HOME WORK V

DUE : MAR 01/2006

(1) Using Taylor expansion, show that

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h} - \frac{h}{2} f''(\xi),$$

for some ξ lying in between x and $x_0 + h$.

(2) Derive an $O(h^4)$ five-point formula to approximate $f'(x_0)$ using nodes $x_0 - h, x_0, x_0 + h, x_0 + 2h$ and $x_0 + 3h$.

(3) Derive an $O(h^4)$ five-point formula to approximate $f'(x_0)$ using nodes $x_0 - 2h, x_0 - h, x_0, x_0 + h$ and $x_0 + 2h$.

(4) Compare two error terms obtained in the above two problems (2) and (3) and decide which is the better.

(5) The forward-difference formula can be expressed as

$$f'(x_0) = \frac{1}{h}\left(f(x_0+h) - f(x_0)\right) - \frac{h}{2} f''(x_0) - \frac{h^2}{6} f'''(x_0) + O(h^3). \tag{0.1}$$

Use extrapolation to derive an $O(h^3)$ formula for $f'(x_0)$.