logic with more variety. The uniqueness condition can be replaced by functorial canonical choices often arising from such additional structure as inf-complete orders on hom sets.

Abstracts presented by title

▶ GUNWON LEE, *ROM: A realization of model.*
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A model described in terms of a theory in a language can be realized. It presupposes that the model in principle is a conceptual model or a conventional device as the models of metaphors in John R. Searle, which I think is not free from the ontological commitments. When a mathematical model given as a theory in the number theories in general has been realized as a physical machine, such as the finite states machine; finite state can be said to be a ROM. A symbolically realizable model of G. Lee; SROM is a model which can be realized as a model usable for the word processing in principle. However if we are restricting the model as the physically realized model PROM, then PROM is a subset of SROM. SROM is not free from the ontological commitment unless we specify the ROM as in practice as in S. Kleene as e-realization, I think. However the fuzziness of the physicality remains as the open question.

▶ ZHOU XUNWEI, *Predicate logic and set theory are the same discipline*.
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In classical logic, logic and set theory are regarded as different disciplines, yet their theorems are strikingly similar. Let's compare theorems in logic and set theory:

	Logic	Set theory
1	$\neg\neg P \iff P$	$\bar{\bar{A}} = A$
2	$P \vee P \iff P$	$A \cup A = A$
3	$P \wedge P \iff P$	$A \cap A = A$
4	$P \vee Q \iff Q \vee P$	$A \cup B = B \cup A$
5	$P \wedge Q \iff Q \wedge P$	$A \cap B = B \cap A$
6	$(P \vee Q) \vee R \iff P \vee (Q \vee R)$	$(A \cup B) \cup C = A \cup (B \cup C)$
7	$(P \wedge Q) \wedge R \iff P \wedge (Q \wedge R)$	$(A \cap B) \cap C = A \cap (B \cap C)$
8	$P \wedge (Q \vee R) \iff (P \wedge Q) \vee (P \wedge R)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
9	$P \vee (Q \wedge R) \iff (P \vee Q) \wedge (P \vee R)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
10	$\neg(P \vee Q) \iff \neg P \wedge \neg Q$	$\overline{A \cup B} = \bar{A} \cap \bar{B}$
11	$\neg(P \wedge Q) \iff \neg P \vee \neg Q$	$\overline{A \cap B} = \bar{A} \cap \bar{B}$
12	$P \vee (P \wedge Q) \iff P$	$A \cup (A \cap B) = A$
13	$P \wedge (P \vee Q) \iff P$	$A \cap (A \cup B) = A$
14	$(P \leftrightarrow Q) \iff (P \rightarrow Q) \wedge (Q \rightarrow P)$	$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
15	$P \vee T \iff T$	$A \cup U = U$
16	$P \wedge F \iff F$	$A \cap \emptyset = \emptyset$
17	$P \vee F \iff P$	$A \cup \emptyset = A$
18	$P \wedge T \iff P$	$A \cap U = A$

19	$P \vee \neg P \iff T$	$A \cup \bar{A} = U$
20	$P \wedge \neg P \iff F$	$A \cap \bar{A} = \emptyset$
21	$(P \to Q) \iff (\neg Q \to \neg P)$	if $A \subseteq B$, then $\bar{B} \subseteq \bar{A}$
22	$P \Longrightarrow P \vee Q$	$A \subseteq A \cup B$
23	$P \wedge Q \Longrightarrow P$	$A \cap B \subseteq A$
24	$(P \to Q) \wedge (Q \to R) \Longrightarrow (P \to R)$	if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$
25	$(P \leftrightarrow Q) \wedge (Q \leftrightarrow R) \Longrightarrow (P \leftrightarrow R)$	if $A = B$ and $B = C$, then $A = C$

Why are logical theorems and theorems in set theory so similar? Because predicate logic and set theory are the same discipline. For instance, it means the same that Aristotle belongs to the set of man and that Aristotle satisfies the predicate of man.

Geometrical logic is constructed by the author. In geometrical logic there is no propositional calculus, predicate calculus is treated the same discipline with set theory. In geometrical logic there are five operators $\sim$, $\sqcap$, $\sqcup$, $\oplus$, $\otimes$ and seven connectives $\sqcap^{-1}$, $\sqcup^{-1}$, $\subseteq^{-1}$, $=^{-1}$, $\subset^{-1}$, X^{-1}, $\oplus^{-1}$. $\sim$ corresponds to $\neg$ in logic and $\overline{}$ in set theory, $\sqcap$ to $\wedge$ in logic and $\cap$ in set theory, $\sqcup$ to $\vee$ in logic and $\cup$ in set theory, $\subseteq^{-1}$ to $\to$ in logic and $\subseteq$ in set theory, $=^{-1}$ to $\leftrightarrow$ in logic and $=$ in set theory, $\subset^{-1}$ to $\to$ in logic and $\subset$ in set theory.

[1] FANG SHICHANG, ***Discrete Mathematics***, Press of Northwestern Communication Engineering Institute, 1987.