**Sgt. Pepper Financial Group**

*Executive Summary*

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## Overview

For this project, our objective is to analyze historical data pertaining to the effect of different economic interest rates and their correlation on the respective lapse rates for specified years. Using this historical data, we are asked to develop a lapse rate formula to predict future lapse rates for policy years 1-15 (issued 2023) of the new annuity product, which will then be implemented into the FA pricing model. We believe our investigation regarding the previously issued fixed annuities will aid in creating a more formula-driven approach for the annuity pricing model.

## Historical Data

We were given past data on the "Pepper Weight Fixed Annuity" issued in the years 1981, 1990, and 1993. Each issue year has its own chart, containing quantitative components that affect full lapse rates. These components are policy year, MVA (Market value adjustments), Mortality Rate, Crediting Rate, 5-year treasury rate, Statutory reserves, 10-year treasury rate and the surrender charge rate. Shown were these factors and their values for a particular range of policy years. Also given is the corresponding full lapse rate for each policy year. With this information, we were tasked to create a lapse rate formula utilizing four of the 9 variables.

## Variable Selection

We decided the four most influential variables out of the 9 would be those that have the potential to impact the value of the annuitant's account value. Under this precedent, we chose MVA, crediting rate, 10-year treasury rate and surrender charge. As MVA is a factor that can change the annuitants account value either negatively or positively when surpassing the free partial withdrawal rate, we considered this to be an important factor. Connected to MVA is the 10-year treasury rate, which is representative of general economic interest rates and inflation. In analyzing the annuity issued in 1993, we observed that for the years in which the 10-year treasury rate was higher than it was the initial policy year, we have a negative MVA as expected. When the opposite is true, the MVA is positive. As these two factors seem to have a direct correlation, we were further inclined to include the 10-year treasury rate in the formula as it does meet our requirement of impacting the value of the account. Like MVA, the surrender charge can negatively affect the annuitants account value within the surrender charge period. However, unlike MVA, the surrender charge cannot positively affect the annuitant and therefore is to always be avoided. Lastly, as the crediting rate is responsible for the yearly accumulation of an annuitant's single premium, we considered this another important variable.

## Generalized Linear Model Selection

Using the four variables discussed to be included in our lapse rate function, we then decided between a logistical regression model and a least-squares linear regression model. Although we found logistical regression to be a potential GLM, it is typically for situations in which the outcome

is considered a classification. As we want to predict percentage values rather than classify our data, we concluded the least-squares linear regression model would work better.

**Base Lapse Rate**

Our base or constant Lapse Rate is *0.1415* or *14.15%*. This value was determined by averaging the values of our Predicted Lapse Rates for Issue Years 1981, 1990, and 1993.

**Dynamic Lapse Rate Formula**

Our Dynamic Lapse Rate Formula is *0.142 + (-0.907 * Crediting Rate Value) + (-0.313 * Market Valued Adjustment Value) + (0.778 * Surrender Charge Fee) + (0.624 * 10 Yr. Treasury Rate)* Our coefficients were determined through Least-Squares Linear Analysis in which coefficients are calculated through reducing the residual value between predicted and historical lapse rates. A positive coefficient indicates a positive correlation in Lapse Rate. A negative coefficient indicates that lapse rates decrease when the factor it is attached to increases. Additionally, when the surrender charge fee ends in Policy Year 9, our dynamic lapse rate formula increases in value, only for that Year, by *0.21*, causing the Predicted Lapse Rate for that year to increase by *21%*. This is to account for a Spike in Lapse Rate noted in the Historical Data.

**Predicted Lapse Rates**

Our Predicted Lapse Rate values range from *4.37%* to *36.87%* and is based on our dynamic lapse rate formula. Our Appendix contains all 15 Lapse Rate values for the future 15 years we calculated.

**Recommendation**

We believe our linear regression model can predict future lapse rates with minimal error. However, we do have suggestions to improve the reliability of the function. Including more independent variables within the formula will allow for a potential greater R squared, thus implying a greater correlation. Allowing for more factors would allow us to predict future lapse rates more accurately. Regarding accuracy, increasing the data set will also provide a lower p-value for each of the variables, again highlighting a better correlation. Although we chose a linear regression model for our formula, we believe with a larger data set that includes whether specific individuals lapsed, we could apply a logistic regression model as we would then have a binary outcome. With increased independent variables alongside individualized lapse rate information, logistical regression would potentially be a better fit. Additionally, removing outliers will lower the residuals within the formula, increasing the accuracy of future predicted lapse rates.

