HOME WORK 6

DUE : MAR 10/2006

Computational Part : Please submit the program you wrote!

(1) Approximate $\int_0^2 x^2 \sin(-x)dx \approx -2.4694834$ by the following quadrature rules to 10^{-6} accuracy and also find the size of h required for each rule.

(a) Composite left point rule.

(b) Composite right point rule.

(c) Composite midpoint rule.

(d) Composite trapezoidal rule.

(e) Composite simpson's rule.

Theoretical Part :

(1) Consider the numerical quadrature rule to approximate $\int_0^1 f(x)dx$ given by

$$\int_0^1 f(x)dx \approx w_1 f(0) + w_2 f(x_1).$$

Find the maximum possible degree of precision you can attain by appropriate choices of x_1, w_1 and w_2. By such choices of x_1, w_1 and w_2, approximate $\int_0^1 x^3\, dx$ and compare with the exact value.

(2) (Optional!!) Show that if n is even, we have

$$\sum_{i=0}^{n} w_i \left(x_i - \frac{a+b}{2} \right)^{n+1} = 0,$$

where $x_i = a + ih$ with $i = 0, \cdots, n$ and

$$w_i = \int_a^b L_k(x)\, dx,$$

where L_k is the k-th basis of Lagrange interpolating polynomial.

(3) Determine constants a, b, c and d that will produce a quadrature formula

$$\int_{-1}^{1} f(x)\,dx = af(-1) + bf(1) + cf'(-1) + df'(1).$$

that has degree of precision 3.