Mathematics 170B – HW7 – Due Tuesday May 15, 2012.

Problems #8,9,10,11 on page 290.

G_1. Recall that X_n converges to X in distribution if the corresponding distribution functions F_n, F satisfy

$$\lim_{n\to\infty} F_n(x) = F(x)$$

for all x so that F is continuous at x.

Show that if X_n converges to 0 in distribution, then X_n converges to 0 in probability.

G_2. Suppose the random variables X_n satisfy $EX_n = 0$, $EX_n^2 \leq 1$, and $Cov(X_n, X_m) \leq 0$ for $n \neq m$. Show that

$$\frac{X_1 + \cdots + X_n}{n}$$

converges to 0 in probability.

Definition. A sequence Y_n of random variables converges to Y a.s. if

$$P(\lim_{n\to\infty} Y_n = Y) = 1.$$

Note: In class, I will show that if $\sum_{n=1}^{\infty} E|Y_n - Y| < \infty$, then Y_n converges to Y a.s.

G_3. Show that in cases (a), (c) and (d) of Problem 5 on page 288, the sequence actually converges a.s. (In case (b) it does not, but you need not show that.)

G_4. Suppose each X_n takes the values ± 1 with probability $\frac{1}{2}$ each. Show that the random series

$$\sum_{n=1}^{\infty} \frac{X_n}{n^p}$$

converges a.s. (which means that the partial sums converge a.s.) if $p > 1$. (Actually, if the X_n's are also independent, then the random series converges if and only if $p > \frac{1}{2}$, but this is harder to prove.)