

### HOME WORK III

DUE : FEB/08/2006

The following homework are mostly from Chapter I, section 2 in your textbook.

- (1) Compute the absolute error and relative error in approximations of  $p$  by  $p^*$  (Use calculator!).
  - a.  $p = \pi$ ,  $p^* = 22/7$
  - b.  $p = \pi$ ,  $p^* = 3.1416$
- (2) Find the largest interval in which  $p^*$  must lie to approximate  $\sqrt{2}$  with relative error at most  $10^{-5}$  for each value for  $p$ .
- (3) Use the 64-bit long real format to find the decimal equivalent of the following floating-point machine numbers.
  - a. 0 10000001010 10010011000000...0
  - b. 1 10000001010 01010011000000...0
- (4) Find the next largest and smallest machine numbers in decimal form for the numbers given in the above problem.
- (5) Use four-digit rounding arithmetic and the formulas to find the most accurate approximations to the roots of the following quadratic equations. Compute the relative error.
  - a.  $\frac{1}{3}x^2 - \frac{123}{4}x + \frac{1}{6} = 0$
  - b.  $1.002x^2 + 11.01x + 0.01265 = 0$ .
- (6) Suppose that  $fl(y)$  is a  $k$ -digit rounding approximation to  $y$ . Show that

$$\left| \frac{y - fl(y)}{y} \right| \leq 0.5 \times 10^{-k+1}.$$

(Hint : if  $d_{k+1} < 5$ , then  $fl(y) = 0.d_1 \cdots d_k \times 10^n$ . If  $d_{k+1} \geq 5$ , then  $fl(y) = 0.d_1 \cdots d_k \times 10^n + 10^{n-k}$ .)