Week 2 Worksheet for Math 61

1. Define the sequence a by $a_0 = 3$ and $a_n = 2 + a_{n-1}$.

   (a) Write out 5 terms of a starting with a_0.

   (b) Calculate $\sum_{i=0}^{4} a_i$.

   (c) Find a formula for a_i.

   (d) Find a formula for $s_n = \sum_{i=0}^{n} a_i$ in terms of n.

   (e) Is a non-decreasing? Is a increasing?

   (f) Is s non-decreasing? Is s increasing?

2. For the following relations R, determine if R is reflexive, symmetric, antisymmetric, transitive, a partial order, and/or an equivalence relation.

   (a) $x, y \in \mathbb{R}$, $xRy \Leftrightarrow xy = 1$.

   (b) $x, y \in \{\text{Rock, Paper, Scissors}\}, xRy \Leftrightarrow x$ beats y.

   (c) $x, y \in \{\text{Rock, Paper, Scissors}\}, xRy \Leftrightarrow x$ beats or ties y.

   (d) $x, y \in \mathbb{Z}, xRy \Leftrightarrow x - y$ is even.

   (e) $x, y \in \mathbb{Z}, xRy \Leftrightarrow x \mid y$. (Recall that $x \mid y$ iff $\exists z \in Z$ such that $y = xz$)

(f) $x, y \in \mathbb{N}, xRy \Leftrightarrow x \mid y$.

3. For each of the following relations. verify that it is an equivalence relation and give one member from each of its equivalence classes. (There many be infinitely many equivalence classes.)

   (a) $x, y \in \mathbb{Z}, xRy \Leftrightarrow 3|x - y$.
   (Recall that $3|x-y$ means "3 divides $x-y$". That is, $\exists z \in \mathbb{Z}$ such that $x-y = 3z$).

   (b) $x, y \in \mathbb{R}, xRy \Leftrightarrow \cos(x) = \cos(y)$.

   (c) $x, y \in \mathbb{R}, xRy \Leftrightarrow x - y \in \mathbb{Z}$.

   (d) $(a, b), (x, y) \in \mathbb{R} \times (\mathbb{R} \setminus \{0\}), (a, b)R(x, y) \Leftrightarrow ay = bx$.

4. Describe the following sets geometrically (or draw a picture)

   (a) $\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$

   (b) $S^1 \times (0, 1)$ where $S^1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ i.e. a circle.

   (c) $([0, 1] \times \{0\}) \cup (\{0\} \times [0, 1]) \cup ([0, 1] \times \{1\}) \cup (\{1\} \times [0, 1])$