**Conclusion**

Through our analysis of the historical lapse rates, we believe our simple linear regression formula limited to 4 variables will assist in pricing the new fixed annuity product. As our predicted lapse rates for the issue years given typically stay within 1 standard error of the data, the lapse rate formula provided can predict future lapse rates which will reflect the price of the fixed annuity.

**Appendix:**

Table 2:

| Table 2: Regression Statistics | |
|---|---|
| Multiple R | 0.7444 |
| R Square | 0.5542 |
| Adjusted R Square | 0.5322 |
| Standard Error | 0.0179 |
| Observations | 86 |

Table 3:

| Table 3: Regression Coefficients and P-values | | |
|---|---|---|
| **Variable** | **Coefficient** | **P-value** |
| Intercept | 0.1415 | 0.0000 |
| MVA | -0.3125 | 0.0003 |
| Crediting Rate | -0.9074 | 0.0023 |
| 10-Yr Treasury rate | 0.6238 | 0.0000 |
| Surrender charge | -0.7778 | 0.0000 |

Dynamic Lapse Rate Formula:

$$\text{Lapse Rate} = \begin{cases} 0.142 + (-0.907 \cdot \text{C.R.}) + (-0.313 \cdot \text{MVA}) + (0.778 \cdot \text{S.C.}) + (0.624 \cdot \text{10YR}), & \text{Policy Years} \neq 9 \\ 0.142 + (-0.907 \cdot \text{C.R.}) + (-0.313 \cdot \text{MVA}) + (0.778 \cdot \text{S.C.}) + (0.624 \cdot \text{10YR}) + 0.21, & \text{Policy Year} = 9 \end{cases}$$

Table 5: Policy Data and Predicted Lapse

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|---|---|---|---|---|---|
| Policy Year | MVA | Crediting Rate | 10-Yr Treasury rate | Surrender charge | Predicted Lapse |
| 1 | 0.10% | 4.50% | 5.87% | 12.00% | 4.37% |
| 2 | 0.20% | 4.50% | 6.00% | 10.00% | 5.97% |
| 3 | -0.45% | 4.50% | 5.80% | 10.00% | 6.05% |
| 4 | -2.35% | 4.70% | 5.00% | 7.00% | 8.30% |
| 5 | -3.04% | 4.80% | 6.35% | 5.00% | 10.82% |
| 6 | -7.96% | 4.80% | 5.26% | 4.00% | 12.46% |
| 7 | -8.15% | 5.00% | 5.65% | 3.00% | 13.35% |
| 8 | -10.63% | 5.00% | 6.03% | 2.00% | 15.14% |
| 9 | -9.98% | 5.00% | 5.02% | 0.00% | 36.87% |
| 10 | -7.08% | 5.20% | 4.61% | 0.00% | 14.53% |
| 11 | -8.03% | 5.20% | 4.01% | 0.00% | 14.45% |
| 12 | -5.39% | 5.50% | 4.27% | 0.00% | 13.51% |
| 13 | -6.34% | 5.50% | 4.29% | 0.00% | 13.82% |
| 14 | -2.47% | 5.50% | 4.80% | 0.00% | 12.93% |
| 15 | 0.30% | 5.50% | 4.63% | 0.00% | 11.96% |

