We started with an induction example from the “What is a Proof?” document I posted on my website and CCLE.

For posterity, here it is:

We say that an integer \( a \) divides an integer \( b \) if there exists an integer \( c \) such that \( a \cdot c = b \). We denote this \( a \mid b \) read as “\( a \) divides \( b \).”

**Claim:** \( 8^n - 3^n \) is divisible by 5 for all \( n \in \mathbb{N} \)

**Proof.**

When \( n = 0 \) we claim that 5 divides 0 which is true because \( 5 \cdot 0 = 0 \).

If \( 8^n - 3^n = 5k \) for some integer \( k \) then

\[
8^{n+1} - 3^{n+1} = 8 \cdot 8^n - 3 \cdot 3^n
\]

\[
= 3 \cdot 8^n + 5 \cdot 8^n - 3 \cdot 3^n
\]

\[
= 3(8^n - 3^n) + 5 \cdot 8^n
\]

\[
= 5k + 5 \cdot 8^n = 5(k + 8^n)
\]

Hence if \( 8^n - 3^n \) is divisible by 5 then \( 8^{n+1} - 3^{n+1} \) is divisible by 5.

By induction the claim is true. \( \square \)

We also did problem 6 from the homework from October 9.

I denote \( 0_V \) as \( \vec{0} \).

(i) Recall that \( 0 + 0 = 0 \). Then \( 0 \cdot x = (0 + 0) \cdot x \overset{VSS8}{=} 0 \cdot x + 0 \cdot x \).

Adding \(-(0 \cdot x)\) to both sides (which exists by VS4) we get:

\[
-(0 \cdot x) + 0 \cdot x = -(0 \cdot x) + (0 \cdot x + 0 \cdot x) \overset{VSS2}{=} -(0 \cdot x) + 0 \cdot x
\]

Then \( \vec{0} = \overrightarrow{0} + 0 \cdot x \overset{VS3}{=} 0 \cdot x \).

(ii) By VS3, \( \vec{0} = \vec{0} + \overrightarrow{0} \).

Then \( a \cdot \vec{0} = a \cdot (\vec{0} + \overrightarrow{0}) \overset{VSS7}{=} a \cdot \overrightarrow{0} + a \cdot \vec{0} \).
Adding $-(a \cdot \vec{0})$ to both sides (which exists by VS4) we get:

$$-(a \cdot \vec{0}) + a \cdot \vec{0} = -(a \cdot \vec{0}) + (a \cdot \vec{0} + a \cdot \vec{0}) \overset{VS2}{=} -(a \cdot \vec{0}) + a \cdot \vec{0} + a \cdot \vec{0}$$

Then by VS4, $\vec{0} = \vec{0} + a \cdot \vec{0} \overset{VS3}{=} a \cdot \vec{0}$

(iii) We start by proving a lemma: $(-1) \cdot x = -x$.

We have $0 \cdot x = (-1 + 1) \cdot x \overset{VS8}{=} -1 \cdot x + 1 \cdot x \overset{VS5}{=} -1 \cdot x + x$

By part (i), $0 \cdot x = \vec{0}$, so $\vec{0} = -1 \cdot x + x$.

Hence $-1 \cdot x = -x$. 