Mathematics 170B – HW3 – Due Tuesday, April 17, 2012.

Problems 17, 19, 22, 24 on pages 249–252, and

C_1. A fair die is tossed n times. Let X be the number of 1's obtained, and Y be the number of 6's obtained. Find the covariance and correlation coefficient of X and Y (without using the joint distribution of X and Y).

C_2. Let $X_1, X_2, ..., X_n$ be i.i.d. random variables with mean μ and variance σ^2, and let

$$\overline{X} = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

be the sample mean. Show that $X_i - \overline{X}$ and $\overline{X}$ are uncorrelated.

C_3. Compute $E(X \mid X + Y)$, where X and Y are independent random variables with the $B(n, p)$ distribution.

C_4. Suppose the continuous random variables X, Y have joint density

$$f(x, y) = \frac{e^{-x/y} e^{-y}}{y}, \quad 0 < x, y < \infty.$$

Find $E(X \mid Y)$.

C_5. Suppose $X_1, X_2, ...$ are independent Bernoulli random variables with parameter p, and let $N = \min\{i : X_i = 1\}$ be the time of the first 1. Compute EN and $\text{var}(N)$ by conditioning on the value of X_1 (i.e., without using the distribution of N).

C_6. Suppose that U is uniform on $[0, 1]$, and that the conditional distribution of X given $U = p$ is $B(n, p)$. What is the distribution of X?