Table 1: Residual Output

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|---|---|---|---|---|---|---|---|---|---|---|---|
| Issue-Year-1981 | | | | Issue-Year-1990 | | | | Issue-Year-1993 | | | |
| Observation | Pred. Full Lapse | Residuals | Res. % | Observation | Pred. Full Lapse | Residuals | Res. % | Observation | Pred. Full Lapse | Residuals | Res. % |
| 1 | 0.1705 | -0.0173 | -1.73% | 31 | 0.0940 | -0.0088 | -0.88% | 59 | 0.0717 | 0.0018 | 0.18% |
| 2 | 0.1636 | 0.0100 | 1.00% | 32 | 0.1141 | 0.0128 | 1.28% | 60 | 0.0854 | -0.0076 | -0.76% |
| 3 | 0.1656 | 0.0193 | 1.93% | 33 | 0.1154 | 0.0052 | 0.52% | 61 | 0.0927 | 0.0248 | 2.48% |
| 4 | 0.1471 | 0.0117 | 1.17% | 34 | 0.0915 | 0.0043 | 0.43% | 62 | 0.0682 | -0.0230 | -2.30% |
| 5 | 0.1562 | 0.0308 | 3.08% | 35 | 0.1341 | -0.0121 | -1.21% | 63 | 0.1267 | 0.0122 | 1.22% |
| 6 | 0.1562 | -0.0009 | -0.09% | 36 | 0.1564 | 0.0222 | 2.22% | 64 | 0.0904 | 0.0075 | 0.75% |
| 7 | 0.1646 | -0.0021 | -0.21% | 37 | 0.1419 | -0.0177 | -1.77% | 65 | 0.1162 | -0.0422 | -4.22% |
| 8 | 0.1695 | -0.0065 | -0.65% | 38 | 0.1339 | 0.0201 | 2.01% | 66 | 0.1309 | 0.0101 | 1.01% |
| 9 | 0.1604 | -0.0060 | -0.60% | 39 | 0.1374 | 0.0238 | 2.38% | 67 | 0.1225 | 0.0106 | 1.06% |
| 10 | 0.1495 | -0.0178 | -1.78% | 40 | 0.1396 | -0.0241 | -2.41% | 68 | 0.1286 | 0.0095 | 0.95% |
| 11 | 0.1446 | -0.0104 | -1.04% | 41 | 0.1451 | 0.0374 | 3.74% | 69 | 0.1361 | 0.0023 | 0.23% |
| 12 | 0.1539 | -0.0169 | -1.69% | 42 | 0.1475 | -0.0077 | -0.77% | 70 | 0.1422 | -0.0273 | -2.73% |
| 13 | 0.1557 | -0.0159 | -1.59% | 43 | 0.1414 | -0.0056 | -0.56% | 71 | 0.1489 | -0.0199 | -1.99% |
| 14 | 0.1530 | -0.0069 | -0.69% | 44 | 0.1427 | 0.0318 | 3.18% | 72 | 0.1459 | -0.0047 | -0.47% |
| 15 | 0.1526 | -0.0040 | -0.40% | 45 | 0.1456 | 0.0163 | 1.63% | 73 | 0.1533 | 0.0071 | 0.71% |
| 16 | 0.1381 | -0.0221 | -2.21% | 46 | 0.1337 | 0.0101 | 1.01% | 74 | 0.1357 | -0.0126 | -1.26% |
| 17 | 0.1376 | -0.0154 | -1.54% | 47 | 0.1292 | -0.0128 | -1.28% | 75 | 0.1421 | -0.0110 | -1.10% |
| 18 | 0.1480 | -0.0083 | -0.83% | 48 | 0.1348 | -0.0025 | -0.25% | 76 | 0.1275 | 0.0037 | 0.37% |
| 19 | 0.1381 | -0.0223 | -2.23% | 49 | 0.1453 | -0.0326 | -3.26% | 77 | 0.1367 | 0.0347 | 3.47% |
| 20 | 0.1370 | -0.0199 | -1.99% | 50 | 0.1455 | 0.0126 | 1.26% | 78 | 0.1308 | 0.0335 | 3.35% |
| 21 | 0.1324 | -0.0033 | -0.33% | 51 | 0.1405 | 0.0100 | 1.00% | 79 | 0.1396 | 0.0013 | 0.13% |
| 22 | 0.1318 | -0.0192 | -1.92% | 52 | 0.1411 | 0.0059 | 0.59% | 80 | 0.1347 | -0.0016 | -0.16% |
| 23 | 0.1447 | -0.0154 | -1.54% | 53 | 0.1288 | 0.0137 | 1.37% | 81 | 0.1272 | 0.0023 | 0.23% |
| 24 | 0.1256 | -0.0150 | -1.50% | 54 | 0.1312 | 0.0067 | 0.67% | 82 | 0.1471 | 0.0356 | 3.56% |
| 25 | 0.1292 | -0.0040 | -0.40% | 55 | 0.1442 | -0.0135 | -1.35% | 83 | 0.1279 | 0.0201 | 2.01% |
| 26 | 0.1348 | -0.0232 | -2.32% | 56 | 0.1394 | 0.0144 | 1.44% | 84 | 0.1180 | -0.0129 | -1.29% |
| 27 | 0.1304 | -0.0033 | -0.33% | 57 | 0.1234 | 0.0391 | 3.91% | 85 | 0.1228 | 0.0022 | 0.22% |
| 28 | 0.1200 | 0.0044 | 0.44% | 58 | 0.1410 | 0.0026 | 0.26% | 86 | 0.1465 | 0.0281 | 2.81% |
| 29 | 0.1377 | -0.0190 | -1.90% | | | | | | | | |
| 30 | 0.1479 | -0.0170 | -1.70% | | | | | | | | |