HOMEWORK 6 (18.314, FALL 2006)

1. Let G be a graph on n vertices and let $\mathcal{C}(G,k)$ be the set of proper colorings of G with k colors. Consider a *move* $C_1 \to C_2$, where $C_1, C_2 \in \mathcal{C}(G,k)$ and C_1 is different from C_2 by a color in one vertex. Denote by d is the maximal degree of vertex in G. Prove that for every two proper colorings $C, C' \in \mathcal{C}(G,k)$ can be connected to each other by a finite sequence of such moves, if

a) $k \geq 2d + 1$.
b) $k \geq d + 2$.

2. Let G be a $m \times n$ grid graph (note that $d = 4$ for $m, n \geq 3$). Solve problem 1 for

a) $k = 5$ colors.
b) $k = 4$ colors.

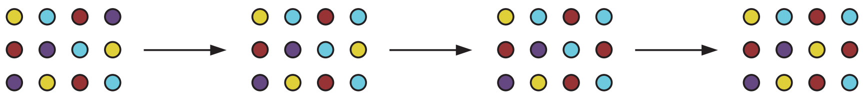

FIGURE. A sequence of moves for proper 4-colorings of 3×4 grid graph.

3. Let $c(n)$ denote the number of proper 3-colorings of a $n \times n$ grid graph. Prove that $c(n) > \alpha^{(A\,n^2)}$ for some $A > 0$ and $\alpha > 1$. *Hint:* fix 1-2 coloring of the boundary and see what happens when n doubles.

4. Two squares of different color are removed from a regular 8×8 chess board. Prove that the remaining part can be tiled by *dominoes* (1×2 and 2×1 squares).

5. Let G be a planar graph of a polytope obtained as in class. Prove that if any two vertices of G are removed, the graph remains connected.

6. Recall the 4-color theorem. Show that it suffices to prove the theorem only for graphs dual to planar 3-regular graphs.

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This Homework is due Wednesday November 1 at 2:05 pm.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX