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Robert J. Blattner

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Bob Blattner was a very good friend of ours, and was an active and resourceful collaborator of ours in the late 1960's and early 1970's. He was a very versatile mathematician whose discoveries went far beyond our joint work.

Bob Blattner was an undergraduate at Harvard, receiving hia AB summa cum laude in 1953. His undergraduate advisor was George W. Mackey whose work was to have a major influence on Bob's first publications. He received his PhD from the University of Chicago in 1953 under the supervision of Irving Segal.

1 Induced representations of groups

The basics of the representation theory of finite groups goes back to Frobenius. If H is a subgroup of G, then every representation of G obviously restricts to a representation of H. Frobenius showed how, starting with a representation of H on can construct a representation of G called the induced representation and proved the Frobenius reciprocity theorem. In today's terminology, this would say that the restriction functor (going from the category of G-representations to H-representations) and the induction functor (going from the category of H representations to the category of G-representations) are adjoint functors. From his construction of induced representations, Frobenius was able to describe the irreducible representations of a semi-direct product.

In 1939, Wigner applied the Frobenius method to determine the irreducible representations of the Poincarë group, and found the amazing result that parameters describing the physically relevant representations are none other than parameters such as mass and spin which enter into the description of elementary particles. As the Poincaré group is far from being finite, the mathematical question arises as how to formulate the notion of induced representations for (say) a locally compact compact group G and a unitary representation L of a closed subgroup H. The key results in this mathematical investigation are all due to Mackey (with some input by Loomis and others) and today the subject is known as the Wigner-Mackey theory of induced representations. Mackey's approach made use of a very heavy dose of deep facts from measure theory and some technical assumptions such as seperability and the existence of "quasi-invariant measures" on G/H.

In a series of papers in the early 1960's, Bob cleaned up this subject enormously. He gives a reformulation of the definition of the induced representation U^L. In this reformulation no mention is made of quasi-invariant measures (and hence is "more invariant") and the separability hypothesis is avoided. A theorem of Mackey (on an isomorphism between the algebra of intertwining operators of representations of H and a subalgebra of the intertwining operators of the induced representations, which Mackey proved under separability assumptions is extended by Bob to eliminate the separability hypotheses. In several other papers Bob streamlines and generalizes other theorems of Mackey.

2 Induced and produced representations of Lie algebras, and the realization theorem for transitive Lie algebras

The study of transitive Lie algebras (at least in low dimensions) goes by to Lie in his **Transformationensgrouppen** where he was interested in using symmetry toward the solution of differential equations. In our paper "An algebraic model of transitive differential geometry" we proved the following "realization theorem': Let $\mathfrak{g}$ be a Lie algebra over a field k of characteristic zero and $\mathfrak{t}$ a sub algebra of finite cxdimension n. Then $\mathfrak{g}$ can be represented as derivations of the $k[x_1, \ldots, x_n]$ in such a way that there are derivations of the form $\frac{\partial}{\partial x_i} + \cdots$, $i = 1, \ldots, n$ in the image, and $\mathfrak{t}$ is represented by such 'formal vector fields" which vanish at the origin. Furthermore, this representation is unique up to formal power series change of variables. Our proof involved finding the required formal vector fields and the coordinate change relating two such realizations recursively "coefficient by coefficient". In 1969 Bob developed the theory of produced and induced representations of Lie algebras and used this to give a beautiful coordinate free proof of this realization theorem. In more detail: Let $U(\mathfrak{g})$ and $U(\mathfrak{t})$ be the universal enveloping algebras of $\mathfrak{g}$ and $\mathfrak{t}$. Given a representation (ρ, V) of $\mathfrak{t}$ (and hence of $U(\mathfrak{t})$), let $W = \mathrm{Hom}_{U(\mathfrak{t})}(U(\mathfrak{g}, V^*)$. Thas the structure of a $U(\mathfrak{g})$ module (hence of a $\mathfrak{g}$ module) called the produced module. It is the Lie algebra version of the induced module from a subgroup. If one takes (ρ, V) to be the trivial representation of $\mathfrak{t}$ on the base field, then W has a natural ring structure on which $\mathfrak{g}$ acts as derivations. Bob proves the realizationn theorem by showing that if k has zero characteristic then W is isomorphic to the power series ring. He then proves some important theorems on these transitive Lie algebras using a Lie algebra produced version of Mackey's imprimitivity theorem.

3 The BKS pairing

Dirac proposed that quantum Hamiltonians should be obtained from classical mechanics via a homomorphism from the Poisson algebra of functions on phase space to the Lie algebra (under commutator) of skew-adjoint operators on Hilbert space. In the case of polynomials of degree two or less on standard phase space, this was successfully carried out giving rise to the famous paper by André Weil on the metaplectic representation (inspired in part by work of Irving Segal). But a "no-go" theorem of Grunwald and van Hove showed that this representation can not be extended so as to include any polynomial of degree higher than two. Also, the above constructed representation is limited to the standard phase space consisting of $T^*\mathbb{R}^n$ whereas the geometry of classical mechanics requires more general symplectic manifolds. At this point, geometric quantization was introduced by Kostant and Souriau. It involves two stages: the first (called pre-quantization) provides a representation of elements of the Poisson algebra of a symplectic manifold M as first order differential operators on a line bundle which certain homological conditions on M are satisfied. n accordance with the metaplectic correction this line bundle is multiplied by the bundle of half-forms.The second stage, called quantization involves a choice of polarization which restricts the class of functions which can be "quantized". Bob, in collaboration with Kostant and Sternberg introduced a pairing between the Hilbert spaces associated with different polarizations under certain hypotheses. The possible applications of this method is still an active area of research.

4 The Blattner conjecture

Among workers in representations of Lie groups, Bob is best known for a conjecture about the K-types of the discrete series, that he made in conversation with Harish-Chandra. As Prof. Varadarajan discusses this in his memorial, we need not go into detail. See also the wikipedia article on the Blattner conjecture.

5 Work on Hopf algebras

We are not competent to write about this work, and rely on Prof. Montgomery, his wife and collaborator, to describe this.