115 Week 2 Notes

Andrew Sack

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We started with an induction example from the "What is a Proof?" document I posted on my website and CCLE.

For posterity, here it is:

We say that an integer a divides an integer b if there exists an integer c such that $a \cdot c = b$. We denote this $a|b$ read as "a divides b."

Claim: $8^n - 3^n$ is divisible by 5 for all $n \in \mathbb{N}$

Proof.
When $n = 0$ we claim that 5 divides 0 which is true because $5 \cdot 0 = 0$.
If $8^n - 3^n = 5k$ for some integer k then

$$\begin{aligned} 8^{n+1} - 3^{n+1} &= 8 \cdot 8^n - 3 \cdot 3^n \\ &= 3 \cdot 8^n + 5 \cdot 8^n - 3 \cdot 3^n \\ &= 3(8^n - 3^n) + 5 \cdot 8^n \\ &= 5k + 5 \cdot 8^n = 5(k + 8^n) \end{aligned}.$$

Hence if $8^n - 3^n$ is divisible by 5 then $8^{n+1} - 3^{n+1}$ is divisible by 5.
By induction the claim is true. $\square$

We also did problem 6 from the homework from October 9.

I denote 0_V as $\vec{0}$.

(i) Recall that $0 + 0 = 0$. Then $0 \cdot x = (0 + 0) \cdot x \overset{VS8}{=} 0 \cdot x + 0 \cdot x$.
Adding $-(0 \cdot x)$ to both sides (which exists by VS4) we get:
$-(0 \cdot x) + 0 \cdot x = -(0 \cdot x) + (0 \cdot x + 0 \cdot x) \overset{VS2}{=} (-(0 \cdot x) + 0 \cdot x) + 0 \cdot x$
Then $\vec{0} = \vec{0} + 0 \cdot x \overset{VS3}{=} 0 \cdot x$.

(ii) By VS3, $\vec{0} = \vec{0} + \vec{0}$.
Then $a \cdot \vec{0} = a \cdot (\vec{0} + \vec{0})\overline{VS7} = a \cdot \vec{0} + a \cdot \vec{0}$

Adding $-(a \cdot \vec{0})$ to both sides (which exists by VS4) we get:
$-(a \cdot \vec{0}) + a \cdot \vec{0} = -(a \cdot \vec{0}) + (a \cdot \vec{0} + a \cdot \vec{0}) \overset{VS2}{=} (-(a \cdot \vec{0}) + a \cdot \vec{0}) + a \cdot \vec{0}$
Then by VS4, $\vec{0} = \vec{0} + a \cdot \vec{0} \overset{VS3}{=} a \cdot \vec{0}$

(iii) We start by proving a lemma: $(-1) \cdot x = -x$.
We have $0 \cdot x = (-1 + 1) \cdot x \overset{VS8}{=} -1 \cdot x + 1 \cdot x \overset{VS5}{=} -1 \cdot x + x$
By part (i), $0 \cdot x = \vec{0}$, so $\vec{0} = -1 \cdot x + x$.
Hence $-1 \cdot x = -x$.