HOME WORK I

DUE : JANUARY/25/2006

Theoretical Part :

(1) Determine a formula which relates the number of iterations, n, required by the bisection method to converge to within an absolute error tolerance of ε, starting from the initial interval (a, b). (Hint : Use the Theorem 2 in my note or Theorem 2.1 in the text book.)

(2) Show that when Newton's method is applied to the equation $x^2 - a = 0$, the resulting iteration function is $g(x) = \frac{1}{2}(x + a/x)$.

(3) Use the Bisection method to find p_3 for $f(x) = \sqrt{x} - \cos(x)$ on $[0, 1]$.

(4) The function $f(x) = \sin(x)$ has a zero on the interval $(3, 4)$, namely, $x = \pi$. Perform three iterations of Newton's method to approximate this zero, using $p_0 = 4$. Determine the absolute error in each of the computed approximations. What is the apparent order of convergence?

Computational Part :

(1) Apply the Newton's method to find the solution to

$$x^3 - x - 3 = 0$$

starting with $p_0 = 0$. Compute $p_1, p_2, p_3, p_4, p_5, p_6$ and p_7 and compare pair of numbers (p_0, p_4), (p_1, p_5), (p_2, p_6) and (p_3, p_7). What can you conclude from this computations?

(2) Use the Bisection method to find solutions accurate to within 10^{-5} (absolute error) for $e^x - x^2 + 3x - 2 = 0$ for $0 \leq x \leq 1$.

(3) Find an approximation to $\sqrt{3}$ correct to within 10^{-4} using the Bisection method (Hint: Consider $f(x) = x^2 - 3$.)