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IN MEMORIAM: RAPHAEL MITCHEL ROBINSON

About a month after his 83rd birthday Raphael Robinson was almost wholly incapacitated by a massive stroke, and 8 weeks later, on January 27, 1995, he died of ensuing complications. Mathematics was his life. He was always working on problems—those brought to him in journals or by colleagues, and others that he invented. Just three days before his death he received word that a paper of his, originating in a published problem, was accepted for publication. His 64 publications spanned a full 6 decades, and included significant papers on number theory, combinatorics, complex analysis, and geometry, in addition to logic and set theory.

Robinson was born on November 2, 1911, in National City, California. He came to Berkeley to study mathematics, obtaining an A.B. in 1932, an M.A. in 1933, and his Ph.D. in December, 1934. His dissertation was in complex analysis. He took 12 graduate courses in a variety of fields, but not one was close to logic or set theory—which was later the setting of about a quarter of his publications.

After completing his studies Robinson served as Instructor for two years at Brown University, then returned for good to Berkeley in 1937, reaching the rank of full Professor in 1949. He taught a great variety of courses and was known as an excellent teacher, but was most interested in problem-solving and research and in 1972, at age 61, he elected early retirement, at considerable financial sacrifice, in order to concentrate on these pursuits.

In Fall 1939 Robinson had a student named Julia Bowman in his number theory class; she had just transferred to Berkeley for her senior year, during which his was one of five mathematics courses that she took. Raphael and Julia began to take long walks during which he explained the various fields of mathematics that interested him. When she obtained the A.B. degree in 1940 he urged her to remain at Berkeley for graduate study, which she did. The walks continued, and their friendship evolved into courtship. She obtained her M.A. in June 1941, and they were married in December of that year. During the next 44 years their shared mathematical interests enriched the work of each; although they never published a joint paper they often wrote on the same topic and in several areas developed ideas introduced earlier by the other. At the end of her life, after she had become the first woman mathematician elected to the National Academy of Sciences and the first woman president of the American Mathematical Society, Julia told her sister Constance Reid, the widely known biographer of mathematicians,

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that she doubted she would have become a mathematician if it had not been for Raphael, adding that he continued to teach her throughout their life together, encouraging and supporting her in many ways.

Robinson's seventeen papers that fall within the area of foundations of mathematics begin with two on axiomatic set theory. In [1937] he published a simplification of von Neumann's axiom system based on *function*, rather than *set*, as the primitive undefined concept. Then, in [1945], he solved a problem raised by Tarski, who had arrived in Berkeley in 1942 and had taken on Julia Robinson as one of his first Ph.D. students. Raphael's paper showed how the notion of a *finite sequence of classes* can be defined in Gödel–Bernays set theory, even though it contains classes that are not members of any class.

Robinson's subsequent foundational work falls mainly into three overlapping areas: Recursive functions and RE sets, decision problems, and problems of definability.

1. Recursive functions and RE sets. In [1947] and [1955], Robinson characterized the set of primitive recursive functions (PR) in a variety of ways, and in [1948] he gave an elegant construction of a function definable by double recursion which is *not* in PR, but can be used to generate PR in a certain way. In [1956] Robinson improved a characterization by Martin Davis of RE sets by Diophantine equations preceded by existential and one bounded universal quantifiers, by reducing to 4 the number of existential quantifiers needed; and then in [1972] he improved this representation by referring to new work of Julia Robinson and Matijasevič.

2. Decision problems. Probably the result of Robinson most widely used by logicians to show the undecidability of many first-order theories is his elegant demonstration that a subset of Peano arithmetic (now known as Robinson's arithmetic Q) consisting of seven simple sentences is essentially undecidable; he first presented this at the 1950 International Congress of Mathematicians [1952], and it was later included in the book [1953]. In [1951], Robinson used the methods of that book to show the undecidability of many classes of rings; in [1964], using methods developed by Julia Robinson the preceding year, he showed that any transcendental extension of a real field is undecidable, in contrast to Tarski's early showing that each real closed field is decidable. In the following decade, Robinson took up a class of decision problems originating with Hao Wang concerning the possibility of completely tiling the plane by translations of a given finite array of tiles (subject to possible conditions defined in terms of their colored edges). The question whether there is such a *periodic* tiling of the plane is clearly decidable, but in [1971] Robinson produced a set of 6 polygonal tiles which can tile the whole plane, but never in a periodic way, and he produced an array of 36 square tiles with colored edges, for which he proved that there is no decision method to tell which partial tilings obtained by finitely many translations of the array can be extended to a tiling of the whole plane.

3. Problems of definability. For many of the structures shown by Robinson to have an undecidable theory, a crucial step of his proof involved showing the definability (in that first-order theory), of the fragment of Peano arithmetic described in [1952] and [1953]. In [1951a] and [1951b] he gave the simplest arithmetical definition of $\mathbb{N}$ in the theory of $\mathbb{Z}$, and discussed the relation between the arithmetical definability of an element x of an algebraic number field F and the invariance of x under automorphisms of F. Then, in [1958], he turned to questions of definability in the monadic second-order theory of some fragments of the theory of $\mathbb{N}$, showing that $+$ is definable in terms of successor and doubling, and then deriving the undecidability of this theory as a corollary.

Robinson's seventeen papers on foundations of mathematics rarely touch on general questions. Each concerns particular problems which he solves by original ideas that often emerge from his deep knowledge of classical analysis, algebra, and above all number theory. He was truly a pioneer in the applications of classical mathematics to problems of metamathematics. In this he reminds me of another pioneer who by chance had the same surname—Abraham Robinson.

In conclusion, I wish to mention what I believe to be Raphael Robinson's most widely known result, even though it is not really in the area of logic and set theory. In [1947] he greatly improved the Banach–Tarski Paradox of 1924, by showing how to partition the points of a solid 3-dimensional sphere into 5 subsets, one of which contains only a single point, such that these sets can be moved by translations and rotations so as to form two solid spheres of the same radius. He proved that 5 is the minimum number that can accomplish this, even if reflections are allowed as well as translations and rotations. He also showed that the surface of the 3-dimensional sphere can be partitioned into 4 subsets, which can be recombined to form two such spherical surfaces of the same radius.

The Robinsons were close friends of mine. Together with four other couples and the first Math Department secretary, the Robinsons and Henkins formed a set of "Foster Parents" who helped support, and exchanged monthly letters with, a sequence of impoverished children who grew to maturity in distant parts of the world. Julia was outgoing, Raphael reticent, but they shared political and social views. On behalf of several of these Julia joined in organized activism, which was not Raphael's style. However, when 5 members of the Math Department were fired in the 1950 Loyalty Oath controversy that racked the University, Raphael joined a group of colleagues who created a "mathfund" to which they pledged 10% of their salaries to support the non-signers, and he served as the Treasurer for the fund.

In his last 10 years, after Julia's death from cancer, Raphael lived alone, taking care of their modest home, shopping for and preparing food. He continued to participate in university colloquia and other mathematics meetings in California, giving talks at some. After his stroke he received much care from Julia's sister, Constance Reid. Shortly before his death he asked her to let people know his wish that those wanting to contribute in remembrance of him, should make the contribution to the Julia Robinson Fund which he had established after her death, to support graduate students in mathematics at the University of California, Berkeley.

CITED WORKS OF R. M. ROBINSON

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